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Bridge Engineering
Roof Trusses

A Manual of

PRACTICAL INSTRUCTION IN THE CALCULATION AND DESIGN OF STEEL TRUSS AND GIRDER BRIDGES FOR RAILROADS AND HIGHWAYS, INCLUDING ALSO THE ANALYSIS AND DESIGN OF ROOF TRUSSES AND OTHER DETAILS OF MILL BUILDING CONSTRUCTION

By FRANK O. DUFOUR, C. E.
Assistant Professor of Structural Engineering, University of Illinois

PART I—BRIDGE ENGINEERING
PART II—ROOF TRUSSES

ILLUSTRATED

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1909

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Foreword

IN recent years, such marvelous advances have been made in the engineering and scientific fields, and so rapid has been the evolution of mechanical and constructive processes and methods, that a distinct need has been created for a series of *practical working guides*, of convenient size and low cost, embodying the accumulated results of experience and the most approved modern practice along a great variety of lines. To fill this acknowledged need, is the special purpose of the series of handbooks to which this volume belongs.

¶ In the preparation of this series, it has been the aim of the publishers to lay special stress on the *practical* side of each subject, as distinguished from mere theoretical or academic discussion. Each volume is written by a well-known expert of acknowledged authority in his special line, and is based on a most careful study of practical needs and up-to-date methods as developed under the conditions of actual practice in the field, the shop, the mill, the power house, the drafting room, the engine room, etc.

¶ These volumes are especially adapted for purposes of self-instruction and home study. The utmost care has been used to bring the treatment of each subject within the range of the com-

mon understanding, so that the work will appeal not only to the technically trained expert, but also to the beginner and the self-taught practical man who wishes to keep abreast of modern progress. The language is simple and clear; heavy technical terms and the formulæ of the higher mathematics have been avoided, yet without sacrificing any of the requirements of practical instruction; the arrangement of matter is such as to carry the reader along by easy steps to complete mastery of each subject; frequent examples for practice are given, to enable the reader to test his knowledge and make it a permanent possession; and the illustrations are selected with the greatest care to supplement and make clear the references in the text.

¶ The method adopted in the preparation of these volumes is that which the American School of Correspondence has developed and employed so successfully for many years. It is not an experiment, but has stood the severest of all tests—that of practical use—which has demonstrated it to be the best method yet devised for the education of the busy working man.

¶ For purposes of ready reference and timely information when needed, it is believed that this series of handbooks will be found to meet every requirement.

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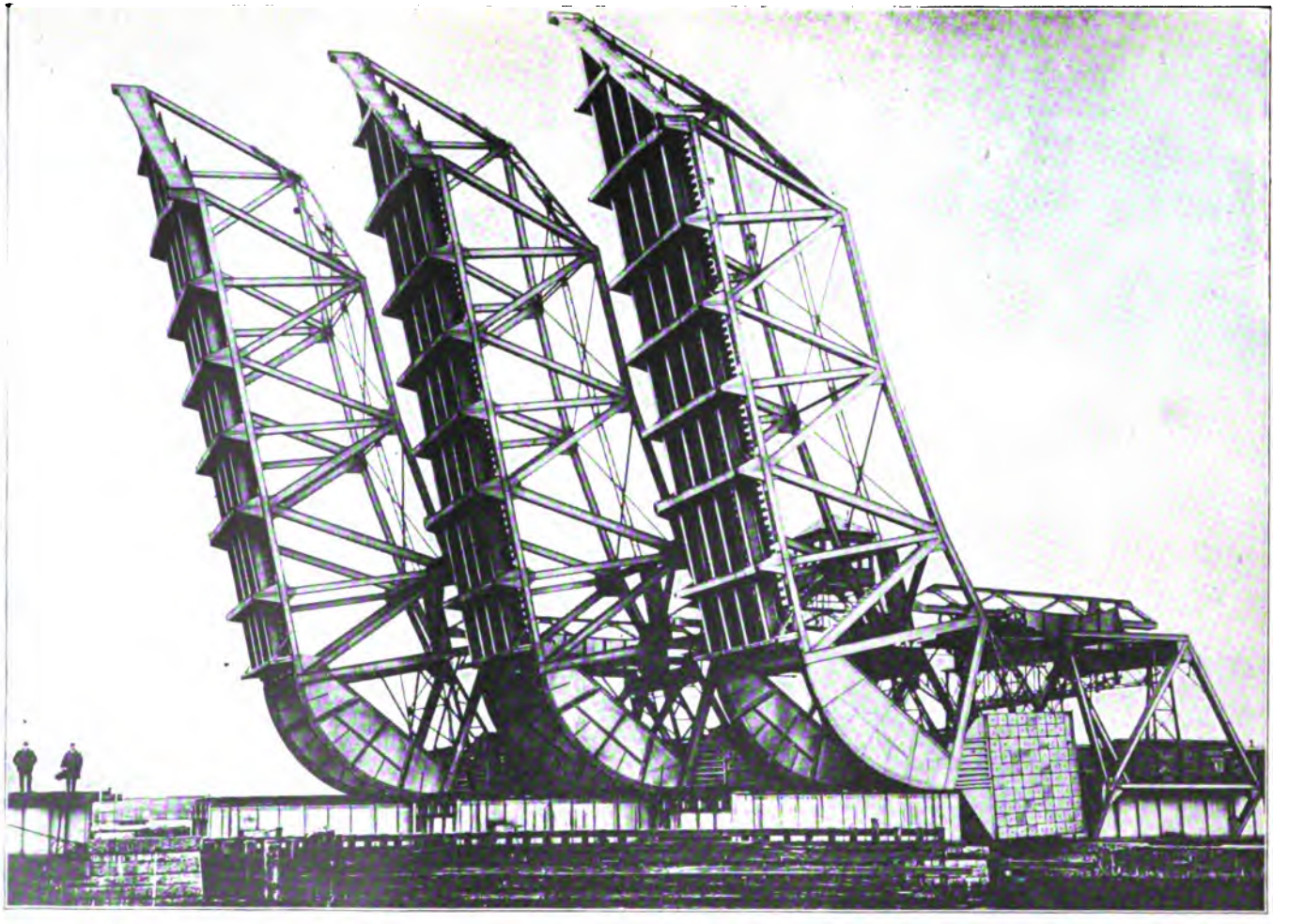

SIX-TRACK ROLLING-LIFT BRIDGE OVER FORT POINT CHANNEL, BOSTON, MASSACHUSETTS

On line of New York, New Haven & Hartford Railroad. Designed by the Scherzer Rolling-Lift Bridge Company, Chicago, Ill.

BRIDGE ENGINEERING

PART I

BRIDGE ANALYSIS

1. **Introduction.** The following treatment of the subject of Bridge Analysis, while not exhaustive, is regarded as sufficiently elaborate to develop and instill the principal theoretical considerations, to illustrate the most convenient and practical methods of analyzing the common forms of trusses and girders, and also to lay a sufficient foundation for the analysis of such other trusses as are not specifically mentioned or treated herein.

The necessary steps and operations required for a proper analysis of the several types of bridges are fully demonstrated by sketches and computations, the numerical values being mechanically obtained by the use of a *slide rule,* which is a handy instrument for quickly performing the operations of multiplication and division, and for squaring and extracting the roots of numbers. The values given may differ from the exact value by one unit in the second decimal place (seldom more) and are sufficiently accurate for the purpose of design. All bridge computers should be proficient in the use of the slide rule.

The problems given in the back of this Instruction Paper, exemplifying the practical application of the subject-matter treated in the various articles, should be solved by the student as each article is mastered.

HISTORY

2. **Early Bridges.** Early bridges were not bridges according to the present conception of the term. They were simple pile trestle bents placed at frequent intervals and connected by wooden beams on which the floor was placed. The *Pons sublicius,* built over the Tiber, at Rome, about 650 years before Christ was born, was of this trestle type. Also the famous bridge built by Cæsar across the Rhine in 55 B. C. was of the same kind of construction. As civilization progressed, the arch type was developed; and in 1390 the great

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bridge at Trezzo over the River Adda was built of one span of 251 feet, which has never been eclipsed or equaled.

3. **Truss Bridge Development.** The first truss bridge is supposed to have been originated by Palladio, an Italian, who used the king-post truss (Fig. 1) about 1570. Its importance was not recognized, and it became entirely forgotten until it was rediscovered in 1798 by Theodore Burr, an American, who used it in his construction. About the same time, Burr invented the truss that bears his name, which was in reality a series of king-post trusses (see Fig. 8). This was found to be unstable under moving loads, and was therefore stiffened by the use of an arch (Fig. 2), or was built somewhat as an arch, there being considerable rise at the center of the span (Fig. 3). By 1830 the principle of the double cross-bracing in the panel was understood; and in quick succession came the patents of Long, Howe, Pratt, and Whipple on forms of trusses which bear their respective names.

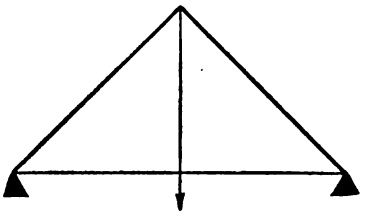


Fig. 1. King-Post Truss.

It remained for Squire Whipple in 1847 to place the science of bridge building on a rational and exact mathematical basis such

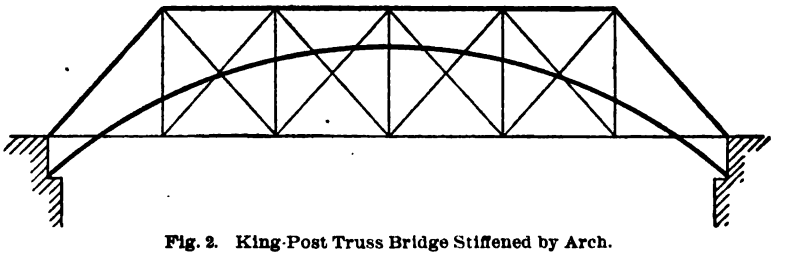


Fig. 2. King-Post Truss Bridge Stiffened by Arch.

as is now used. Previous to this time, and indeed several years afterwards—for Whipple's work did not become generally known until a much later date—bridges were built, not from previously computed strains, but by "judgment." All parts of a bridge were made of the same size, and if one started to fail it was replaced by a larger one; or small models were made and loaded proportionally, broken members being replaced by larger ones. There is no doubt

that many of the bridges built at this period were very weak as well as very strong. The failures are not remembered; but the sound judgment of many of our earlier bridge engineers is recorded in the wooden structures they left behind them, some of which have stood the demands of traffic for over a century. After 1850, bridges were built from computed stresses; wood was discarded; and the development became rapid, until about 1870, when the introduction of sub-diagonal systems brought the truss system to practically what it is to-day.

DEFINITIONS AND DESCRIPTIONS

4. **Trusses.** A truss is a series of members taking stress in the direction of their length, placed together so as to form a triangle

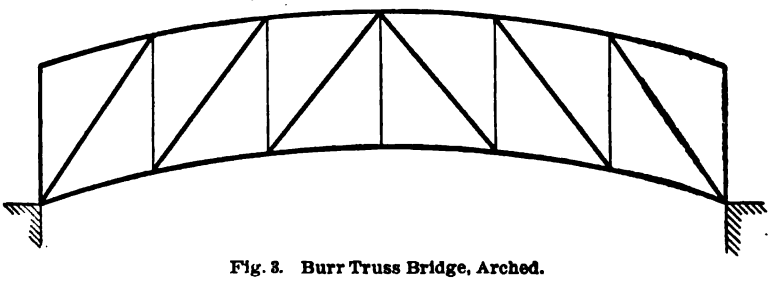


Fig. 3. Burr Truss Bridge, Arched.

or system of triangles, which, when placed upon supports a certain distance apart, will, in addition to their own weight, sustain certain loads applied at the points where the members intersect. These points are called *panel points.*

5. **Bridge Trusses.** A bridge truss is one in which the members that carry the superimposed loads are in the same plane. Usually this plane is vertical.

6. **Truss Bridges.** A truss bridge is a structure consisting of two or more—usually two—bridge trusses connected by a system of beams called the *floor system,* which transfer to panel points the load for which the trusses are designed.

7. **Girders.** These are beams consisting of a wide, thin plate, called a *web plate,* with shapes, usually angles and narrow, thin plates called *flanges,* at the top and bottom edges. All are firmly riveted together. (See Part IV, "Steel Construction.")

8. **Girder Bridges.** These consist of usually two, sometimes

three, girders connected as in the case of truss bridges by a system of beams.

9. **Deck Bridges.** In cases where the floor system connects the trusses at their tops, the bridge is called a *deck bridge,* since the traffic moves on a *deck,* as it were (see Fig. 4).

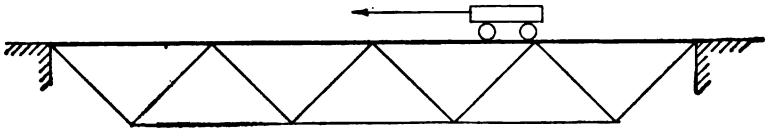


Fig. 4. Deck Bridge.

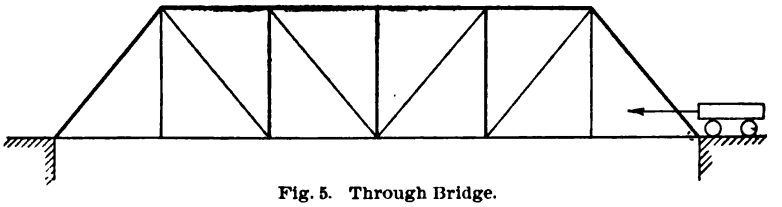


Fig. 5. Through Bridge.

10. **Through Bridges.** In cases where the floor system connects the bottoms of the trusses, the bridge is called a *through bridge,* as the traffic moves *through* the space between the trusses (see Fig. 5).

11. **Members of a Truss.** Each truss consists of a *top* and *bottom chord, end-posts,* and *web members.* The web members are further divided into *hip verticals, intermediate posts,* and *diagonals.* Fig. 6 shows these various classes, *A-A* being top chord, *B-B*

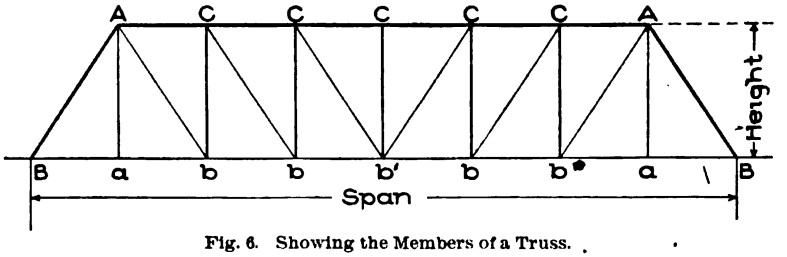


Fig. 6. Showing the Members of a Truss.

bottom chord, *A-B* end-posts, vertical members *C-b** intermediate posts, *A-a* hip verticals, and *A-b* and *C-b'* diagonals.

12. **Pony-Truss Bridges.** When the height of the trusses of a through bridge is less than the height of the loads that go over them, they are called *pony trusses,* and the bridge a *pony-truss bridge.*

13. **Lateral Bracing.** In all deck bridges, and in all through bridges except pony-truss bridges, the chords which are not connected by the floor system are connected by a horizontal truss system called the *lateral bracing*. In all bridges the chords which are connected by the floor system are connected by a horizontal truss system, also called the *lateral bracing*. One of these systems is called the

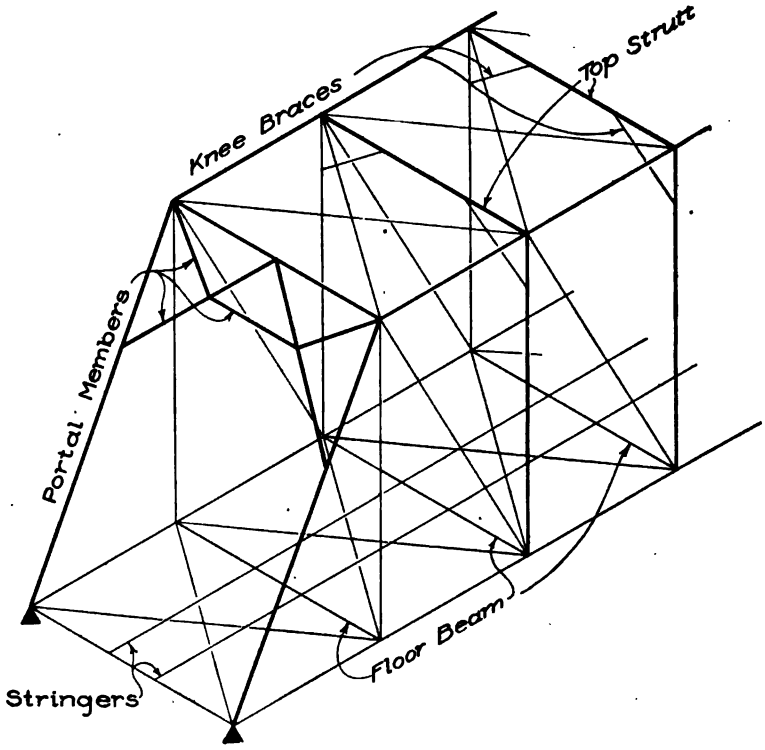


Fig. 7. Through Bridge, Showing Top and Bottom Systems of Lateral Bracing, also Portal Bracing and Floor System.

top lateral system, as it connects the top chords; and the other is called the *bottom lateral system*, as it connects the bottom chords (see Fig. 7).

14. **Portals.** In through bridges, the end-posts of the pair of trusses are connected by a system of braces in order to preserve the rectangular cross-section of the bridge. This is called the *portal bracing* (see Fig. 7).

15. **Sway Bracing and Knee-Braces.** These serve the same purpose as the portal braces, and are either small struts or systems

of cross-bracing placed at the intermediate posts. The former are called *knee-braces*, and the latter *sway bracing*.

16. **Floor Systems.** In both highway and railway bridges, there are beams running from the intermediate posts or hip verticals across to the like members opposite. These are called *floor-beams*. In highway bridges, there are smaller beams running parallel to the trusses and resting at their ends upon the floor-beams. These are called *floor-joists*, and the plank or other floor rests directly upon them. In railway bridges, two beams or girders per track run parallel to the trusses and are connected at their ends to the floor-beams. These are called *track stringers* (or simply *stringers*). The ties rest directly upon them. The various members of the floor system of a railway bridge are shown in Fig. 7. The diagonals connecting the top chords, and those connecting the bottom chords, are the *top* and *bottom laterals* respectively.

CLASSES OF TRUSSES

17. **Names.** Trusses may be classified according to their names, the character of their chords, and the system of webbing. Table I gives the classification of the more important of these according to name.

TABLE I

Chronological List of Trusses

Name	Year of Origin	Inventor	Country	Illustrated in
King-Post	1570	Palladio	Italy	Fig. 1
King-Post	1798	Theodore Burr	America	Fig. 1
Burr	1798	Theodore Burr	America	Fig. 8
Warren	1838		England	Fig. 9
Howe	1840	William Howe	America	Fig. 10
Pratt	1844	Thos & Caleb Pratt	America	Fig. 11
Whipple	1847	Squire Whipple	America	Fig. 12
Bowstring	1847	Squire Whipple	America	Fig. 13
Baltimore	1877	Penn. R. R.	America	Fig. 14

Of the types of trusses listed in Table I, the Warren, Howe, Pratt, Bowstring, and Baltimore are now built; and of these constructions probably 90 per cent are Pratt trusses. The Baltimore truss is used for long spans only.

18. **Chord Characteristics.** In most types of bridges the

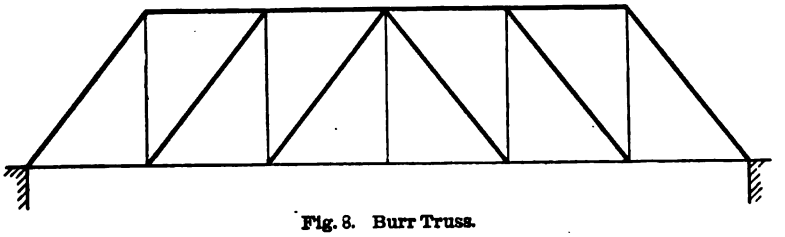


Fig. 8. Burr Truss.

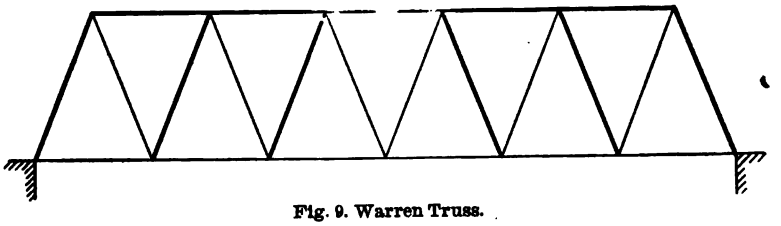


Fig. 9. Warren Truss.

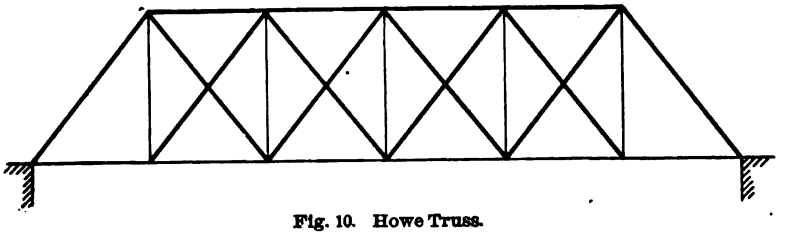


Fig. 10. Howe Truss.

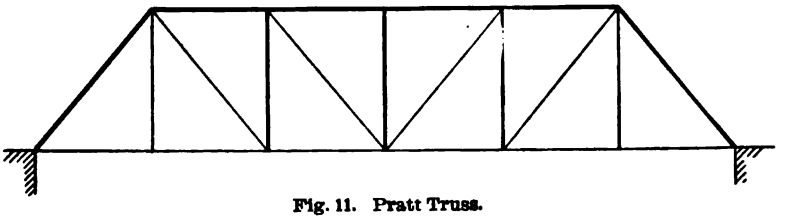


Fig. 11. Pratt Truss.

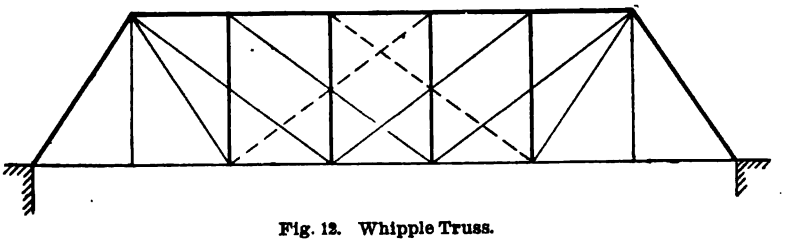


Fig. 12. Whipple Truss.

chords are parallel. When such is the case, the stresses increase from the end toward the center, and there is a considerable difference between any two adjacent panels of the same chord. This necessitates different areas for each section. When the chords are not parallel, as in the bowstring truss, the stresses in the chords are so nearly equal that the same area is used throughout or nearly throughout the entire chord. Also, the stresses in the diagonals are nearly equal. These conditions would seem to indicate that this was a very economical form of truss. Theoretically it is; but practical considerations—such as the beveled joints and the posts which must be constructed to withstand reversals of stress—customarily limit this type to the longer spans.

19. **Web Characteristics.** The web systems of the Burr, Warren, Howe, Pratt, and Bowstring trusses are called *single* systems; that of the Whipple truss is a *multiple* system; while those of the Baltimore trusses are examples of webbing with *sub-systems.* As the maximum economical panel length has been found to be about 25 feet, which makes the economical height of the truss about 30 feet, and as the length of the span should not be more than ten times the depth, the span for trusses with simple systems of webbing is limited to about 300 feet. In order to increase the limiting span, multiple systems like that of the Whipple or similar ones were introduced. Calculations of stresses in members of the Whipple truss are somewhat unreliable on account of the fact that we are unable to tell just how the effects of the loads are distributed. For this reason, that type has gone out of use, and the sub-systems are used instead. These allow spans of twice the above limit; and, indeed, trusses with this type of webbing have been built up to and over 600 feet. This style of webbing can be applied to the bowstring truss, almost all long-span bridges being of this type with sub-systems of webbing.

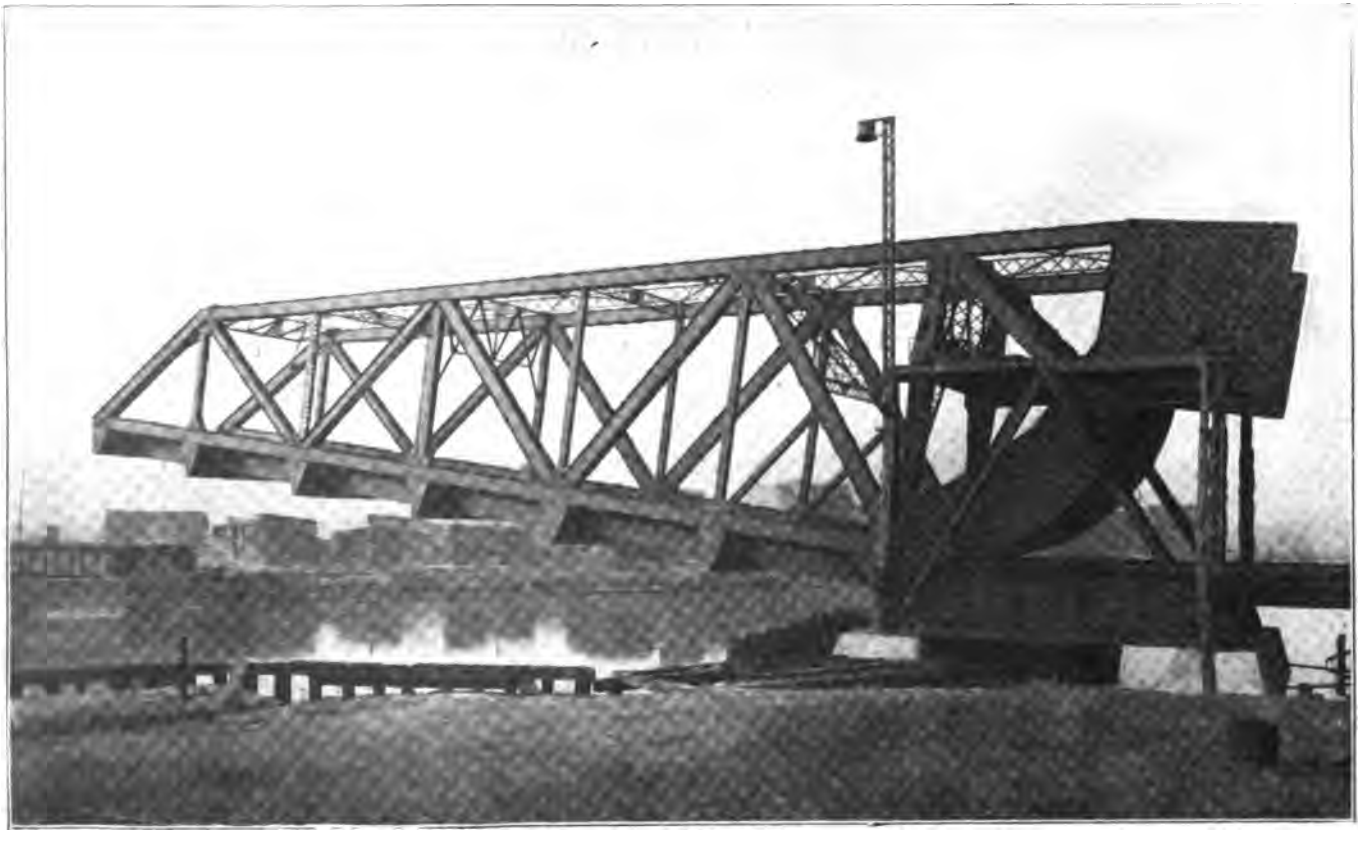
WEIGHTS OF BRIDGES

20. **Formulæ.** In order to obtain the stresses due to the weight of the structure, the latter quantity must be known. As this weight can be determined only after the structure has been designed, it is evident that an assumption as to the weight must be made. The best method is to use the actual weight of a similar structure of like span which has been built. As the necessary data

BRIDGE OVER ALLEGHENY RIVER, PITTSBURG, PENNSYLVANIA

On line of Pittsburg, Fort Wayne & Chicago Railroad. Built in 1901. Consists of the following four-track double-deck through pin-connected spans: One of 157 ft., one of 333 ft., and three of 160 ft. each, with a double-track viaduct at one end, 816 ft. long.

Courtesy of American Bridge Company.



SCHERZER TYPE BASCULE BRIDGE OVER THE CUYAHOGA RIVER, CLEVELAND, OHIO

On line of Newburgh & South Shore Railroad. Built in 1904-05. Double-track; one leaf, 160 ft.

Courtesy of American Bridge Company.

for this is not always available, it is customary to use formulæ to derive an approximate weight of sufficient accuracy for purposes of computation. Table II gives some of the most reliable formulæ.

TABLE II

Formulæ for Weights of Bridges

Class of Bridge	Weight of Steel per Linear Foot of Span	Author*
Heavy Interurban Riveted	$w = 600 + 1.8l + 27b + \frac{1}{12}bl\left(1 + \frac{1}{1{,}000}l\right)$	E. S. Shaw
First-Class Highway Riveted	$w = 300 + l + 22b + \frac{1}{15}bl\left(1 + \frac{1}{1{,}000}l\right)$	E. S. Shaw
First-Class Highway Pin	$w = 34 + 22b + 0.16bl + 0.7l$	J. A. L. Waddell
Light Country Highway	$w = 250 + 2.5l$	Author
Railroad Truss E 50	$w = (650 + 7l)$	F. E. Turneaure
Railroad Truss E 40	$w = \frac{7}{8}(650 + 7l)$	F. E. Turneaure
Railroad Truss E 30	$w = \frac{3}{4}(650 + 7l)$	F E. Turneaure
Railroad Deck Girder E 50	$w = 124.0 + 12.0l$	Author
Railroad Deck Girder E 40	$w = 123.5 + 10.0l$	Author
Railroad Deck Girder E 30	$w = 111.0 + 8.8l$	Author

In the above formulæ, w = Weight of steel per linear foot of span; l = Length of span in feet; b = Breadth of roadway, including sidewalks.

In using the formulæ of Table II, remember that a span has two trusses. The weights for highway bridges do not include the weight of the wooden floor, which may be assumed as 10 pounds per square foot of floor surface All highway bridges have steel joists. The weights of railroad spans do not include the weight of the ties and rails, which may be assumed at 400 pounds per track per linear foot of span. If solid steel floors are to be used, 700 pounds per linear foot of span are to be added to the weights computed from the table.

All the weights given for railroad spans are for single track. Double-track truss-spans are about 95 per cent heavier; and double-track girder-spans are 100 per cent heavier. Through girder spans are about 25 per cent heavier than deck girder spans; and through spans are about 15 to 40 per cent heavier than deck spans.

The spans on which Table II is based are of medium steel. Bridges built of soft steel or wrought iron will weigh 10 to 15 per cent more.

* The author is indebted to the distinguished engineers whose names appear in Table II, for permission in this connection to make use of the formulæ given opposite their names.

In order to give an idea of the relative weights of steel in different classes of bridges, let it be required to compute the dead weight of a 100-foot span of each class. For heaviest highway bridges to carry heavy interurban cars:

$$w = 600 + 180 + 27 \times 16 + \frac{16 \times 100}{12}\left(1 + \frac{100}{1\,000}\right) = 1\,358 \text{ lbs. per linear ft.}$$

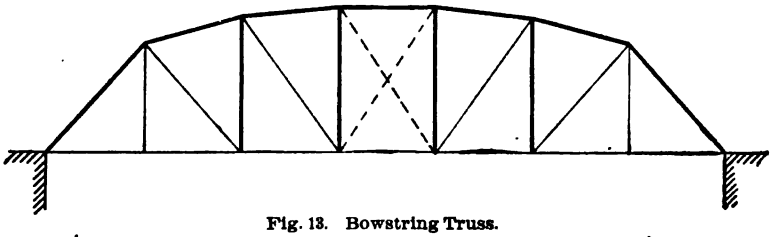


Fig. 13. Bowstring Truss.

For heavy riveted highway bridges to carry heavy farm engines:

$$w = 300 + 100 + 22 \times 16 + \frac{16 \times 100}{15}\left(1 + \frac{100}{1\,000}\right) = 870 \text{ lbs. per linear foot.}$$

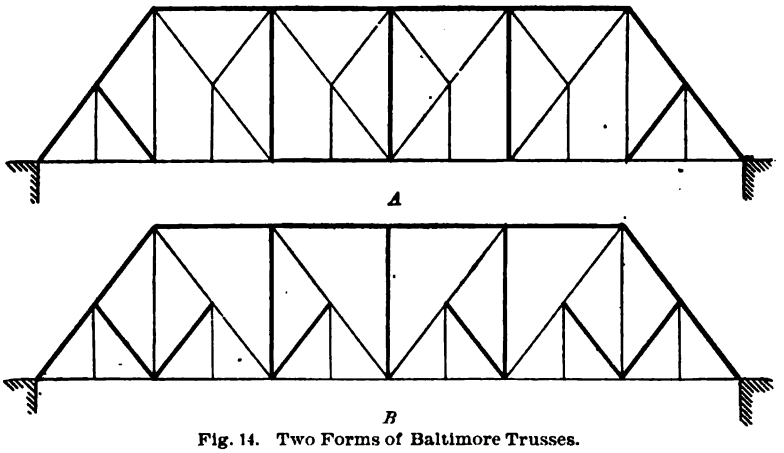


Fig. 14. Two Forms of Baltimore Trusses.

For heavy pin-connected highway bridges to carry heavy farm or traction engines:

$$w = 34 + 22 \times 16 + 0.16 \times 16 \times 100 + 0.7 \times 100 = 710 \text{ lbs. per linear ft.}$$

For light country highway bridges to carry 100 pounds per square foot of floor surface:

$$w = 250 + 2.5 \times 100 = 500 \text{ lbs. per linear foot.}$$

If the total weight is required, the weight of the wooden floor must be added. Take, for example, the last bridge:

Weight of steel = 500 × 100 = 50 000 pounds.
" " floor = 100 × 16 × 10 = 16 000 pounds.
Total dead load = 66 000 pounds.

The weight per linear foot for a railroad truss bridge of 100-foot span is:

$$w = 650 + 7 \times 100 = 1\,350 \text{ lbs. per linear foot.}$$

This is about the same as that for a heavy interurban bridge The reason for this is that in addition to the heavy rolling stock of the electric road, the heavy highway traffic must be provided for. A deck girder of 100-foot span weighs:

$$w = 124 + 12 \times 100 = 1\,324 \text{ lbs. per linear foot.}$$

21. **Actual Weights of Railroad Spans.** In case actual weights can be obtained, a more exact analysis can be made. The weights of bridges indicated in the accompanying tables and diagrams, are based on actual constructions recently erected. These bridges represent the very best modern practice of engineers and manufacturers

The weights of through truss-spans made of medium steel and designed for E 50 loading, are given in Fig. 15. The weights include the weight of the ordinary open steel floor, and they also include the weight of the ties and rails, which is taken at 400 pounds per linear foot per track.

The weight of steel in medium steel deck plate-girder spans designed for E 50 loading, is given in Table III.

TABLE III

Weights of Deck Plate-Girders, Medium Steel

Loading E 50

Span (in feet)	Weight (in pounds)	Span (in feet)	Weight (in pounds)
15	5 300	70	59 500
20	7 800	75	67 300
25	11 800	80	76 300
30	14 500	85	94 200
35	18 800	90	105 500
40	23 300	95	114 200
45	27 400	100	123 600
50	32 400	105	146 000
55	38 800	110	161 700
60	45 500	115	174 900
65	51 500	120	187 000

The spans are the distance center to center of bearings; and the weights do not include the weight of the ties and rails, which is to be taken at 400 pounds per linear foot per track. Intermediate spans may be interpolated.

22. **Actual Weights of Highway Spans.** The actual weights of highway spans for heavy interurban trolley-cars and traffic, should preferably be computed from the formulæ of Shaw or Waddell (Table II). The weights of country bridges, including floor, may be taken from the diagram of Fig. 16.

LOADS

23. **Classes of Loads.** Those weights just given constitute what is called the *dead load* of the bridge. The traffic which passes

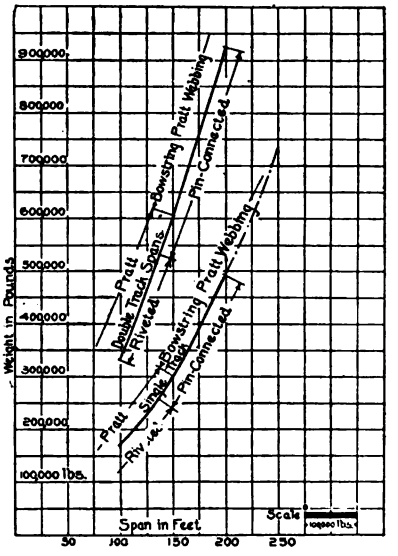


Fig. 15. Weights of Through Truss-Spans. Medium Steel, E 50 Loading.

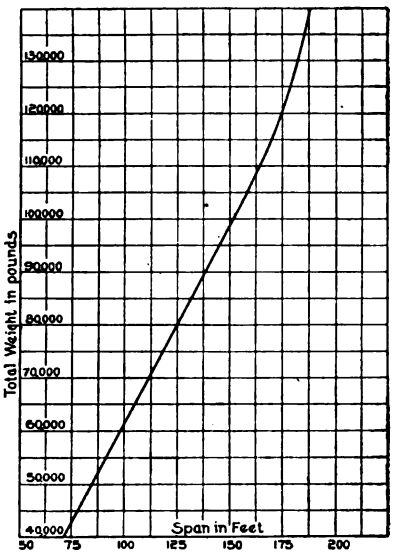


Fig. 16. Weights of Country Bridges, In cluding Floor.

over the bridge is called the *live* or *moving load.* In addition to the two classes mentioned, is the effect of the wind, which is designated as the *wind load.* These loads vary with the class of bridge, be it highway or railway, and with the purpose for which it is intended.

24. **Live Loads for Highway Bridges.** Highway bridges are usually divided into several classes according to the traffic, which may be that of heavy interurban cars, light trolley-cars, farm engines, road rollers, teams, human beings, or some combination of these loadings. The standard specifications of J. A. L. Waddell or of Theodore Cooper are obtainable for a very small sum. Their pur-

chase is advised, and the reader is referred to them for further information.

The trusses of country highway bridges are usually designed for a live load of 100 pounds per square foot of roadway. This may be considered good practice; and it is the law in some States. The floor system of these same bridges should be of sufficient strength to sustain 100 pounds per square foot of roadway, or a 12-ton farm engine having 4 tons on the two rear wheels, which are 12 inches wide and 6 feet apart, and 2 tons on each of the front wheels, which are 6 inches wide and 5 feet apart. The axles of this engine are spaced 8 feet center to center.

25. **Live Loads for Railway Bridges.** The loads for any particular railroad bridge are not always the same, on account of the great variation in the weights and wheel spacings of engines and cars. It is customary to design the bridge for the heaviest in use at the time of construction, or for the heaviest that could reasonably be expected to be built thereafter.

As the computations with engines were formerly somewhat laborious on account of the different weights and spacing of wheels, it has been proposed by some engineers to use a uniform load, called the *equivalent load*, which would give stresses the same, or very nearly the same, as those obtained by the use of engine loads. However, as these loads are different for each weight of engine, and also different for the chord members, the web members, and the floor-beam reaction of each different length of span, and as the labor of the computations, using engine-wheel loads, has been greatly reduced by means of diagrams, it does not seem as if this method would ever come into very general favor except for long-span bridges, where the live load is much smaller than the dead load.

The equivalent loads for Cooper's E 40 (see Fig. 85) are given in Table IV.

Most railways specify that their bridges shall be computed by using two engines and tenders followed by a train. The spacing of the wheels, and the load which comes on each wheel of the engines and tenders, are fixed by the railway company. The train is represented by a uniform load. Formerly there was a great diversity of practice among the different roads in regard to the engine and train loads specified; but practice has of late years become quite uniform,

TABLE IV

Equivalent Uniform Loads

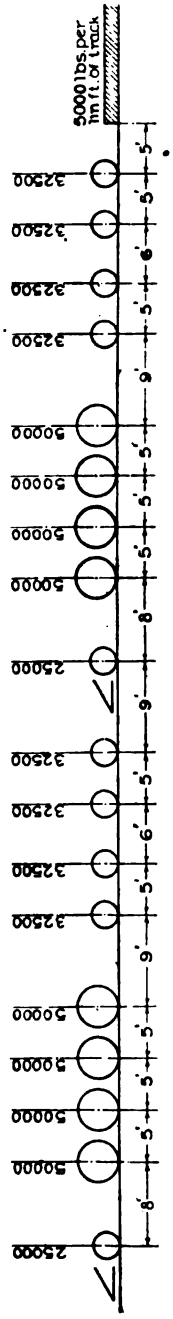
Loading E 40

Span (in feet)	Equivalent Uniform Load			Span (in feet)	Equivalent Uniform Load		
	Chords	Web	Floor-Beam		Chords	Web	Floor-Beam
10	9 000	12 000	8 200	46	6 330	7 240	5 240
11	9 310	11 640	7 960	48	6 220	7 140	5 200
12	9 340	11 330	7 830	50	6 110	7 060	5 140
13	9 340	11 080	7 600	52	6 040	6 940	5 130
14	9 210	10 860	7 460	54	5 960	6 820	5 120
15	9 030	10 670	7 330	56	5 880	6 720	5 110
16	8 850	10 500	7 120	58	5 800	6 620	5 090
17	8 650	10 350	6 940	60	5 730	6 530	5 080
18	8 430	10 240	6 780	62	5 690	6 490	5 080
19	8 220	10 100	6 630	64	5 700	6 450	5 070
20	8 000	10 000	6 500	66	5 620	6 450	5 070
21	8 040	9 780	6 390	68	5 560	6 380	5 060
22	8 040	9 580	6 290	70	5 510	6 340	5 060
23	8 010	9 400	6 200	72	5 490	6 320	5 030
24	7 960	9 230	6 120	74	5 460	6 300	5 010
25	7 890	9 080	6 040	76	5 440	6 290	4 990
26	7 780	8 930	5 970	78	5 420	6 270	4 970
27	7 660	8 790	5 900	80	5 400	6 250	4 950
28	7 540	8 660	5 830	82	5 370	6 230	4 930
29	7 420	8 540	5 770	84	5 340	6 200	4 910
30	7 300	8 430	5 720	86	5 310	6 180	4 890
31	7 220	8 320	5 680	88	5 270	6 150	4 870
32	7 140	8 190	5 650	90	5 250	6 130	4 860
33	7 050	8 080	5 620	92	5 250	6 110	4 830
34	6 960	7 980	5 600	94	5 210	6 090	4 810
35	6 870	7 890	5 570	96	5 170	6 060	4 780
36	6 820	7 820	5 530	98	5 150	6 040	4 760
37	6 760	7 750	5 500	100	5 140	6 020	4 740
38	6 700	7 690	5 460	125	5 100	5 770	4 720
39	6 630	7 630	5 430	150	5 010	5 570	4 700
40	6 560	7 570	5 400	175	4 890	5 350	4 686
42	6 530	7 450	5 340	200	4 740	5 240	4 660
44	6 470	7 340	5 300	250	4 510	5 030	4 640

with an apparent tendency to standardize in accordance with the classes of loading specified by Cooper. Cooper's Class E 50, which represents the heaviest engines now in common use, was invented by Theodore Cooper, a consulting engineer of New York City. It is given in Fig. 17.

Lighter loadings for light traffic on the same general plan are advocated by Mr. Cooper, and are given at length in his "General Specifications for Iron and Steel Railway Bridges and Viaducts" (1906 edition).

26. Wind Loads. Some designers require that the stresses due



Cooper's E 50 Loading.

Fig. 17. Diagram of Cooper's E 50 Loading - Representing Heaviest Engines in Common Use.

to wind shall be computed by using 30 pounds per square foot of actual truss surface. This requires that you know the size of the members of the bridge before it is designed—which is evidently an impossibility; or that an assumption as to their size be made—which allows a chance for a mistake in judgment, especially in an inexperienced person. A more logical method, and one used to a great extent, is to assume a force of so many pounds per linear foot to act on the top and bottom chords and on the traffic as it moves across the bridge.

In highway through bridges, it is the usual practice to take the wind load as 150 pounds per linear foot of top and bottom chords, and 150 pounds per linear foot of the amount of live load which is on the bridge.

For railroad bridges, it is customary to use considerably higher values than those used in highway practice—not that the wind blows harder on railroad than on highway bridges, but so that the bracing designed by the use of these values may be sufficiently strong to stiffen the bridge not only against the wind, but also against the vibrations caused by the rapidly moving traffic. Good practice for through bridges is to use 150 pounds per linear foot of the top chord, 150 pounds per linear foot of the bottom chord, and 450 pounds per linear foot of live load on the bridge. This latter force is supposed to act at a line 8.5 feet above the base of the rail.

For deck bridges, for both highway and railway use, the unit-loads on the moving or live load remain the same, but the unit-loads on the top and bottom chords are reversed.

In computations involving the live load, it is always assumed that the live load moves over the bridge from right to left.

THEORY

27. **Principles of Analysis.** The stresses in bridge trusses may be determined by both algebraic and graphic methods. In some

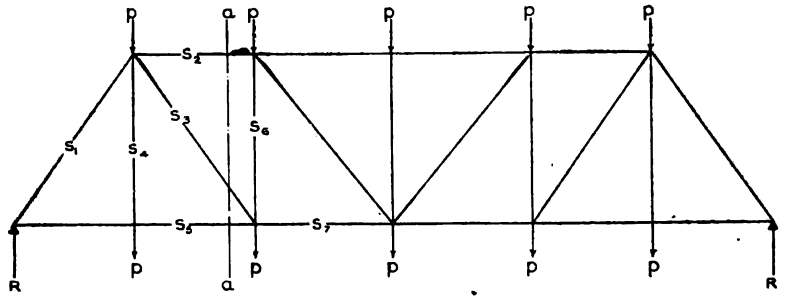


Fig. 18. Truss under Loads, Illustrating Principles of Analysis.

cases, one is more expeditious than the other. Algebraic methods alone will be considered in this text.

The analysis of stresses is based upon the fact that the interior stresses in a member or group of members hold in equilibrium the exterior forces. That this is a fact, can easily be understood. Consider a man pulling on a rope which is fastened at one end to an immovable object. There will be a stress in the rope equal to, and opposite in direction to, the pull exerted by the man. In order to prove this, cut the rope and apply a force equal and opposite to the pull exerted by the man, where the cut is made; and the rope and man will be in equilibrium. Also, suppose that a truss under loads, as indicated by the arrows, Fig. 18, were cut along the section *a-a*, and that forces F_2, F_3, F_5 equal to the stresses S_2, S_3, and S_5 were placed at the ends of the members as indicated in Fig. 19, then that portion of the truss to the left of the section would be in equilibrium. The interior stresses, represented by F_2, F_3, and F_5, would hold in equilibrium the exterior forces p and R.

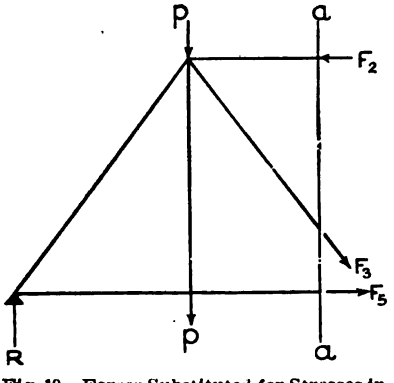


Fig. 19. Forces Substituted for Stresses in Truss of Fig. 18.

From inspection of Fig. 19, it will appear evident that, as the position of the truss to the left of the section is in equilibrium, the following statements are true:

1. The algebraic sum of the moments of the exterior forces and the stresses in the members cut by the section, is equal to zero. This is true of the moments taken about any or all points; for, if it were not, the portion of the truss would begin to rotate about some point, and would continue until equilibrium was established.

2. In a vertical plane, the algebraic sum of the components of the exterior forces and the stresses in the members cut by the section is equal to zero; for, if such were not the case, the portion of the truss shown would move up or down with a constant acceleration.

3. The algebraic sum of the components of the exterior forces and the stresses in the members cut by the section in a horizontal plane, is equal to zero; for, if such were not the case, the portion of the truss would move either to the right or to the left, with a constant acceleration.

4. From 2 and 3, above, it is evident that the algebraic sum of the components of the exterior forces and the stresses in the members cut by the section is equal to zero in any and all planes.

The section is not necessarily required to be a vertical line as in

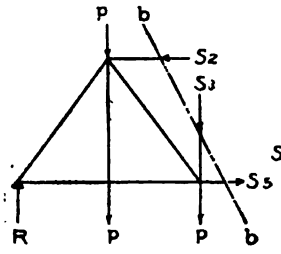


Fig. 20.
Oblique Section Cutting Truss.

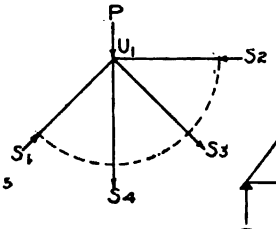


Fig. 20a.
Circular Section Cutting Truss.

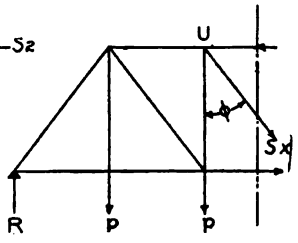


Fig. 21.
Illustrating Resolution of Forces.

Fig. 19. It may be oblique, as in Fig. 20; or it may be a circular section, as shown in Fig. 20*a*. When the latter is the case, it is said that the sum of the components of the forces around the point U_1 is in equilibrium in any plane that may be taken.

It is also evident that the forces in the members cut by the section, and the exterior forces to the right, are in equilibrium. This condition is very seldom utilized in the determination of stresses, as that portion of the truss to the left of the section is almost always considered.

28. **Resolution of Forces.** This method is one of the simplest

and at the same time least laborious. The forces are generally resolved into their horizontal and vertical components, or parallel and perpendicular to some member. In cases where two unknown stresses occur, two equations can usually be formed, and these solved.

It should be assumed that the unknown stress acts *away from* the section which cuts it. It will then solve out, with the proper sign indicating the character of the stress—that is, whether it is tensile or compressive. Tensile stresses are indicated by placing

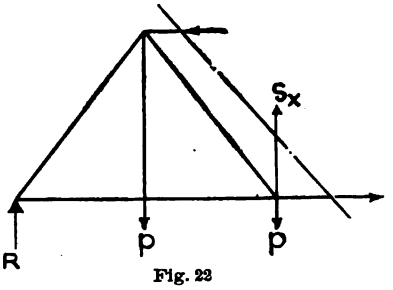


Fig. 22

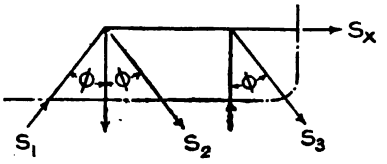


Fig. 23.

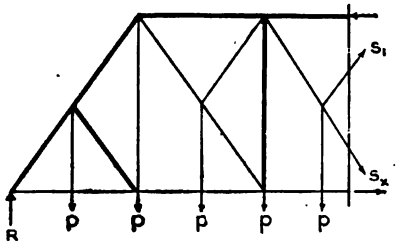


Fig. 24.

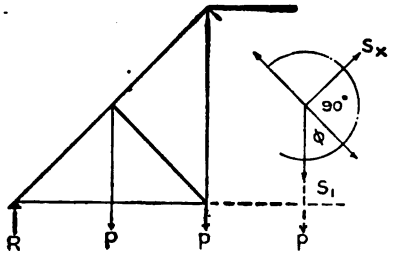


Fig. 25.

Diagrams Illustrating Application of Method of Resolution of Forces in Analysis of Trusses.

the plus (+) sign before them, while a minus (−) sign indicates compression.

A few equations showing the application of the method of the resolution of forces can be written after inspection of Figs. 21 to 25 inclusive. In all cases, S_x is the unknown stress, and is assumed to be acting away from the section. The other stresses S_1, S_2, etc., are known, and their direction given them accordingly, it being toward the section if the member is in compression, and away from the section if the member is in tension. Forces or components acting upward or to the right are considered plus; those acting down-

ward or to the left are considered minus. For a fuller explanation, see the instruction paper on Statics, Articles 17 to 23 inclusive.

In Fig 21, the sum of the vertical components is taken, and the equation is:

$$+R - p - p - S_x \cos \phi = 0;$$

whence,

$$S_x = -(+R - p - p) \sec \phi.$$

In Fig. 22, the section is oblique, and the sum of the vertical components is taken:

$$+R - p - p + S_x = 0;$$

whence.

$$S_x = -(+R - p - p).$$

In both of the above cases, it will be noted that the chord stresses do not enter into the equation, as their vertical components are zero.

In Fig. 23, the sum of the horizontal forces is used in determining the stress S_x. Note that the exterior forces R and p do not enter the equation, as they are not to the left of the section, and also their horizontal components are zero.

$$+S_1 \sin \phi + S_2 \sin \phi + S_3 \sin \phi + S_x = 0;$$

whence,

$$S_x = -(S_1 + S_2 + S_3) \sin \phi.$$

In Fig. 24, the sum of the vertical forces is again used. Here the section cuts the member with the known tensile stress S_1.

$$+R - p - p - p - p - p + S_1 \cos \phi - S_x \cos \phi = 0;$$

whence,

$$S_x = +(R - 5p) \sec \phi + S_1.$$

In Fig. 25, use is made of the fact that the sum of the components of the forces about a point is zero when they are resolved in any plane. Here they will be resolved perpendicular to the diagonal.

$$-S_1 \sin \phi + S_x = 0.$$

$$S_x = +S_1 \sin \phi.$$

These are some of the most common cases which occur in the determination of stresses in simple trusses. In all cases, follow this method of procedure:

1. Pass a section cutting as few members as possible, one of which must be the one whose stress is desired.
2. The stress in all the members cut, with but one exception, must be known.
3. Write your equation, always placing it equal to zero.
4. Solve for your stress.

29. **Method of Moments.** The stresses in all members of a truss can be determined by this method. By section 1 of Art. 28, the point about which the moments are considered can be taken anywhere. Fig. 26 represents the point as taken somewhere outside of the truss at a distance a above the point U_1. The equation will then be:

$$-S_1 \times a - S_2 \times b - S_3 (a + h) + Rp \pm P_1 \times 0 + P_2 \times p = 0.$$

This involves three unknown quantities, and therefore two other points should be taken, and two more equations written. By the use of the three equations, the stresses can be determined.

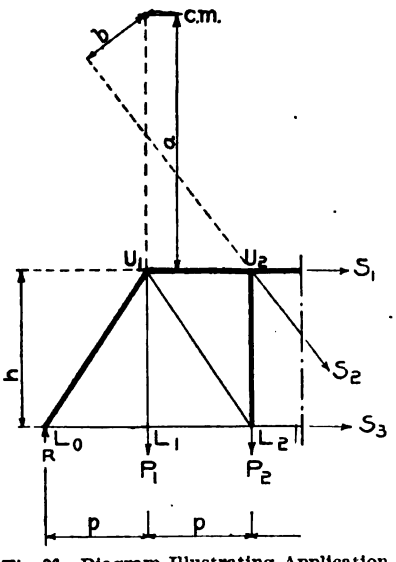


Fig. 26. Diagram Illustrating Application of Method of Moments in Analysis of Trusses.

It is customary to assume the center of moments at such a place that the moments of all the unknown stresses, with one exception, are zero. This condition requires that their lines of action pass through the center of moments. Let it be required to determine the stress S_3. If the center of moments is taken at U_2, then, as the lines of action of S_1 and S_2 pass through this point, their moments will be zero, and the following is true:

$$+R \times 2p - P_1 \times p \pm P_2 \times 0 - S_3 \times h = 0.$$

whence,

$$S_3 = \frac{1}{h} (+R \times 2p - P_1 \times p).$$

Likewise, if the top chord is curved, the center of moments can be taken in such a position that only the unknown stress will enter into the equation. If it is desired to determine the stress S_2, Fig. 27, the equation would be:

$$-S_2 \times l - R \times a + P_1 (a + p) + P_2 (a + 2p) = 0,$$

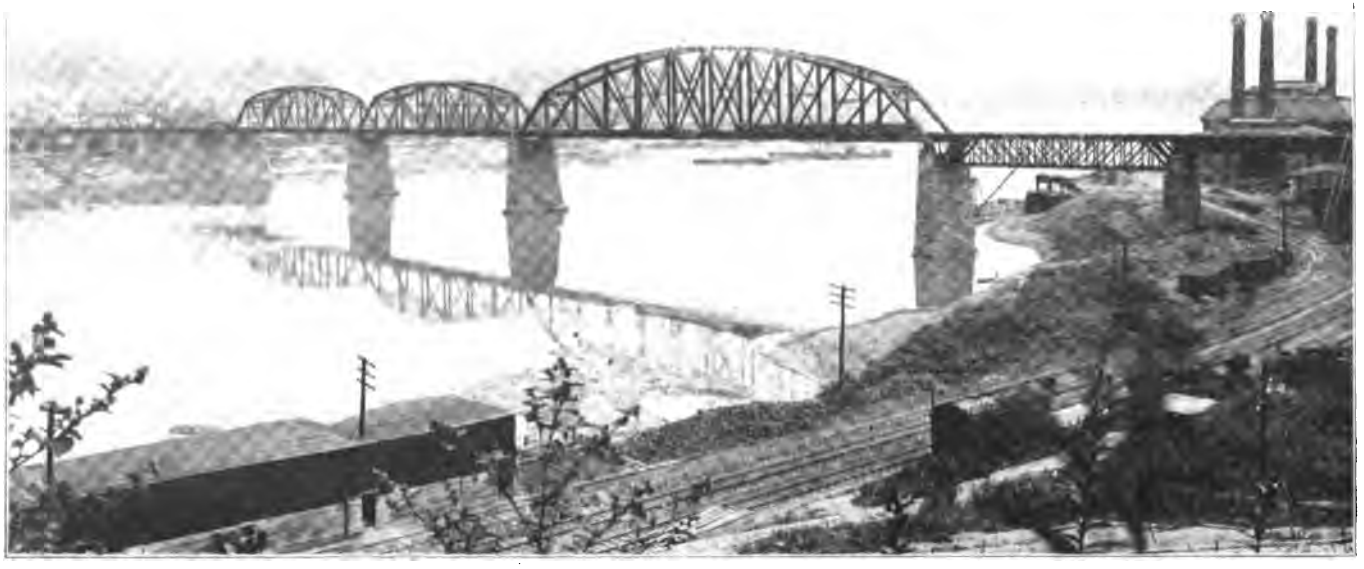
the center of moment being at O, the intersection of the lines of stress of S_1 and S_3. Solving the equation just stated,

$$S_2 = \frac{1}{l} \left\{ -Ra + P_1 (a + p) + P_2 (a + 2p) \right\}.$$

BRIDGE OVER SUSQUEHANNA RIVER BETWEEN HAVRE DE GRACE AND PERRYVILLE, MARYLAND

On line of Philadelphia, Baltimore & Washington Railroad. Built in 1905. Consists of the following double-track spans: One deck pin-connected fixed span, 192 ft.; eight deck pin-connected spans, each 255 ft.; and one through lattice draw-span, 277 ft. 2 in. Clear height above mean low tide at draw-span, 54½ ft.

Courtesy of American Bridge Company.



BRIDGE OVER ALLEGHENY RIVER AT BRILLIANT, PENNSYLVANIA

On line of Pennsylvania Railroad. Built in 1902-03. Double-track, consisting of two deck plate-girder spans, each 90 ft. 6. in; two plate-girder spans, each 87 ft. 4 in.; two through pin-connected spans, each 218 ft. 6 in.; and one through pin-connected span, 407 ft.

Courtesy of American Bridge Company.

30. **Stresses in Web Members.** By reference to Articles 28 and 29, it is seen that several methods are presented for the solution of stresses in web members. Each should be adapted to the case in hand. The simplest method, and the one which is commonly used in all trusses with parallel chords, is by the resolution of the vertical forces. Fig. 21 is to be referred to. The equation given on page 19 is:

$$+R - p - p - S_x \cos \phi = 0.$$

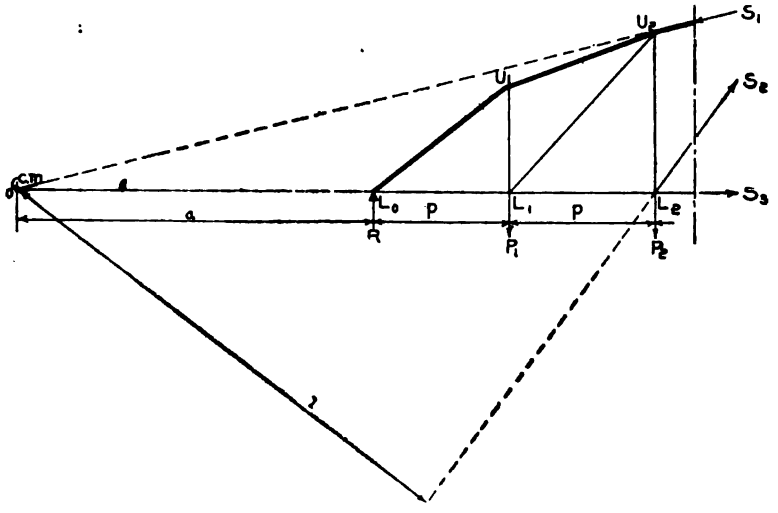


Fig. 27. Diagram Illustrating Application of Method of Moments in Analysis of Trusses. Top chord curved.

But $R - p - p$ is equal to V, the vertical shear at the section, and so the equation may now be written:

$$V - S_x \cos \phi = 0 \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

whence the following important rule is deduced:

The algebraic sum of the vertical shear at the section and the vertical components of the stress in all of the members cut by the section, is equal to zero.

In trusses with horizontal chords and a simple system of webbing, the equation may be put in the form:

$$S_x = +V \sec \phi;$$

and the statement that the stress in any web member is equal to the shear *times* the secant of the angle that it makes with the vertical is

true. The practice of using this latter statement is not to be encouraged, as it leads to confusion in the signs of the stresses. Equation (1) should be written in all cases, and the stress will then solve with its correct characteristic sign, indicating that the stress is either tensile or compressive.

As an example, let it be required to determine the stresses in the web members S_2 and S_3 of the Pratt truss shown in Fig. 28, the loads being in thousands of pounds. First, a section should be passed, cutting that member and as few others as possible. Next, the shear at that section should be computed. Then the vertical components of all the stresses cut by the section, and the vertical shear, should be

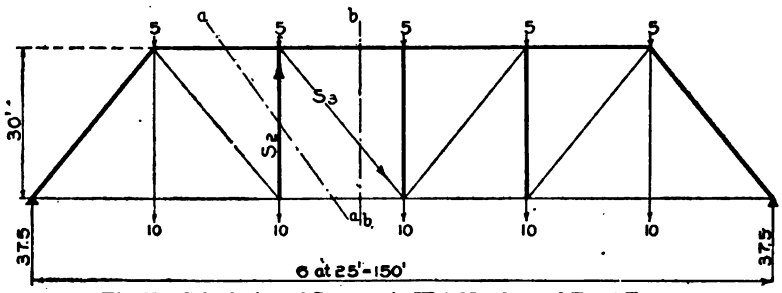


Fig. 28. Calculation of Stresses in Web Members of Pratt Truss.

equated to zero. Finally, solve the equation. Remember that the unknown stress is to be assumed as acting away from the section, and that forces or resultants acting downward are considered negative, while those acting upward are considered positive.

To determine S_2:

The vertical shear at the section $a-a$ is:

$$+37.5 - 2 \times 10 - 5 = +12.5.$$

As the chord stresses do not exert a vertical component, the equation is:

$$+12.5 + S_2 = 0$$

$$S_2 = -12.5, \text{ which is a compressive stress of 12,500 pounds.}$$

Note that in this case the angle which the member makes with the vertical is zero, and the cosine and secant are unity.

To determine S_3:

The vertical shear at the section $b-b$ is

$$+37.5 - 2 \times 10 - 2 \times 5 = +7.5.$$

The equation is:

$$+7.5 - S_3 \cos \phi = 0$$

$$S_3 = +7.5 \sec \phi.$$

Sec ϕ is equal to $\sqrt{30^2 + 25^2} \div 30$, which is equal to 1.302; and therefore,

$$S_2 = +7.5 \times 1.302$$
$$= +9.765, \text{ which is a tensile stress of 9,765 pounds.}$$

31. **Stresses in Chord Members.** The stresses in chord may be obtained by either the method of moments or the method of resolution of forces, this latter being usually the resolution of horizontal forces.

In accordance with the text of Article 29, the following rule may be stated with regard to the solution of stresses in chord members by the method of moments:

Pass a plane section cutting the member whose stress is to be computed, and as few others as possible; then take the center of moments at such a point

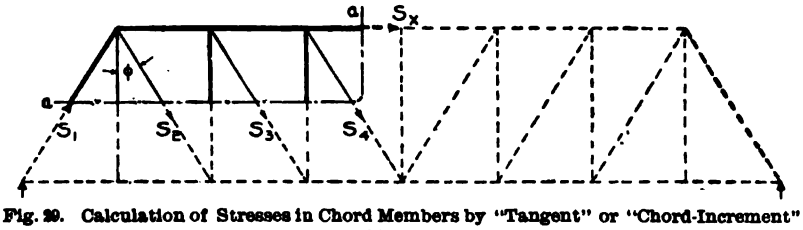


Fig. 29. Calculation of Stresses in Chord Members by "Tangent" or "Chord-Increment" Method.

that the lines of action of as many forces as possible, the unknown one excepted, pass through that point; write an equation of the moments about this point of the known loads and forces to the left of the section, assuming the unknown force to act away from the section, and taking the known forces to act as given, the tensile stresses to act away from the section, and the compressive stresses to act towards the section; place the equation equal to zero, and solve.

The stress will solve out with its correct characteristic sign.

In the majority of cases a section can be made to cut three members only, one of the three being the one whose unknown stress is desired. In such cases, take the center of moments at the intersection of the other two, and proceed as before. As examples of this latter case, note the centers of moments at U_2, Fig. 26, and O, Fig. 27, and also the equations resulting therefrom.

When the method of resolution of forces is used, it is usually designated as the *tangent method* or the *chord increment method.* The simplest application of this method is to trusses with horizontal chords and vertical posts in the web members. Then the stress in any chord member is equal to the product of the sum of the shears

in the panels up to that section, and the tangent of the angle which the diagonals make with the vertical.

This can readily be proved by reference to Fig. 29. Let it be required to determine the stress in the chord member S_x. Pass the section $a-a$. The stresses S_1, S_2, S_3, and S_4 are now computed, and are $S_1 = -V_1 \sec \phi$; $S_2 = +V_2 \sec \phi$; $S_3 = +V_3 \sec \phi$; and $S_4 = +V_4 \sec \phi$. Now noting the directions of the known stresses and

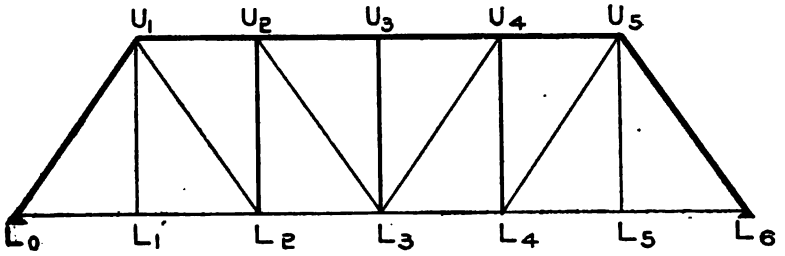


Fig. 30. Illustrating Method of Notation of Stresses and Members in a Through Bridge.

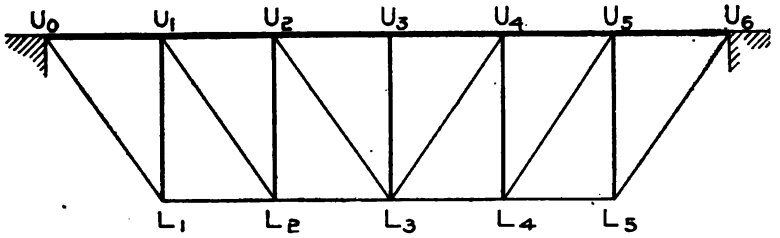


Fig. 31. Illustrating Method of Notation of Stresses and Members in a Deck Bridge.

assuming S_x to act away from the section, the equation of the horizontal component is:

$$+S_1 \sin \phi + S_2 \sin \phi + S_3 \sin \phi + S_4 \sin \phi + S_x = 0.$$

Now, substituting the values of S_1, S_2, etc., and remembering that $\sec \phi = \dfrac{1}{\cos \phi}$, the equation becomes:

$$V_1 \frac{\sin \phi}{\cos \phi} + V_2 \frac{\sin \phi}{\cos \phi} + V_3 \frac{\sin \phi}{\cos \phi} + V_4 \frac{\sin \phi}{\cos \phi} + S_x = 0,$$

from which,

$$S_x = -(V_1 + V_2 + V_3 + V_4) \tan \phi;$$
$$S_x = -\Sigma V \tan \phi.$$

From inspection of Fig. 29, it will be noticed that the stress in any section of the chord is equal to that in the section to the left of

it, *plus* the increment (horizontal component) of the diagonal; hence the name *chord increment method.*

32. **Notation.** The practice hitherto used in designating stresses by S_1, S_2, etc., will now be discontinued, as it is inconvenient in the extreme; moreover, it is not the method used in practical work. The notation to be used is that given in Figs. 30 and 31, the former being for a through and the latter for a deck truss.

The practical advantages of this system are very great. When $U_1 U_2$ is noted, it is at once known to be the top chord of the second panel; $U_2 L_2$ is known to be the second vertical; while $U_2 L_3$ is at once recognized as the diagonal in the third panel. A stress in a member, as well as the member itself, is designated by the subscript letters at its ends. Thus $U_1 L_2$ may mean the member

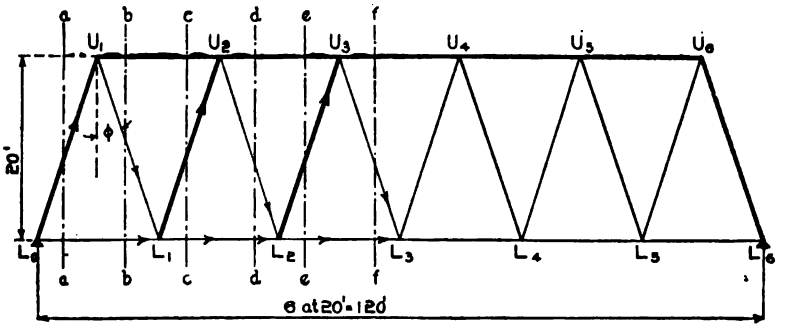


Fig. 32. Calculation of Stresses in a Six-Panel Warren Truss Through Bridge.

itself or the stress in the member. The text will clear this up. In analysis, the stress would be implied, while in design the member itself would be intended.

33. **Warren Truss under Dead Loads.** The Warren truss has its web members so built of angles and plates or of channels, that they can take either tension or compression. The top chord is of structural shapes, while the lower chord may be of built-up shapes or simply of bars.

Let it be required to determine all of the stresses in the six-panel truss of a through Warren highway 120-foot span for country traffic. The height is to be 20 feet. The outline is given in Fig. 32. According to Fig. 16, the total weight of the span, including wooden floor, is 76 000 pounds. Each truss carries one-half of this, or 76 000

$\div 2 = 38\ 000$ pounds. As there are six panels, each panel load is $38\ 000 \div 6 = 6\ 333$ pounds. This means that we must compute the stresses in the above truss by considering that a load of 6 333 pounds is at points L_1, L_2, L_3, L_4, and L_5. Of course there is some weight at L_0 and L_6; but this does not stress the bridge, as it is directed over the abutments or supports. The reactions at L_0 and L_6 are each equal to $(5 \times 6\ 333) \div 2 = 15\ 833$ pounds.

The shears are next computed, and are:

$$V_1 = +15\ 833 - 0 = +15\ 833$$
$$V_2 = +15\ 833 - 6\ 333 = +9\ 500$$
$$V_3 = +15\ 833 - 2 \times 6\ 333 = +3\ 167$$

It is unnecessary to go past the center of the bridge, as it is symmetrical. The V_1 represents the shear on any section between L_0 and L_1; V_2 represents the shear on any section between L_1 and L_2; and V_3 represents the shear on any section between L_2 and L_3. The secant of the angle ϕ is:

$$\left(\sqrt{\overline{(20)}^2 + \overline{(10)}^2}\right) \div 20 = 1.12.$$

The stresses in the web members are computed as follows:

For $L_0\,U_1$. Pass section $a-a$. Assume stress acting away from the section, as shown. Then,

$$V_1 + L_0U_1 \cos \phi = 0;$$
$$L_0U_1 = -V_1 \sec \phi;$$
$$L_0U_1 = -15\ 833 \times 1.12 = -17\ 700 \text{ pounds},$$

which shows that $L_0\,U_1$ has a compressive stress of 17 700 pounds.

For $U_1\,L_1$. Pass section $b-b$. Assume stress acting away from the section, as shown. Then,

$$V_1 - U_1L_1 \cos \phi = 0;$$
$$U_1L_1 = +V_1 \sec \phi;$$
$$U_1L_1 = +15\ 833 \times 1.12 = +17\ 700 \text{ pounds},$$

which shows that U_1L_1 has a tensile stress of 17 700 pounds.

For L_1U_2. Pass section $c - c$. Then,

$$V_2 + L_1U_2 \cos \phi = 0;$$
$$L_1U_2 = -V_2 \sec \phi;$$
$$L_1U_2 = -9\ 500 \times 1.12 = -10\ 640 \text{ pounds}.$$

For U_2L_2. Pass section $d - d$. Then,

$$+9\ 500 - U_2L_2 \cos \phi = 0;$$
$$U_2L_2 = +9\ 500 \times 1.12 = +10\ 640.$$

For L_2U_3. Pass section $e - e$. Then,

$$+3\,167 + L_2U_3 \cos \phi = 0;$$
$$L_2U_3 = -3\,167 \times 1.12 = -3\,540.$$

For U_3L_3. Pass section $f - f$. Then,

$$+3\,167 - U_3L_3 \cos \phi = 0;$$
$$U_3L_3 = +3\,167 \times 1.12 = +3\,540.$$

The computation of the stresses in the chords is made by the method of moments, and is as follows:

For L_0L_1. Section $b - b$ cuts U_1L_1 and U_1U_2, besides the member whose stress is desired, and therefore the center of moments will be taken at their intersection U_1. The equation is:

$$+15\,833 \times 10 - L_0L_1 \times 20 = 0,$$

whence,

$$L_0L_1 = (+15\,833 \times 10) \div 20;$$
$$= +7\,917 = \text{a tension of } 7\,917 \text{ pounds.}$$

For L_1L_2. Either section $c - c$ or $d - d$ may be used, and each shows the center of moments to be at U_2. The equation is:

$$+15\,833 \times 30 - 6\,333 \times 10 - L_1L_2 \times 20 = 0;$$
$$L_1L_2 = (+15\,833 \times 30 - 6\,333 \times 10) \div 20;$$
$$= +20\,583 = \text{a tensile stress of } 20\,583 \text{ pounds.}$$

For L_2L_3. Either section $e - e$ or $f - f$ may be used, and each shows the center of moments to be at U_3. The equation is:

$$+15\,833 \times 50 - 6\,333 \times 30 - 6\,333 \times 10 - L_2L_3 \times 20 = 0;$$

whence,

$$L_2L_3 = +26\,917.$$

The center of moments for U_1U_2 is at L_1; for U_2U_3, it is at L_2; and for U_3U_4, it is at L_3. The following equations can now be written:

$+20 \times U_1U_2 + 20 \times 15\,833 = 0$; whence $U_1U_2 = -15\,833$;
$+20 \times U_2U_3 + 40 \times 15\,833 - 20 \times 6\,333 = 0$; whence $U_2U_3 = -25\,333$;
$+20 \times U_3U_4 + 60 \times 15\,833 - 40 \times 6\,333 - 20 \times 6\,333 = 0$; whence $U_3U_4 = -28\,500$.

A diagram of half of the truss should now be made, and all the stresses placed upon it. The dimensions should also be put upon this diagram. The student should cultivate this habit, as it shows him at a glance the general relation of stresses and the general rules of their variations. Fig. 33 gives the half-truss, together with the stresses and dimensions. The stresses in the members of the right half of the truss are the same as those in the corresponding members of the left half.

From inspection of the above diagram, it is seen that the chord stresses increase from the end toward the center; that the web stresses decrease from the end toward the center; and that all members slanting the same way as the end-post L_0U_1 have stresses of that sign,

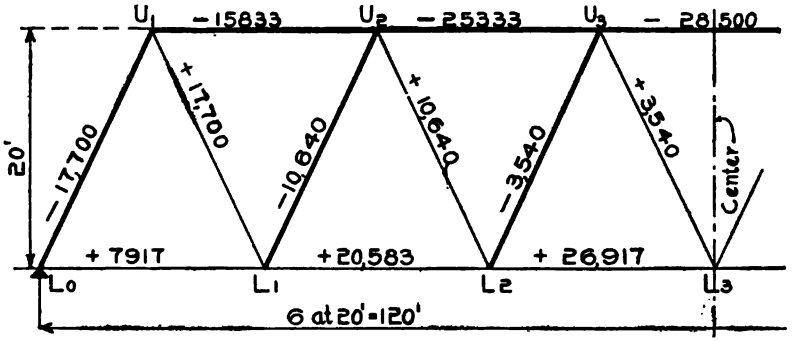


Fig. 33. Dimensions and Stress Diagram of Half a Six-Panel Warren Through Truss.

while all that slant a different way have an opposite sign. These relations are true of all trusses with parallel chords and simple systems of webbings.

34. **Position of Live Load for Maximum Positive and Negative Shears.** The dead load, by reason of its nature, is an unchangeable load. The stresses due to it are the same at any and at all times.

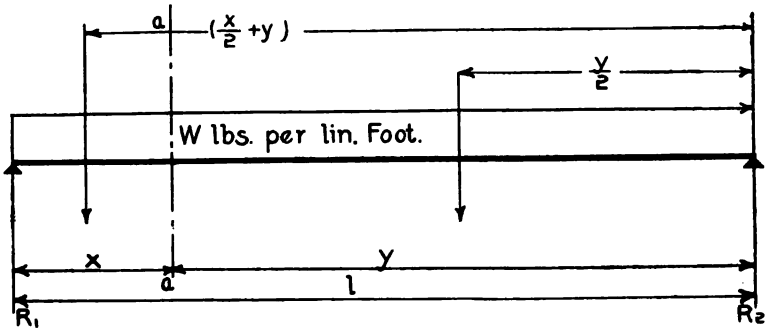


Fig. 34. Calculating Maximum Positive and Negative Shears in Simple Beam under Live Load. Conventional Method.

With the live load, the case is different. The live load represents the movement of traffic upon the bridge. At certain times there may be none on the bridge, while at other times it may fill the bridge partially or entirely. In such cases the shears due to live load will vary.

Conventional Method. It has been found that *the maximum positive shear at any section of a simple beam occurs when the beam is loaded from that section to the right support,* and that *the maximum negative shear occurs at the same section when this beam is loaded from the section to the left support.* This can be proved as follows:

Let a beam be as in Fig. 34, and let $a - a$ be the section under consideration. The reaction R_1 is due to the load wy on the part y, and to the load wx on the part x. That is,

$$R_1 = \left\{wx\left(\frac{x}{2} + y\right) + wy\frac{y}{2}\right\} \div l.$$

Now the shear at the section $a - a$ is $R_1 - wx$; or,

$$\left\{wx\left(\frac{x}{2} + y\right) + wy\frac{y}{2}\right\}\frac{1}{l} - wx = V_{a \cdot a}$$

$$wx\left(\frac{\frac{x}{2} - y}{l}\right) + \frac{wy^2}{2l} - wx = V_{a \cdot a}$$

$$\left\{wx\left(\frac{\frac{x}{2} - y}{l}\right) - wx\right\} + \frac{wy^2}{2l} = V_{a \cdot a}$$

From inspection of this last equation, it is seen that $wx\left(\frac{\frac{x}{2} - y}{l}\right)$ is the amount that is added to the reaction by loading the part x. Also, that $\frac{\frac{x}{2} - y}{l}$ is less than unity, is evident. The amount in brackets in the last equation represents the effect of the loading of the segment x of the beam. As this is negative and will only reduce the positive valued term $\frac{wy^2}{2l}$, it is therefore proved that to get the largest positive shear the beam should be loaded from the section to the right support.

From further inspection of the equation, it will be seen that the term in brackets, which represents the effect of the load on the segment x on the shear, is always negative; and that the term $\frac{wx^2}{2l}$, which represents the effect of the load on the segment y on the shear, is always positive. Hence, to get the largest negative shear at the section, the load should be on the segment x. That is, the loading should be from the section to the left support.

In a truss, the loads are placed at the panel points; and the above rules in application, should be formulated as follows:

To get the maximum positive shear at a section or in a panel, load all panel points to the right of it.

To get the maximum negative shear at a section or in a panel, load all panel points to the left of it.

Example. Determine the maximum positive and the maximum negative shears in the panels of the 7-panel Pratt truss shown in Fig. 35, the

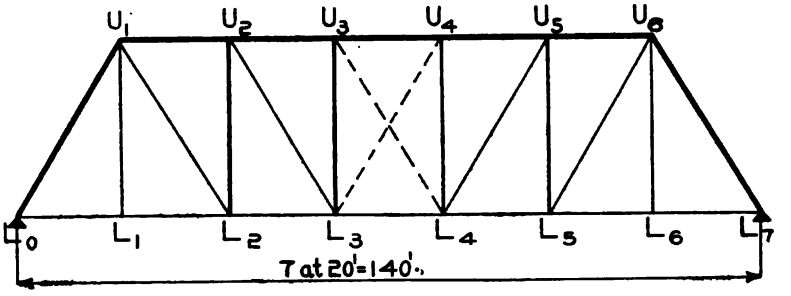


Fig. 35. Calculation of Shears in Panels of 7-Panel Pratt Truss.

live panel load being 40 000 pounds. (It will be noticed that the height of the truss is not required.)

For maximum	$+V$ in	1st panel,	load	L_1, L_2, L_3, L_4, L_5 and L_6.
" "	$+V$ "	2d "	"	" L_2, L_3, L_4, L_5, and L_6.
" "	$+V$ "	3d "	"	" L_3, L_4, L_5, and L_6.
" "	$+V$ "	4th "	"	" L_4, L_5, and L_6.
" "	$+V$ "	5th "	"	" L_5 and L_6
" "	$+V$ "	6th "	"	" L_6.
" "	$+V$ "	7th "	"	" no panel points at all.

The reaction produced by each of the loadings is equal to the shear for that particular case, since the shear at any section or in any panel is equal to the reaction *minus* the loads to the left of the section or panel, and, according to the method of loading, there are no loads to the left of the section; therefore the reaction is equal to the shear.

For the first panel, the computation is made as follows, the center of moments being, of course, at L_7:

$$(R_1 = +V_1) \times 7 \times 20 = 40 \times 20 + 40 \times 2 \times 20 + 40 \times 3 \times 20 + 40 \times 4 \times 20 + 40 \times 5 \times 20 + 40 \times 6 \times 20.$$

It will be seen that as 20 occurs in all terms of this equation, it can be factored out by dividing both sides by 20, and the result will be the same. The equation can now be written:

$$+V_1 \times 7 = 40 + 40 \times 2 + 40 \times 3 + 40 \times 4 + 40 \times 5 + 40 \times 6,$$

and can still be simplified by writing:

$$+V_1 = \frac{40}{7}(1 + 2 + 3 + 4 + 5 + 6) = +120.00,$$

which is the form customarily used, the panel length being taken as a unit of measurement. The other shears are now easily computed in a similar manner:

$$+V_2 = \frac{40}{7}(1 + 2 + 3 + 4 + 5) = +85.71$$

$$+V_3 = \frac{40}{7}(1 + 2 + 3 + 4) = +57.14$$

$$+V_4 = \frac{40}{7}(1 + 2 + 3) = +34.28$$

$$+V_5 = \frac{40}{7}(1 + 2) = +17.14$$

$$+V_6 = \frac{40}{7}(1) = +5.71$$

$$+V_7 = \frac{40}{7}(0) = +0$$

In computing the maximum negative shears, sometimes called the *minimum shears*, the reaction is not the same as the shear, as there are loads to the left of the section, and these must be subtracted. The loadings are:

For maximum	$-V$	in 1st	panel,	load no points.
" "	$-V$	" 2d	"	" L_1.
" "	$-V$	" 3d	"	" L_1 and L_2.
" "	$-V$	" 4th	"	" L_1, L_2, and L_3.
" "	$-V$	" 5th	"	" L_1, L_2, L_3, and L_4.
" "	$-V$	" 6th	"	" L_1, L_2, L_3, L_4, and L_5.
" "	$-V$	" 7th	"	" L_1, L_2, L_3, L_4, L_5, and L_6.

It is evident that the maximum $-V_1$ is equal to zero, there being no loads on the span. The maximum negative shear in the second panel is equal to the reaction produced by loading the panel point L_1, and the load at L_1. Thus,

$$7R_1 = 40 \times 6$$

$$R_1 = \frac{40}{7}(6)$$

$$-V_2 = R_1 - \text{load at } L_1$$

$$= \frac{40}{7}(6) - 40$$

$$= -5.71$$

The other shears are next computed as follows:

$$-V_3 = \frac{40}{7}(6 + 5) - 2 \times 40 = -17.14$$

$$-V_4 = \frac{40}{7}(6 + 5 + 4) - 3 \times 40 = -34.28$$

$$-V_5 = \frac{40}{7}(6 + 5 + 4 + 3) - 4 \times 40 = -57.14$$

$$-V_6 = \frac{40}{7}(6 + 5 + 4 + 3 + 2) - 5 \times 40 = -85.71$$

$$-V_7 = \frac{40}{7}(6 + 5 + 4 + 3 + 2 + 1) - 6 \times 40 = -120.00$$

The maximum positive and the maximum negative live-load shears should now be written side by side, and inspected, in order to observe any existing relations which might help to lessen the labor of future computations. The values are given in thousands of pounds below:

Location	Max. + Live-Load Shear	Max. − Live-Load Shear
V_1	+120.00	− 0.00
V_2	+85.71	− 5.71
V_3	+57.14	−17.14
V_4	+34.28	−34.28
V_5	+17.14	−57.14
V_6	+5.71	−85.71
V_7	+0.00	−120.00

It is at once seen that the negative shears are numerically equal in value to the positive ones, but that they occur in reverse order. This simplifies the labor required in the derivation of the negative shears; for, after computing the maximum positive shears, these may be written in reverse order, and the negative sign prefixed; the result will be the maximum negative shears.

The above method for maximum live-load shears is called the *conventional method.* It is the one that is almost universally used, and its use will be continued throughout this text.

Exact Method. On account of the fact that the floor stringers or joists transfer the loads to the panel points, it would be impossible to have a full panel live load at one panel point and no load at the panel point ahead or behind. In order to have a full panel load at one point, the stringers in the panels on both sides of the point must

be full-loaded, and this would give a load at the panel point ahead, provided the bridge was fully loaded up to and not beyond the panel point ahead, equal in value to one-half of a full panel load (see Fig.

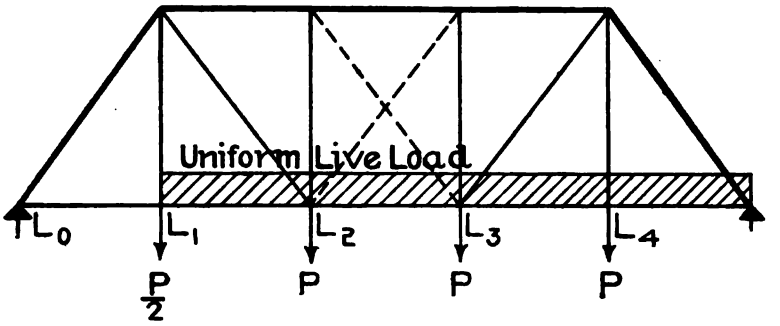


Fig. 36. Illustrating "Exact" Method of Calculating Live-Load Shears in Panels.

36). The uniform live load, in order to produce full panel loads at L_2, L_3 and L_4, will also produce one-half a panel load at L_1.

By the methods of differential calculus, it can be proved that the true maximum positive live-load shear occurs in a panel when the

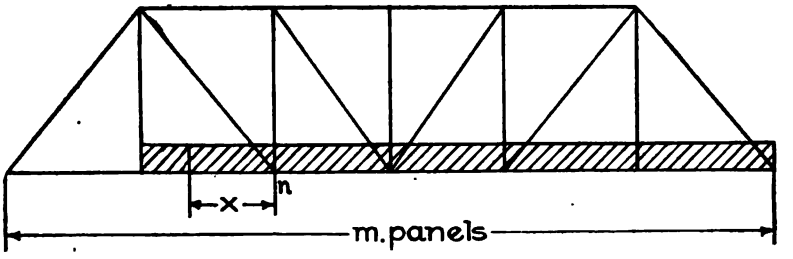


Fig. 37. Calculating Maximum Positive Live-Load Shear in Panel.

live load extends from the panel point to the right into that panel an amount (see Fig. 37) equal to

$$x = \frac{n}{m-1}\,p,$$

in which,

n = Number of the panel point to the left of the panel under consideration, counting from the right;
m = Total number of panels in the bridge;
p = Panel length.

Let the truss of Fig. 35 be considered, the live load being 2 000

pounds per linear foot of truss, and let it be required to determine the true maximum positive live-load shear in the 5th panel *from the right end.*

$$x = \frac{4}{7 - 1} \times 20 = 13.333 \text{ feet.}$$

There will now be $(4 \times 20 + 13.333) \times 2\,000 = 186\,666$ pounds on the truss; and the left reaction will be $\{186\,666 \times (4 \times 20 + 13.333) \div 2\} \div 140 = 62\,200$ pounds. From this must be subtracted the amount of the load on the 13.333 feet, which is transferred to the point L_2. This is equal to the reaction of a beam of a span equal to the panel length, loaded for a distance of 13.333 feet from the right support with a uniform load of 2 000 pounds per linear foot. This amounts to $(13.333 \times 2\,000 \times \frac{13.333}{2}) \div 20 = 8\,890$ pounds. The true shear is now:

$$+V_3 = +62\,200 - 8\,890 = +53\,310 \text{ pounds.}$$

The $+V_3$, as computed by the conventional method, was $+57\,140$, making a difference of 3 730 pounds between the two. If the true shears were computed and compared with the conventional shears, it would be found that the V_1 would be the same, and that the remainder of the conventional shears would be greater than the corresponding true shears. The difference between any two corresponding shears would increase from the left to the right end; that is, the difference between the conventional and exact shears would be greatest in the panel L_5L_6.

To get the maximum negative shear in any panel, load from the left support and out into the panel under consideration an amount $p - x$, and proceed in a manner similar to that above described.

As this method of exact or true shears is seldom employed, problems illustrating its application will here be omitted.

35. **Position of Live Load for Maximum Moments.** *In order to obtain the maximum moment at any point, the live load must cover the entire bridge.* Let the beam of Fig. 34 be considered, and let it be required to obtain the maximum moment at the section $a - a$. The reaction, as before computed, is:

$$R_1 = \left\{ wx \left(\frac{x}{2} + y \right) + \frac{wy^2}{2} \right\} \frac{1}{l},$$

all terms of which are positive. The moment at the section is:

$$M = R_1 \times x - wx\frac{x}{2};$$

and substituting for R_1 its value,

$$M = \frac{wx^2}{l}\left(\frac{x}{2} + y\right) + \frac{wy^2x}{2l} - \frac{wx^2}{2};$$

$$= \frac{wx^2}{l}\left(\frac{x}{2} + y\right) - \frac{wx^2}{2} + \frac{wy^2x}{2l};$$

$$= wx^2\left(\frac{x}{2l} + \frac{y}{l} - \frac{1}{2}\right) + \frac{wy^2}{2l}.$$

But $y = l - x$; therefore,

$$M = wx^2\left(\frac{x}{2l} + \frac{l-x}{l} - \frac{1}{2}\right) + \frac{wy^2x}{2l};$$

$$= wx^2\left(\frac{2l-x}{2l} - \frac{1}{2}\right) + \frac{wy^2x}{2l}.$$

The first term of this equation represents the effect of the load on the portion x, and the second term represents the effect of the load on the portion y. The value of M will always be positive. The quantity x varies between 0 and l. When $x = 0$, M is equal to 0. When $x = l$, the moment is equal to $+ wy^2x \div 2$. For all values of x between 0 and l, the first term is positive; and the second term being positive in all cases, it is therefore proved that for maximum live-load moments at any point, the entire span should be loaded, as loads on both segments add positive values to the moment value.

36. Warren Truss under Live Load. In order to analyze a truss intelligently, it is necessary to know its physical structure; that is, it must be known what character of stress can be withstood by the different members. The top chords of all trusses are built to take only compression, and the bottom chords are built to take only tension; while some web members of some trusses are built for tension stresses, some for compression stresses, and some for both. The characteristic of the Warren truss is that the web members are built so as to be able to withstand either tension or compression.

Let it be required to determine the live-load stresses in the Warren truss of Fig. 32. Let the live load per square foot of roadway, which is assumed to be 15 feet wide, be 100 pounds. The live panel load is then $100 \times 15 \times 20 \div 2 = 15\,000$ pounds, and the live-load reaction under full load is $2\frac{1}{2} \times 15\,000 = 37\,500$ pounds.

As the live load must cover the entire bridge to give maximum moments—and therefore maximum chord stresses, as the chord stress is equal to the moment divided by the height of the truss—a simple method for the determination of live-load chord stresses presents itself. The live load and the dead load being applied at the same points, and being different in intensity, the stresses produced will be proportional to the panel loads. The maximum live-load chord stresses (see Fig. 33) will then be equal to the dead-load chord stresses multiplied by 15 000 ÷ 6 333 = 2.371, and they are as follows:

$$L_0U_1 = -2.371 \times 17\,700 = -42\,000$$
$$U_1U_2 = -2.371 \times 15\,833 = -37\,530$$
$$U_2U_3 = -2.371 \times 25\,333 = -60\,050$$
$$U_3U_4 = -2.371 \times 28\,500 = -67\,600$$
$$L_0L_1 = +2.371 \times 7\,917 = +18\,770$$
$$L_1L_2 = +2.371 \times 20\,583 = +48\,800$$
$$L_2L_3 = +2.371 \times 26\,917 = +63\,850$$

The next step in order is to determine the maximum positive shears, and from these write the maximum negative shears. This is done as follows:

	+ Live-Load V	− Live-Load V
$V_1 = \frac{15\,000}{6}(1 + 2 + 3 + 4 + 5)$	= +37 500	0
$V_2 = \frac{15\,000}{6}(1 + 2 + 3 + 4)$	= +25 000	−2 500
$V_3 = \frac{15\,000}{6}(1 + 2 + 3)$	= +15 000	−7 500
$V_4 = \frac{15\,000}{6}(1 + 2)$	= + 7 500	−15 000
$V_5 = \frac{15\,000}{6}$	= + 2 500	−25 000
$V_6 =$	+ 0	−37 500

The stresses produced by the positive shears are called the *maximum live-load stresses*, and are:

$$+L_0U_1 \cos\phi + 37\,500 = 0 \qquad \therefore L_0U_1 = -37\,500 \times 1.12 = -42\,000$$
$$-U_1L_1 \cos\phi + 37\,500 = 0 \qquad \therefore U_1L_1 = +37\,500 \times 1\,12 = +42\,000$$
$$+L_1U_2 \cos\phi + 25\,000 = 0 \qquad \therefore L_1U_2 = -25\,000 \times 1.12 = -28\,000$$
$$-U_2L_2 \cos\phi + 25\,000 = 0 \qquad \therefore U_2L_2 = +25\,000 \times 1.12 = +28\,000$$
$$+L_2U_3 \cos\phi + 15\,000 = 0 \qquad \therefore L_2U_3 = -15\,000 \times 1.12 = -16\,800$$
$$-U_3L_3 \cos\phi + 15\,000 = 0 \qquad \therefore U_3L_3 = +15\,000 \times 1.12 = +16\,800$$

The stresses produced by the negative shears are called the *minimum live-load stresses*, and are:

NEW BLACKWELL'S ISLAND CANTILEVER BRIDGE OVER THE EAST RIVER, NEW YORK, UNDER CONSTRUCTION

View looking east.

Copyright, 1907, by Underwood & Underwood, New York.

BRIDGE AT 52D STREET OVER FREIGHT YARD OF THE PENNSYLVANIA RAILROAD, PITTSBURG, PA.

One single-track, through, pin-connected span of 387 feet 10 inches. Built in 1902.

Courtesy of American Bridge Company.

$+L_0U_1 \cos\phi + 0 = 0$	$\therefore L_0U_1 = 0$	
$-U_1L_1 \cos\phi + 0 = 0$	$\therefore U_1L_1 = 0$	
$+L_1U_2 \cos\phi - 2\,500 = 0$	$\therefore L_1U_2 = +2\,500 \times 1.12 = +2\,800$	
$-U_2L_2 \cos\phi - 2\,500 = 0$	$\therefore U_2L_2 = -2\,500 \times 1.12 = -2\,800$	
$+L_2U_3 \cos\phi - 7\,500 = 0$	$\therefore L_2U_3 = +7\,500 \times 1.12 = +8\,400$	
$-U_3L_3 \cos\phi - 7\,500 = 0$	$\therefore U_3L_3 = -7\,500 \times 1.12 = -8\,400$	

These stresses, together with the dead-load stresses, should now be placed together as a half-diagram, as is done in Fig. 38, the stresses being rounded off to the nearest ten pounds and then expressed in thousands of pounds. No minimum live-load stress is given for the chords, as this will evidently be zero in all cases, since no position of the live load will cause a reversal of stress. It will be seen that the stresses produced by the negative shears are of opposite

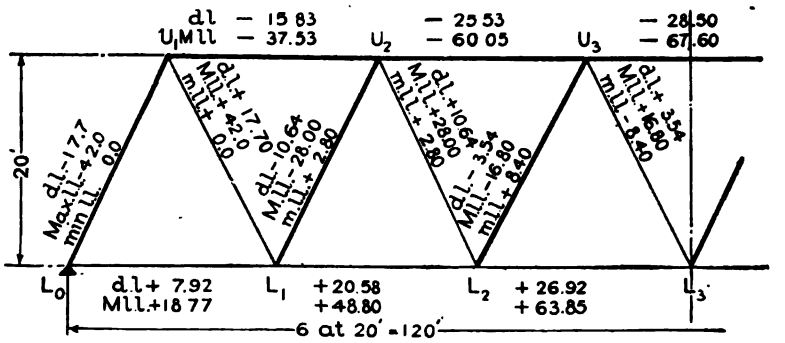


Fig. 38. Dimension and Stress Diagram of Warren Half-Truss under Live Load.

sign from the stress produced by the dead load, and these tend to decrease the dead-load stress by that amount; and in some cases (see L_2U_3 and U_3L_3, Fig. 38) it will be so large as to overcome the dead-load stress and therefore change the total stress from one kind to another. Do not forget, in considering any combination of the above stresses, that the dead load occurs with either the maximum or the minimum live load, but *not with both at the same time.*

37. **Counters.** By reference to U_3L_3 (Fig. 38), it is seen that when the live load is on the panel points L_1 and L_2 the total stress in the member is $+ 3.54 + (- 8.40) = -4.86$, a compressive stress of 4 860 pounds; whereas, under dead load alone, the stress was $+ 3.54$, a tensile stress of 3 540 pounds. If the member U_3L_3 had been built of long, thin bars which could take only tension, and which consequently would have doubled up under the resultant compression

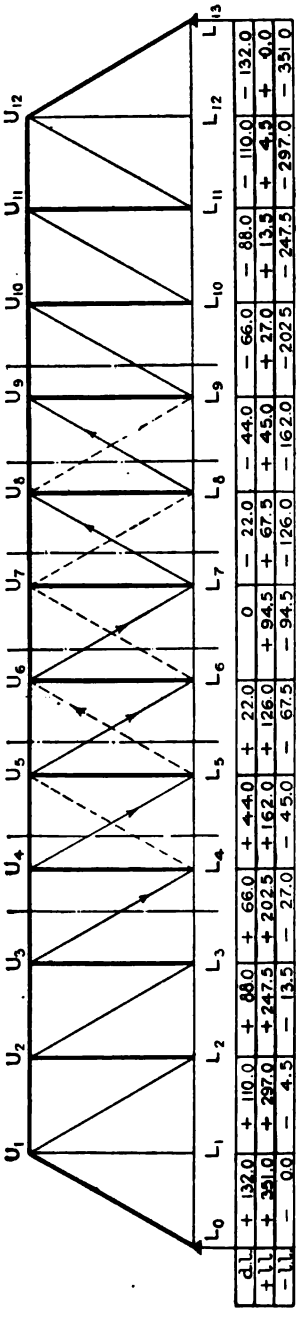


Fig. 39. Load Diagram of 13-Panel Pratt Truss.

brought upon them by the combined action of the dead and minimum live-load stresses, then this member could not be used in this case, but some other arrangement would be necessary in order to insure the stability of the truss.

In the Warren truss, no special arrangement is necessary, as the web members are built so as to take either tension or compression; but with the Pratt and Howe trusses some special arrangement is necessary, as the diagonals are built to take one kind of stress only. The case of the Pratt will be considered first.

The Pratt truss has the diagonals made of long bars which take tension only, and the intermediate posts are constructed so as to be able to take compression only. It is not necessary to consider the intermediate posts, for the action of the web members is such that the resulting stresses are always compressive.

Let the 13-panel Pratt truss of Fig. 39 be considered. The panel length is 18 feet, the height 25 feet, the dead panel load 22 000 pounds, and the live panel load 58 500 pounds. The secant is $(18^2 + 25^2)^{\frac{1}{2}} \div 25 = 1.231$. The dead-load shears and the maximum and minimum live-load shears are placed directly below their respective panels. Only those members are shown full-lined in Fig. 39 which act under the dead load. Note that the dead-load shears in the center

panel being zero, the dead-load stress in the diagonals in the center panel would be $0 \times \sec \phi = 0$.

In the first four panels from either end, the live-load shear, which is of a different sign from that of the dead-load shear, is smaller than the dead-load shear, and therefore will not cause a reversal of stress in the member in that panel. For example, take $U_3 L_4$; then, for dead-load stress,

$$-U_3L_4 \cos \phi + 66.0 = 0 \qquad \therefore U_3L_3 = +66.0 \times 1.231 = +81.20$$

For live-load stress,

$$-U_3L_4 \cos \phi - 27.0 = 0 \qquad \therefore U_3L_3 = -27.0 \times 1.231 = -33.25$$

The total stress $= +81.20 - 33.25 = +47.95$, which is still tension.

Considering $L_9 U_{10}$, the stress equations are:
For dead-load stress,

$$+L_9U_{10} \cos \phi - 66.0 = 0 \qquad \therefore L_9U_{10} = +81.20$$

For live-load stress,

$$+L_9U_{10} \cos \phi + 27.0 = 0 \qquad \therefore L_9U_{10} = -33.25$$

The total stress, as before, is $+47.95$, or a tension of 47 950 pounds.

An inspection of the center panel and the two panels on each side of it, shows that the live-load shear is of a different sign from the dead-load shear, and is also greater in value than the dead-load shear. If the members shown in Fig. 39 were the only ones in the panels, then the dead-load stresses would be:

$$-U_4L_5 \cos \phi + 44.0 = 0 \qquad U_4L_5 = +54.20$$
$$-U_5L_6 \cos \phi + 22.0 = 0 \qquad U_5L_6 = +27.10$$
$$+L_7U_8 \cos \phi - 22.0 = 0 \qquad L_7U_8 = +27.10$$
$$+L_8U_9 \cos \phi - 44.0 = 0 \qquad L_8U_9 = +54.20$$

and the live-load stresses caused by the shear of opposite sign from that of the dead-load shear, are:

$$-U_4L_5 \cos \phi - 45.0 = 0 \qquad U_4L_5 = -55.40$$
$$-U_5L_6 \cos \phi - 67.5 = 0 \qquad U_5L_6 = -83.10$$
$$+L_7U_8 \cos \phi + 67.5 = 0 \qquad L_7U_8 = -83.10$$
$$+L_8U_9 \cos \phi + 45.0 = 0 \qquad L_8U_9 = -55.40$$

As no diagonal acts under dead load in the center panel, we may assume that $U_6 L_7$ acts under live load. The stresses which occur in this are:

$$+U_6L_7 \cos \phi + 94.5 = 0 \qquad U_6L_7 = +116.30$$
$$-U_6L_7 \cos \phi - 94.5 = 0 \qquad U_6L_7 = -116.30$$

The above shows that compressive stresses will occur in the diagonals which were built for tension only. These stresses are:

$$U_4L_5 = +54.20 - 55.40 = -1\,200 \text{ pounds}$$
$$U_5L_6 = +27.10 - 83.10 = -56\,000 \text{ "}$$
$$L_7U_8 = +27.10 - 83.10 = -56\,000 \text{ "}$$
$$L_8U_9 = +54.20 - 55.40 = -1\,200 \text{ "}$$
$$U_6L_7 = 0 - 116.30 = -116\,300 \text{ "}$$

If some provision were not made for these stresses, they would cause the members to crumple up and the truss to fail. In order to allow for them, diagonals are placed in the panels, as shown by the dotted and dashed lines. These members will take up the above stress; and moreover, as they slope the opposite way from the main members, they will be in tension.

In order to prove this, assume L_5U_6 to act when the live load is on points L_5, L_4, L_3, L_2, and L_1. Now, U_5L_6 will not be regarded, as its stress will be zero. Then the stresses will be:
For dead load,

$$+L_5U_6 \cos \phi + 22.0 = 0 \qquad L_5U_6 = -27.10.$$

For live load,

$$+L_5U_6 \cos \phi - 67.5 = 0 \qquad L_5U_6 = +83\ 10;$$

and the total stress in L_5U_6 will be $-27.10 + 83.10 = +56.00$.

In a similar manner, the stresses in the other members are: $L_4U_5 = +1.2$; $L_6U_7 = +116.30$; $U_7L_8 = +56.00$; and $U_8L_9 = +1.2$. These diagonals are called *counters* or *counter-bracing*.

From a consideration of the foregoing, it is evident that:

(*a*) *If the live-load shear in any panel is of opposite sign and greater than the dead-load shear in the same panel, then a counter is required.*

(*b*) *The stress in a counter is equal to the algebraic sum of the dead-load shear and the live-load shear of opposite sign times the secant of the angle it makes with the vertical.*

This is true for any truss with horizontal chords and a simple system of webbing with diagonals and verticals.

38. **Maximum and Minimum Stresses.** Some specifications require the member to be designed for the maximum stress, while others take into account the range of stress. In this latter case it is necessary to determine the minimum as well as the maximum stress. Except where a reversal of stress occurs—and this does not happen in trusses with horizontal chords—few specifications require any

but the maximum stresses to be computed. For that reason, little space will here be devoted to the minimum stresses, their computation in succeeding articles being thought to illustrate them sufficiently.

(*a*) *The maximum stress in a member is equal to the sum of the dead-load stress and the live-load stress of the same sign.*

(*b*) *The minimum stress is equal to the sum of the dead-load stress and the live-load stress of the opposite sign,* or *to the dead-load stress alone,* according to which gives the smallest value *algebraically.* By this latter statement it should be seen that if the maximum stress is -58.60, then 0 or $+18.00$ would be smaller than -3.00.

(*c*) It is evident that the minimum in all counters and in all main members in panels where counters are employed will be zero, for when the counter is acting the main member is not, and therefore its stress is zero. The reverse is also true.

(*d*) An exception to *a* is seen in the case of the counters. Here it is evident that the maximum stress is equal to the algebraic sum of the dead-load shear and the live-load shear of opposite sign *times* the secant of the angle which the counter makes with the vertical.

While it is true that in trusses with horizontal chords the loading for maximum shears will give the maximum live-load stress to be added to the dead load for *the maximum stress,* it is not always true that the loading for minimum live-load shears will give the stress to add to the dead-load stress to get the *minimum stress.* However, the loading for the minimum live-load shears will give the live-load stress to be added to the dead-load stress for the minimum stress, *except* in the case

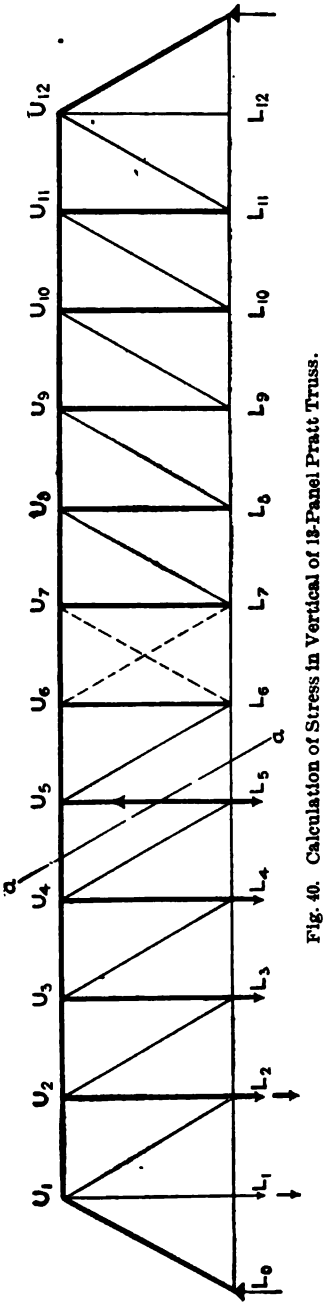
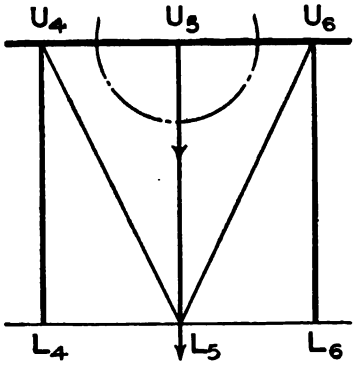


Fig. 40. Calculation of Stress in Vertical of 13-Panel Pratt Truss.

of verticals placed between panels each of which contains counters, and in that case it may or may not do so. In such cases a loading must be assumed—preferably the one for minimum shears—and the shears in the panels on each side of the vertical must be computed *for the loading assumed.* If the resultant shear is the same sign as the live load, then the main diagonal acts; if it is of different sign, then the counter acts.



d.l. 22.0
l.l. 58.5

d.l.v.	+ 44.0	+ 22.0
l.l.v.	− 9.0	− 67.5
Total v.	+	−

Fig. 41. Stress Diagram for Vertical in Truss of Fig. 40.

As an example, let it be required to find the minimum stress in the vertical U_5L_5 of the truss of Figs. 39 and 40. It is assumed that the loading for minimum shears will give the result. The section $a-a$ is then passed, and the live load placed on L_5 and all points to the left. The shears will then be as shown in Fig. 41. To obtain the shear in the panel L_4L_5, *under this loading,* it must be remembered that a load is at L_5; and so the shear is the shear in the panel L_5L_6 with the panel load at L_5 added, or, $-67.5 + 58.5 = -9.0$. The diagonals now act as indicated by Fig. 41, and the total stress in U_5L_5 is determined by passing a circular section around U_5, and it is :

$$-\text{Load at } U_5 - U_5L_5 = 0.$$

As there is no load at U_5, the stress in U_5L_5 is $= 0$. The same result will occur if points L_4 or L_3 and to the left are loaded; but if points L_2 and to the left are loaded, the members U_4L_5 and U_5L_6 will act, and the stress in U_5L_5 will then be equal to the shear on the section $a-a$. The stresses are: Dead-load, -22.0; and live-load, $+13.5$, which gives a total of -8.5; but as the maximum stress is $-22.0 - 126.0 = -148.0$, it is evident that 0 and not -8.5 is the minimum.

The computation of the maximum stress is as follows:

Load points L_6 and to the right. The shear on $a-a$ is, for

dead load, +22.0; and for + live load, +126.0; and the equations of the stresses are:

$$+ 22.0 + U_5L_5 = 0 \qquad U_5L_5 = -22.0$$
$$+126.0 + U_5L_5 = 0 \qquad U_5L_5 = -126.0$$
$$\text{Max.} = -148\,0$$

TRUSSES UNDER DEAD AND LIVE LOADS

39. **The Pratt Truss.** The Pratt truss is used to perhaps a greater extent than any other form; probably 90 per cent of all simple truss spans are of this kind.

Let it be desired to determine the stresses in the 8-panel 200-foot single-track span shown in Fig. 42, the height being 30 feet, the dead panel load being 30 000 pounds, and the live panel load 62 400 pounds. The secant is $\left(\overline{25}^2 + \overline{30}^2\right)^{\frac{1}{2}} \div 30 = 1.302$, and the cosine is 0.7685. The dead-load reaction is $3\frac{1}{2} \times 30.0 = 105.0$.

The dead-load shears are:

$$V_1 = +105.0$$
$$V_2 = + 75.0$$
$$V_3 = + 45.0$$
$$V_4 = + 15.0$$
$$V_5 = - 15.0$$

The dead-load chord stresses may be tabulated as follows (see Articles 27 and 29):

Dead-Load Chord Stresses

Member	Section	Center of Moments	Stress Equation	Stress
$L_0L_1 = L_1L_2$	$a-a$	U_1	$+ 105.0 \times 25 - L_1L_2 \times 30 = 0$	+ 87.5
L_2L_3	$b-b$	U_2	$+105.0 \times 50 - 30.0 \times 25 - L_2L_3 \times 30 = 0$	+150.0
L_3L_4	$c-c$	U_3	$+105.0 \times 75 - 30.0\,(25 + 50) - L_3L_4 \times 30 = 0$	+187.5
U_1U_2	$a-a$	L_2	$+105.0 \times 75 - 30.0 \times 25 + U_1U_2 \times 30 = 0$	−150.0
U_2U_3	$b-b$	L_3	$+105.0 \times 75 - 30.0\,(25 + 50) + U_2U_3 \times 30 = 0$	−187.5
U_3U_4	$c-c$	L_4	$+105.0 \times 100 - 30.0\,(25 + 50 + 75) + U_3U_4 \times 30 = 0$	−200.0

In determining dead-load stresses in web members, it is customary to assume one-third of the dead panel loads as applied at the

upper chord points. This, as will be seen, makes no difference in the stresses in the chords or in the diagonals, the stresses in the verticals only being different from what is the case when all the dead load is taken on the lower chord.

The stresses in the diagonals (see Articles 27, 28, and 30) are:

Dead-Load Stresses in Diagonals

Member	Section	Shear on Section	Stress Equation	Stress
L_0U_1	$o-o$	+105.0	$+105.0 + L_0U_1 \times 0.7685 = 0$	−136.70
U_1L_2	$a-a$	+ 75.0	$+ 75.0 - U_1L_2 \times 0.7685 = 0$	+ 97.60
U_2L_3	$b-b$	+ 45.0	$+ 45.0 - U_2L_3 \times 0.7685 = 0$	+ 58.60
U_3L_4	$c-c$	+ 15.0	$+ 15.0 - U_3L_4 \times 0.7685 = 0$	+ 19.53

In determining the stresses in the verticals, it is to be remembered that one-third the dead panel load (or 10.0) is at the panel

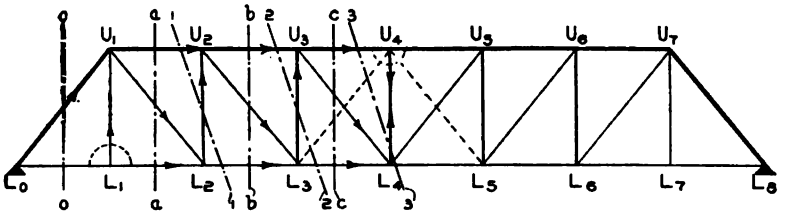


Fig. 42. Outline Diagram of 8-Panel Single-Track Pratt Truss Span.

points of the upper chord, and two-thirds (or 20.0) is at the lower chord. The stress in the hip vertical U_1L_1 is determined by passing a circular section around L_1. It is solved thus:

$$-20.0 + U_1L_1 = 0 \qquad U_1L_1 = +20.0$$

In a similar manner the stress in U_4L_4 is found to be:

$$-10 - U_4L_4 = 0 \qquad U_4L_4 = -10.0$$

In order to find the stress in the remaining verticals, sections 1 − 1 and 2 − 2 are passed, cutting them, and the shears on these sections computed. The shears are:

$$V_{1-1} = + 105.0 - 2 \times 20 - 1 \times 10 = +55.0$$
$$V_{2-2} = + 105.0 - 3 \times 20 - 2 \times 10 = +25.0$$

The stress equations are written, remembering that as the verticals

make an angle of zero with the vertical, their cosine is equal to unity. These equations are:

$$+U_2L_2 + 55.0 = 0 \qquad U_2L_2 = -55.0$$
$$+U_3L_3 + 25.0 = 0 \qquad U_3L_3 = -25.0$$

The live-load chord stresses will be proportional to the dead-load chord stresses, as both loads cover the entire truss in exactly the same manner. The ratio of the panel loads by which the dead-load chord stresses are multiplied in order to get the live-load chord stresses, is $\frac{62\ 400}{30\ 000} = 2.08$, and the chord stresses are:

$$L_0L_1 = L_1L_2 = +\ 87.5 \times 2.08 = +182.0$$
$$L_2L_3 = +150.0 \times 2.08 = +312.0$$
$$L_3L_4 = +187.5 \times 2.08 = +390.0$$
$$U_1U_2 = -150.0 \times 2.08 = \ \ 312.0$$
$$U_2U_3 = -187\ 5 \times 2.08 = -390.0$$
$$U_3U_4 = -200.0 \times 2.08 = -416.0$$

As the entire bridge is to be loaded to get the maximum stress in L_0U_1, it is therefore equal to the dead-load stress *times* the above ratio; or $L_0U_1 = -136.70 \times 2.08 = -284.20$.

The maximum live-load stress in U_1L_1 is determined by passing a circular section around L_1, and is solved (see Fig. 43) from the equation:

$$+U_1L - 62.4 = 0 \quad \therefore U_1L_1 = +62.4$$

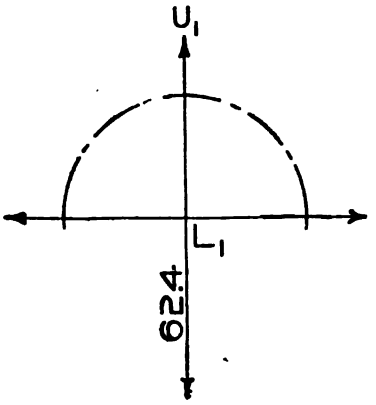


Fig. 43. Calculation of Maximum Live-Load Stress in a Vertical of Span of Fig. 42.

For U_1L_2, the section $a - a$ is passed, and the points L_2 and to the right are loaded. The maximum shear is:

$$+V_2 = +\frac{62.4}{8}(1 + 2 + 3 + 4 + 5 + 6) = +163.8;$$

and the stress equation is:

$$+163.8 - U_1L_2 \times 0.7685 = 0 \qquad \therefore U_1L_2 = +213.2.$$

In a similar manner, pass section $b - b$, and load points L_3 and to the right, and the shear and the stress equations for U_2L_3 are:

$$+V_3 = +\frac{62.4}{8}(1 + 2 + 3 + 4 + 5) = +117.0$$
$$+117.0 - U_2L_3 \times 0.7685 = 0 \qquad \therefore U_2L_3 = +152.4$$

For U_3L_4, the section $c - c$ is passed, and the panel points to the right are loaded. The shear and stress equations are:

$$+V_4 = +\frac{62.4}{8}(1 + 2 + 3 + 4) = +78.0$$

$$+78.0 - U_3L_4 \times 0.7685 = 0 \qquad \therefore U_3L_4 = +101.6$$

For the maximum stresses in the verticals, sections 1−1, 2−2, and 3−3 are passed, and in each case the panel points to the left of these loaded. The shears are

$$V_{1-1} = \frac{62.4}{8}(1 + 2 + 3 + 4 + 5) = +117.0$$

$$V_{2-2} = \frac{62.4}{8}(1 + 2 + 3 + 4) = +78.0$$

$$V_{3-3} = \frac{62.4}{8}(1 + 2 + 3) = +46.8$$

The stress equations for U_2L_2 and U_3L_3 are simple, as only three members are cut. They are:

$$+117.0 + U_2L_2 = 0 \qquad \therefore U_2L_2 = -117.0$$
$$+78.0 + U_3L_3 = 0 \qquad \therefore U_3L_3 = -78.0$$

It is seen that the section 3−3 cuts the member L_4U_5, and therefore the stress in this must be determined before the stress equation can be written, as its vertical component will enter into it. However, by comparing the dead-load shear in that panel, which is −15.0, and the live-load shear V_{3-3}, which is +46.8, it is seen that the resultant shear is +; and, as this is of opposite sign from the dead-load shear, a counter is required and is acting. The stress in U_5L_4 is zero, and the diagonals act as in Fig. 44, the section 3 − 3 then cutting three members. The stress equation is $+46.8 + U_4L_4 = 0$, from which $U_4L_4 = -46.8$.

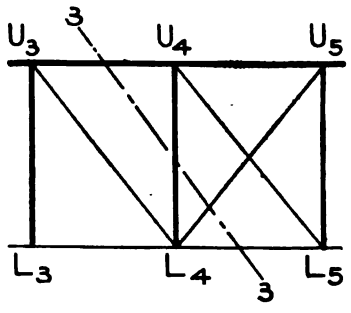


Fig. 44. Calculation of Stress in Diagonal of Span of Fig. 42.

Care should be taken not to add to this −46.8 the −10.0 derived as dead-load stress on page 44, in order to get the maximum stress, as the −10.0 previously derived was the dead-load stress in U_4L_4 when U_3L_4 and L_4U_5 were acting. The dead-load stress which goes with the live-load stress of −46.8 acts simultaneously with it,

and is the dead-load stress in U_4L_4 when the members U_3L_4 and U_4L_5 are acting as in Fig. 44. The dead-load shear on the section 3 – 3 would then be the left reaction *minus* the loads at points U_1, U_2, U_3, L_1, L_2, L_3 and L_4; or,

$$V_{3-3} = +105.0 - 3 \times 10 - 4 \times 20 = -5.0;$$

and

$$-5.0 + U_4L_4 = 0 \qquad \therefore U_4L_4 = +5.0.$$

Remember that this +5.0 can act only when the live load tends to produce a stress of −46.8; and thus the total stress in U_4L_4 with live load in that position is −46.8 + 5.0 = −43.8, while with dead load only in the truss the stress is −10.0.

The dead-load shears and the maximum + and − live-load shears should now be written for inspection, in order to investigate for counters and then for the minimum stresses. Those whose derivation has not been given should be easily computed by the student at this time. The shears are

Dead-Load		+ Live-Load	− Live-Load
V_1	+105.0	+218.4	0
V_2	+ 75.0	+163.8	− 7.8
V_3	+ 45.0	+117.0	−23.4
V_4	+ 15.0	+ 78.0	−46.8

From a study of these it is seen that a counter is required in the 4th panel according to rule *a*, Article 37; and according to rule *b* of the same article, the maximum stress is (−46.8 + 15.0) × 1.302 = +41.4, the minimum stress for it and also U_3L_4 being zero according to the same article. A counter is also required in panel 5, as the truss is symmetrical.

The minimum live-load stress in U_1L_1 is zero, and occurs when no live load is at the point L_1.

The minimum live-load stresses in the diagonals U_1L_2 and U_2L_3 occur when the truss is loaded successively to the left of the sections $a - a$ and $b - b$, in which case the shears are −7.8 and −23.4 respectively. The stress equations are

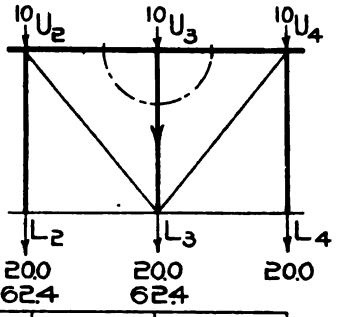
$$-U_1L_2 - 7.8 \times 0.7685 = 0 \qquad \therefore U_1L_2 = -10.16$$
$$-U_2L_3 - 23.4 \times 0.7685 = 0 \qquad \therefore U_2L_3 = -29.15$$

The minimum live-load stress in U_2L_2 is obtained by passing

section 1 — 1 and loading the panel points to the left. The live-load shear is the same at this section as it is at the section $b-b$ —namely, -23.4. The stress equation is

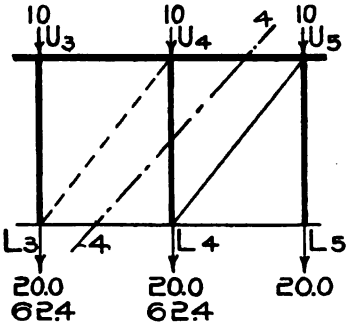
$$+U_2L_2 - 23.4 = 0. \qquad \therefore U_2L_2 = +23.4$$

To determine the minimum live-load stress in U_3L_3, proceed as indicated on page 42. By loading points L_3 and to the left, the live-load shear in the 4th panel will be -46.8, and in the 3d panel *under this same loading* it will be $-46.8 + 62.4 = +15.6$. The sign of the total shear in the two adjacent panels, and the members acting, are shown in Fig. 45. The stress in U_3L_3 is then determined by using a



d.l.v.	+ 45.0	+ 15.0
l.l.v.	+ 15.6	−46.8
Total v.	+	—

Fig. 45.



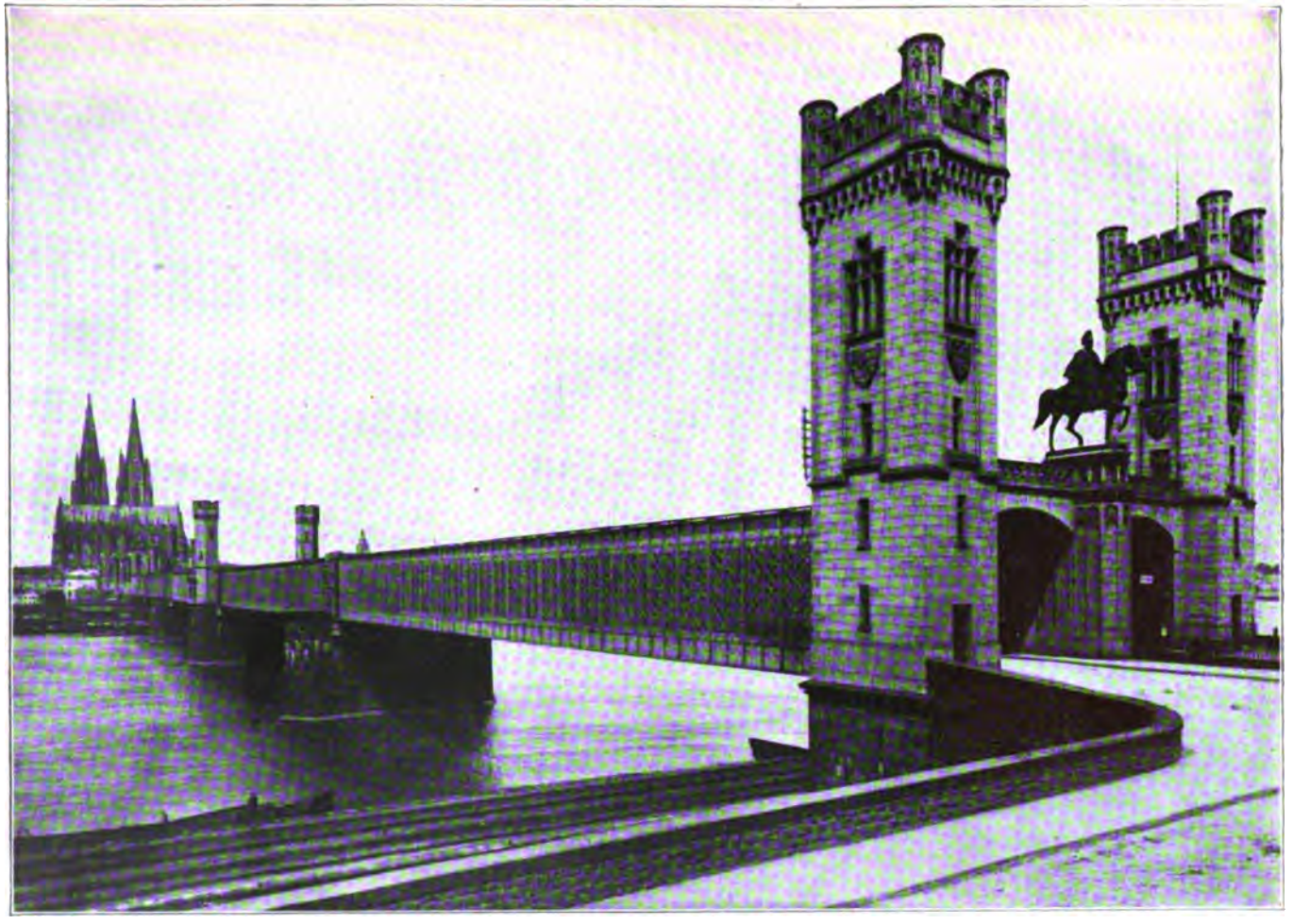
d.l.v.	+ 15.0	− 15.0
l.l.v.	− 15.6	−78.0
Total v.	—	—

Fig. 46.

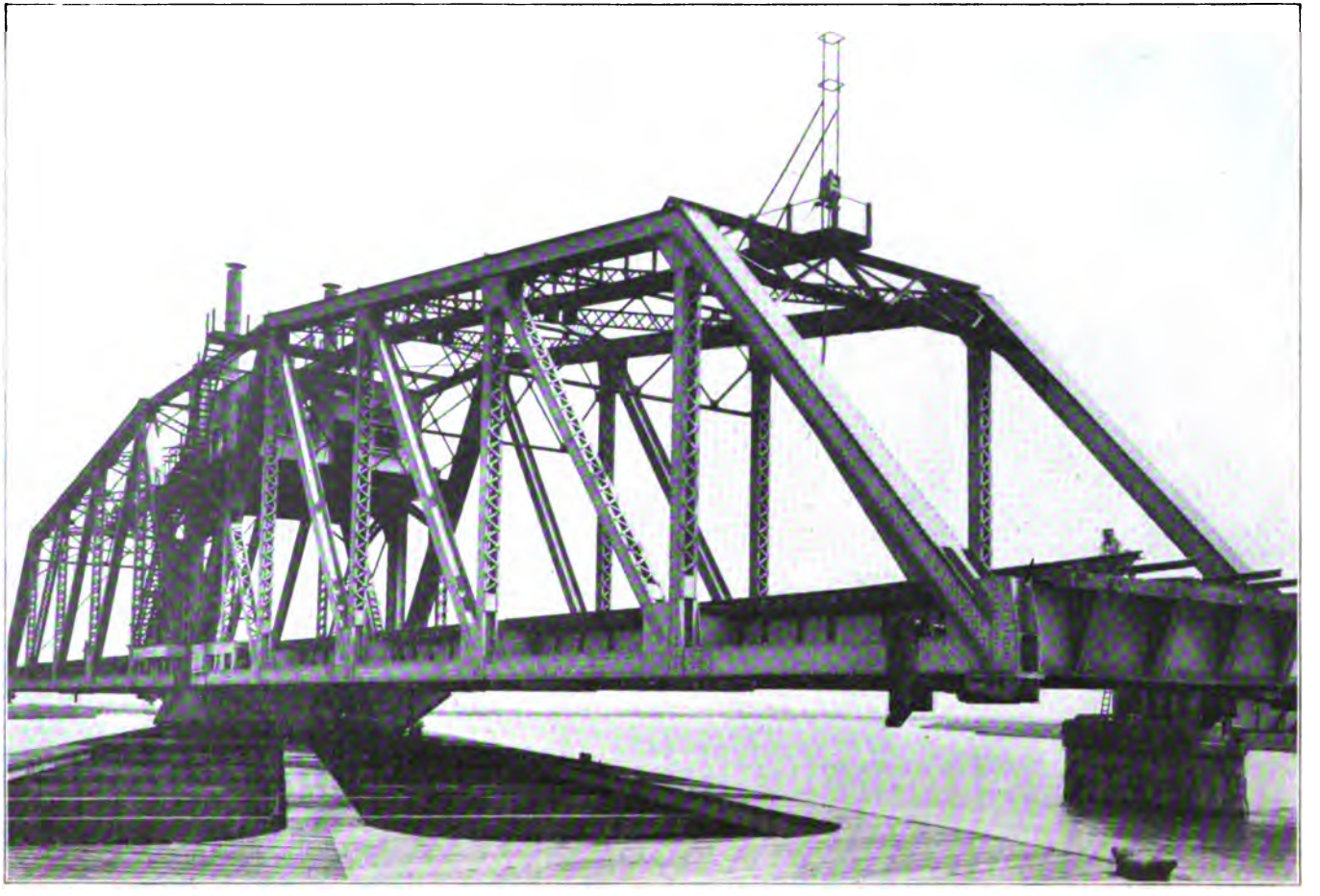
Stress Diagrams for Verticals in Span of Fig. 42.

circular section around U_3, and is simply the dead load at U_3, there being no live-load stress in the member when the bridge is loaded as has been done.

In finding the minimum live-load stress and also the minimum stress in U_4L_4, the same method of procedure will be followed. Let L_4 and to the left be loaded. Then the shear in the 5th panel is -78.0, and *under this same loading* the shear in the 4th panel is $-78.0 + 62.4 = -15.6$. The sign of the total shear in each of the adjacent panels is given in Fig. 46. It should be remembered that a resultant shear with the same sign as the dead-load shear causes the main diagonal to act, while a resultant shear of opposite sign to that of the dead-load shear causes the counter to act. The members



BRIDGE OVER THE RIVER RHINE AT COLOGNE, GERMANY
The famous Cologne Cathedral is shown at left, in background.



BRIDGE OVER THE PASSAIC RIVER AT NEWARK, NEW JERSEY

On line of Pennsylvania Railroad. Built in 1901. Consists of the following double-track spans: One riveted draw, 277 ft.; one deck plate-girder, 80 ft.; and four deck plate-girder spans, each 100 ft.

Courtesy of American Bridge Company.

acting are shown, and a section 4 — 4 can be passed. The dead-load shear at this section is $105 - 3 \times 20 - 4 \times 10 = +5.0$; and accordingly,

$$-U_4L_4 + 5.0 = 0.$$

Therefore,

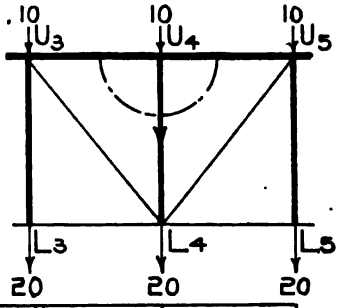
$$U_4L_4 = +5.0 = \text{Dead-load stress in this case.}$$

The live-load stress which acts at the same time is:

$$-U_4L_4 - 15.6 = 0 \qquad \therefore U_4L_4 = -15.6,$$

the term -15.6 representing the live-load shear on the section 4 — 4. This is not the minimum stress, as will next be shown, but it illustrates the fact that the loading for minimum live-load shears does not always give the minimum live-load stress.

By loading L_1, the live-load shear in the second panel, and likewise all others from this to the right support, will be -7.8. The total shears, together with their sign, and also the members they cause to act, are given in Fig. 47. The minimum live-load stress in U_4L_4 is found to be zero, and the dead-load stress is -10, as is derived by passing a circular section around U_4, the equation being as follows:



d.l.v.	+ 15.0	−15.0
l.l.v.	− 7.8	− 7.8
Total v.	+	−

Fig. 47. Stress Diagram for Vertical in Span of Fig. 42.

$$-\text{Live load at } U_4 - U_4L_4 = 0 \qquad \therefore U_4L_4 = 0 \text{ for live load.}$$
$$-\text{Dead load at } U_4 - U_4L_4 = 0 \qquad \therefore U_4L_4 = -10.0 \text{ for dead load.}$$

A diagram of half the truss should now be made, and all dead and live load stresses placed upon it, and these should be combined so as to form the maximum and the minimum stresses. Such a diagram, together with all stresses, is given in Fig. 48.

The stresses are written in the following order: Dead load, maximum live load, minimum live load, the maximum, and the minimum. In the chord and end-post stresses, there is no minimum live-load stress recorded, it being zero. Where pairs of stresses occur simultaneously, a bent arrow connects them.

40. The Howe Truss. The physical make-up of the Howe truss

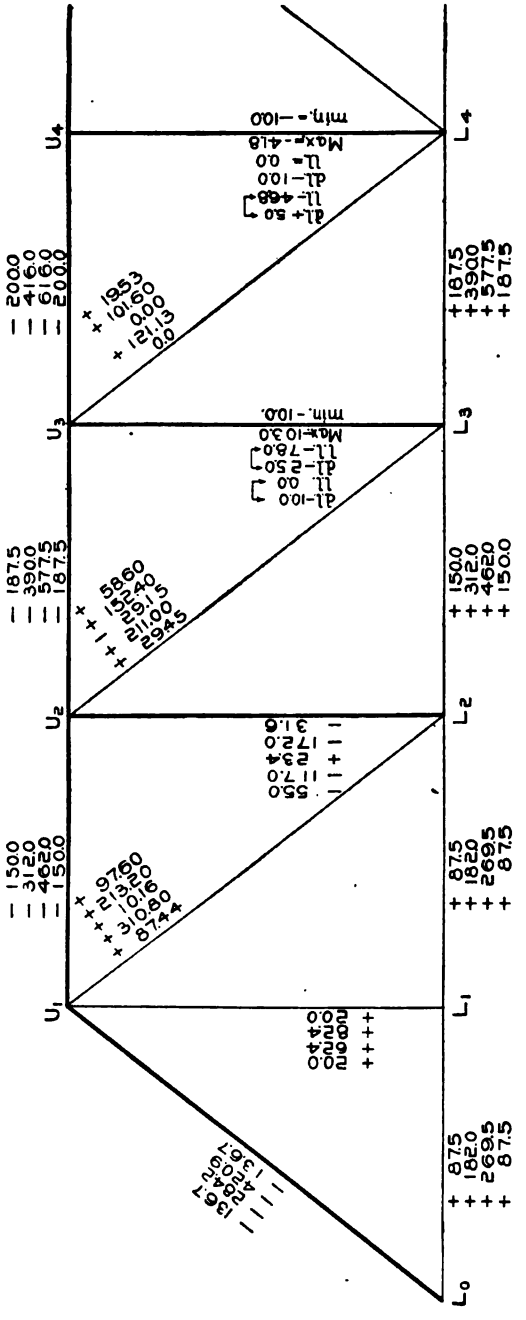


Fig. 48. Stress Diagram of Half-Span of Fig. 42, Pratt Truss, Single Track.

differs from that of the Pratt in that the diagonals are made to stand compression only, and the verticals can stand tension only. In the Pratt truss it was found that none of the intermediate posts could be brought into tension by any loading. In the Howe truss it will be found that none of the verticals can be brought into compression.

Let it be required to determine the stresses in a Howe truss of the same span, height, and loading as the Pratt truss of Article 39. An outline diagram is given in Fig. 49.

The dead-load shears and the maximum and minimum live-load shears will be the same as for the Pratt truss, and they are:

Dead-Load V	+ Live-Load V	− Live-Load V
V_1 +105.0	+218.4	− 0
V_2 + 75.0	+163.8	− 7.8
V_3 + 45.0	+117.0	−23.4
V_4 + 15.0	+ 78.0	−46.8
V_5 − 15.0	+ 46.8	−78.0

Inspection of these shows that counters are required in the 4th and 5th panels (see Article 37).

The dead-load lower chord stresses will be computed by the

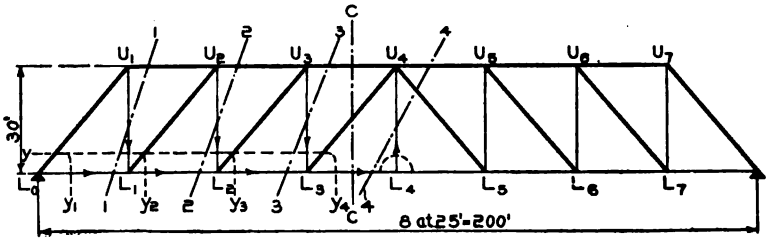


Fig. 49. Outline Diagram of 8-Panel Single-Track Howe Truss Span.

tangent method (see Article 31), the section being $y - y_1$, etc. The tangent of ϕ is $25 \div 30 = 0.8333$. The stresses may be conveniently tabulated as follows:

Dead-Load Chord Stresses (Lower Chord)

Member	Section	Stress Equation	Stress
L_0L_1	$y - y_1$	$-105.0 \times 0.8333 + L_0L_1 = 0$	+ 87.5
L_1L_2	$y - y_2$	$-(105.0 + 75.0)\,0.8333 + L_1L_2 = 0$	+150.0
L_2L_3	$y - y_3$	$-(105.0 + 75.0 + 45)\,0.8333 + L_2L_3 = 0$	+187.5
L_3L_4	$y - y_4$	$-(105.0 + 75.0 + 45.0 + 15.0)\,0.8333 + L_3L_4 = 0$	+200.0

A simple method for the determination of the upper chord stresses, is to pass a section and to equate the sum of the horizontal forces. Pass section 1 − 1. The only horizontal forces are the stresses in L_0L_1 and U_1U_2; and as these are parallel, one must be equal and opposite to the other. In a like manner the stresses in the other sections of the top chord are found. The stresses are:

$$U_1U_2 = -L_0L_1 = -(+\ 87.5) = -\ 87.5$$
$$U_2U_3 = -L_1L_2 = -(+150.0) = -150.0$$
$$U_3U_4 = -L_2L_3 = -(+187.5) = -187.5$$

A consideration of the Pratt truss shows that this method can be applied to it in determining the chord stresses.

As it is known that the diagonal web members are in compression under the dead load which produces a positive shear in the left half of the truss, it is evident that positive live-load shears will produce compressive stresses, and negative live-load shears tensile stresses, in the diagonals in the left half of the truss. Also, from Article 30, the stress in a diagonal is $V \sec \phi$. The stresses can now be written directly without the aid of the stress equation:

$$L_0U_1 = -105.0 \times 1.302 = -136.70$$
$$L_1U_2 = -\ 75.0 \times 1.302 = -\ 97.60$$
$$L_2U_3 = -\ 45.0 \times 1.302 = -\ 58.60$$
$$L_3U_4 = -\ 15.0 \times 1.302 = -\ 19.53$$

Likewise the stresses in the verticals can be written directly, remembering that here the secant is unity, and that the shear at the section cutting the member is to be used, not forgetting that $\frac{1}{3}$ of the dead panel load is applied at the top panel points. The shears and stresses are:

$$V_{1-1} = +105.0 - 10 = +95.0 \qquad U_1L_1 = +95.0$$
$$V_{2-2} = +105.0 - 20 - 2 \times 10 = +65.0 \qquad U_2L_2 = +65.0$$
$$V_{3-3} = +105.0 - 2 \times 20 - 3 \times 10 = +35.0 \qquad U_3L_3 = +35.0$$

The member U_4L_4 cannot be easily determined by passing a section 4 – 4, for this cuts four members. It is determined by passing a circular section about the point L_4, the equation being $+U_4L_4 - 20.0 = 0$, from which $U_4L_4 = +20.0$, which is equal to the dead panel load at the point L_4.

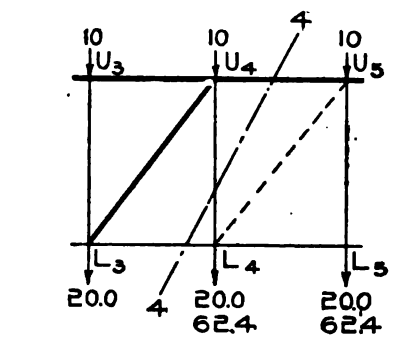
The live-load chord stresses are determined by multiplying the dead-load chord stresses by the ratio of the live to the dead loads. This has been found to be equal to 12.08. The live-load chord stresses are found to be:

$$L_0L_1 = +182.0 \qquad U_1U_2 = -182.0$$
$$L_1L_2 = +312.0 \qquad U_2U_3 = -312.0$$
$$L_2L_3 = +390.0 \qquad U_3U_4 = -390.0$$
$$L_3L_4 = +416.0$$

As the character of the stresses which can be taken by the diagonals and the verticals is known, the maximum and minimum live-load stresses can be written without first writing the stress equations. The maximum live-load stresses are:

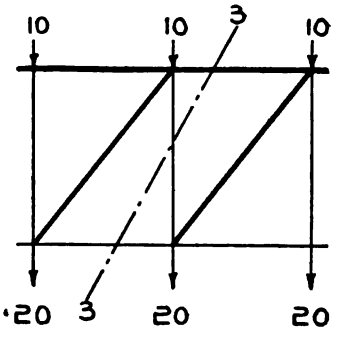
$$L_0U_1 = -218.4 \times 1.302 = -284.36 \qquad U_1L_1 = +218.4$$
$$L_1U_2 = -163.8 \times 1.302 = -213.27 \qquad U_2L_2 = +163.8$$
$$L_2U_3 = -117.0 \times 1.302 = -152.33 \qquad U_3L_3 = +117.0$$
$$L_3U_4 = -\ 78.0 \times 1.302 = -101.56 \qquad U_4L_4 = +\ 78.0$$

It should be noted that when L_4 and all panel points to the right are loaded, the shears and the members acting are as shown in Fig. 50. The dead-load shear on the section $4-4$ is $+15.0$, less the load at U_4, or $+15.0 - 10.0 = +5.0$; and the equation of stress is $-U_4L_4 + 5.0 = 0$, from which $U_4L_4 = +5.0$. Thus it is seen that in this



d.l.v.	+ 15.0	−15.0
l.l.v.	− 7.8	+15.6
Total v.	+	+

Fig. 50.



d.l.v.	+ 45.0	+ 15.0
l.l.v	− 7.8	− 7.8
Total v.	+	+

Fig. 51.

Stress Diagrams for Members of Howe Truss Span of Fig. 49.

case the dead-load stress is $+5.0$ when the live-load stress is $+78.0$.

The maximum stresses in the counters (see Article 37) are:

$$(-46.8 + 15.0)\ 1.302 = -41.4.$$

The minimum live-load stresses are now written as follows:

$$L_1U_2 = +\ 7.8 \times 1.302 = +10.16 \qquad U_1L_1 = 0$$
$$L_2U_3 = +23.4 \times 1.302 = +29.15 \qquad U_2L_2 = -7.8$$
$$L_3U_4 = 0 \qquad U_3L_3 \left\{ \text{See discussion following.} \right.$$
$$U_3L_4 = 0 \qquad U_4L_4 \left\{ \right.$$

If live panel loads were placed at points L_1, L_2, and L_3 the live-load shear in $c - c$ would be -46.8; and the dead-load shear being $+15$, the counter would act, and the stress in U_3L_3 would be tensile and equal to the sum of the dead and live panel loads which are at its lower end L_3. If points L_1 and L_2 had live panel loads on them,

the resultant shear in $c - c$ would be $-23.4 + 15.0 = -8.4$; the counter would act, and the stress in U_4L_4 would be tensile and equal to the dead panel load which is at L_3. There being no live panel load at L_3, the live-load stress in U_3L_3 would be zero under this loading. If a live panel load be placed at L_1 only, then the shears and the members acting will be as shown in Fig. 51, and V_{3-2} for dead load = $+45.0$ − the load at U_3, or $= 45 - 10 = +35.0$. The V_{3-3} for live load $= -7.8$, and the stress equation $- U_3L_3 - 7.8 = 0$, from which $U_3L_3 = -7.8$. So this live-load compression stress of 7 800 pounds occurs at the same time as the dead-load tensile stress of 45 000 pounds.

By loading various groups of panel points in succession and determining the resulting live-load stresses in U_4L_4, it will be found that under no loading can a negative live-load stress be produced. The minimum live-load stress is therefore zero, and occurs when there is no live load on the bridge.

The stresses should now be placed on an outline diagram similar to that of Fig. 48, and the stresses in corresponding members compared with those in that figure. This is left for the student.

41. **Bowstring and Parabolic Trusses.** A bowstring truss is shown in Fig. 13, the full lines representing the main members, which are the members under stress by the dead load. The dotted members represent counters which may be stressed by the action of the live load.

As before mentioned, the stresses in the chords and also in the webbing are quite uniform. When the end supports and the panel points lie on the arc of a certain curve, called a parabola, then, under full load, the stresses in all panels of the lower chord are equal; the stress in all verticals is tensile and is equal to the panel load at the lower end; and the stress in all diagonals is zero. Under partial load, the stresses in the webbing are exceedingly small, and the chord stresses remain almost equal.

If it is desired to have a parabolic truss, first decide upon the length of span, the number of panels, and the height at the center. The height of any vertical post is given by the formula:

$$h = H - \frac{4Hd^2}{l^2},$$

in which,

H = Approximate height at center;
d = Distance of vertical post from center;
l = Span;
h = Height of vertical post sought.

All distances are in feet. Suppose, as an example, that it was desired to determine the heights of the vertical posts in an 8-panel parabolic truss of a height approximately equal to 24 feet. One-half

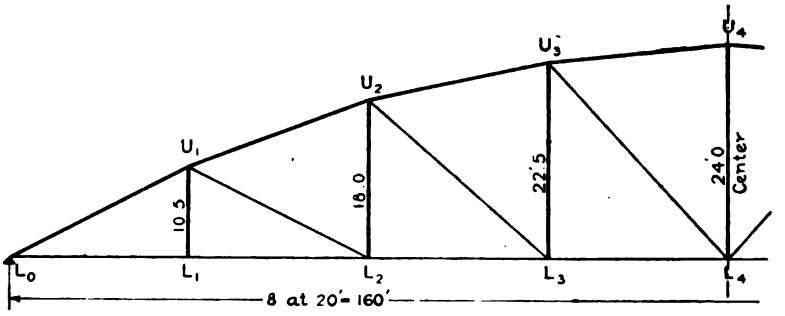


Fig. 52. One-Half of 8-Panel Parabolic Truss.

the truss is shown in Fig. 52. At the center, $d = 0$, and the equation reduces to $h = H$, which is 24 feet. For U_3L_3, $d = 20$; then,

$$h = 24 - \frac{4 \times 24 \times 20^2}{160^2},$$

from which,

$$h = 22.5 \text{ feet.}$$

For U_2L_2,

$$d = 40$$

$$h = 24 - \frac{4 \times 24 \times 40^2}{160^2} \qquad h = 18.0 \text{ feet.}$$

For U_1L_1,

$$d = 60$$

$$h = 24 - \frac{4 \times 24 \times 60^2}{160^2} \qquad h = 10.5 \text{ feet.}$$

Inspection of the above results shows that the span or the center height must become quite great before the clearance at U_1L_1 will be sufficient to allow the traffic to pass under a portal bracing at this point. For this reason these trusses are usually built as through trusses with bracing on the outside of the truss, which connects to the floor-beams extended.

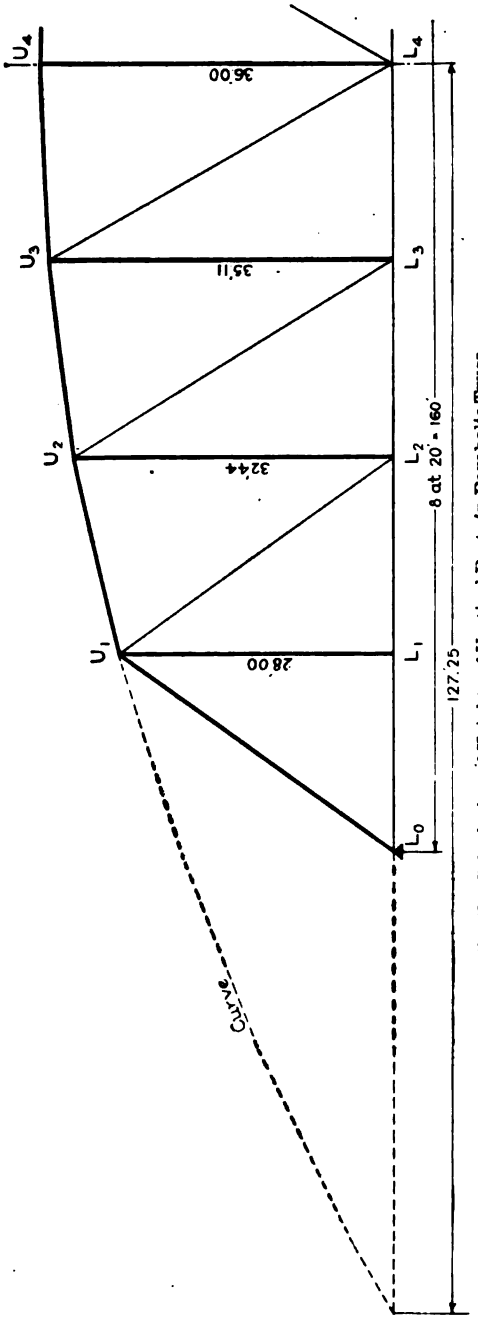


Fig. 53. Calculation of Heights of Vertical Posts in Parabolic Truss.

In the bowstring truss, the panel points of the top chord usually lie on the arc of a parabola which does not pass through the supports. For example, suppose that it was decided to have the span and panels the same as shown in Fig. 52, but the height at L_1 was to be 28 feet, and at L_4 36 feet. By substituting these values in the equation just given, and solving for l, the place will be determined where the parabolic curve cuts the lower chord extended, and the lengths of the vertical posts may be computed as before. Substituting these results:

$$28 = 36 - \frac{4 \times 36 \times \overline{60^2}}{l^2}$$

$$(-36 + 28)\, l^2 = -4 \times 36 \times 60^2$$

$$l = \sqrt{\frac{4 \times 36 \times 60^2}{8}}$$

$$= 254.5,$$

which shows that the arc cuts the lower chord extended at a point $254.5 \div 2 = 127.25$ feet from the center of the span (see Fig. 53).

The other vertical posts are:

U_3L_3 $\quad h = 36 - \dfrac{4 \times 36 \times 20^2}{254.5^2} = 35.11$ feet;

U_2L_2 $\quad h = 36 - \dfrac{4 \times 36 \times 40^2}{254.5^2} = 32.44$ feet;

U_1L_1 $\quad h = 36 - \dfrac{4 \times 36 \times 60}{254.5^2} = 28.00$ feet, which checks.

The analysis of a bowstring truss will now be given. Both the maximum and minimum stresses will be determined, as reversal of

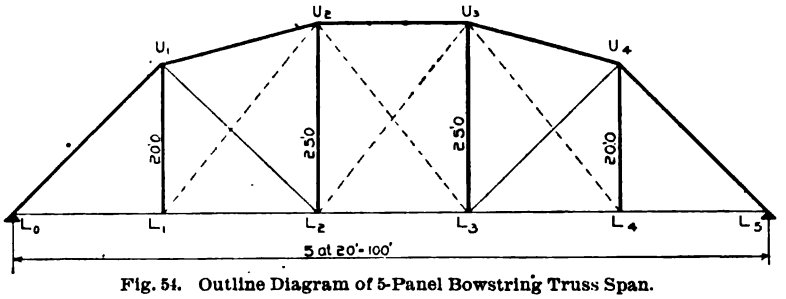


Fig. 54. Outline Diagram of 5-Panel Bowstring Truss Span.

stresses is liable to occur in the intermediate posts. The loading for minimum live-load stresses can be ascertained only by trial, care being taken to compute the dead-load stresses for the arrangement of web members caused by that particular live loading.

Let it be required to determine the maximum stresses in the 5-panel 100-foot bowstring truss shown in Fig. 54, remembering that the diagonals take only tension. The height of U_1L_1 is 20 feet, and of U_2L_2 25 feet. The dead panel load is 17 200 pounds, and the live panel load is 50 000 pounds. The full lines show the main members which act under dead-load stress, and the dotted lines show the counters which may act under the action of the live load. One-third of the dead panel load, or 5 730 pounds, is taken as acting at the upper panel points, while the remainder, 11 470 pounds, acts at the lower

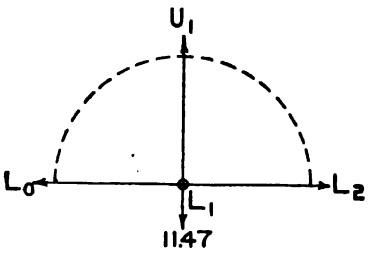


Fig. 55. Resolution of Forces around Panel Point in Bowstring Truss of Fig. 54.

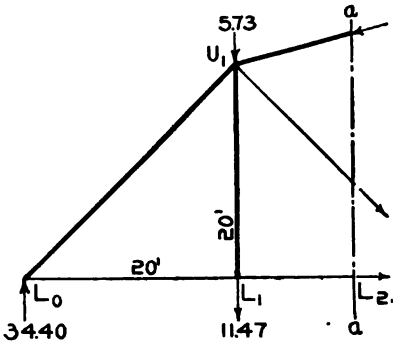


Fig. 56.

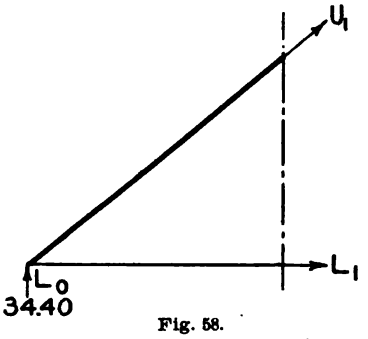


Fig. 58.

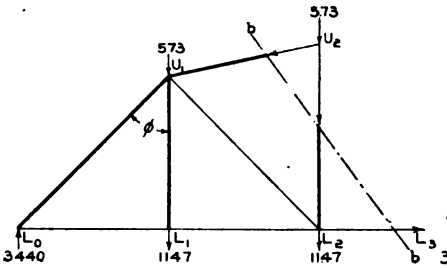


Fig. 57.

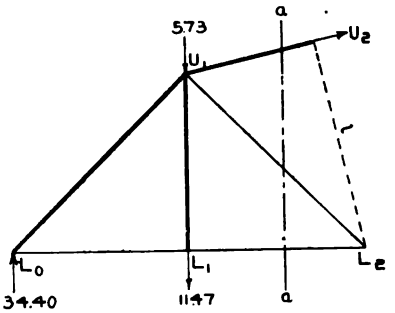


Fig. 59.

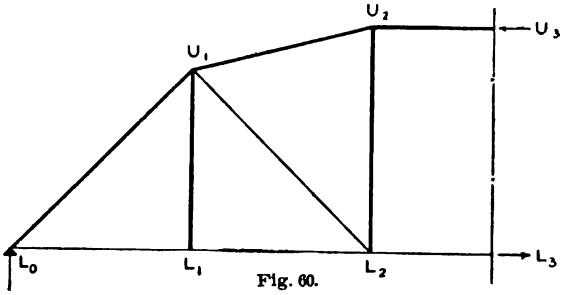


Fig. 60.

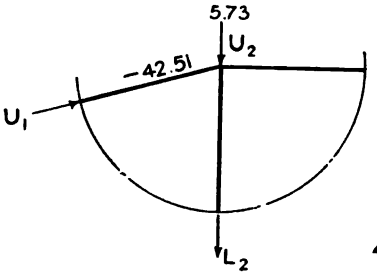


Fig. 61.

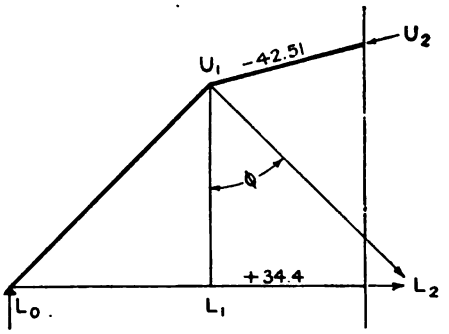


Fig. 62.

Analysis of Stresses in Various Members of the Bowstring Truss of Fig. 54.

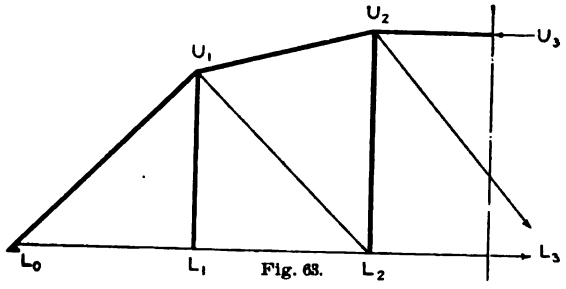


Fig. 63.

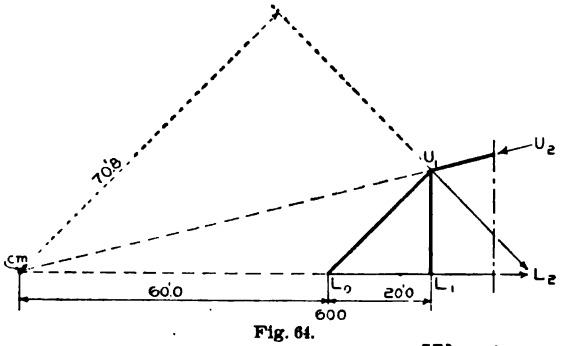


Fig. 64.

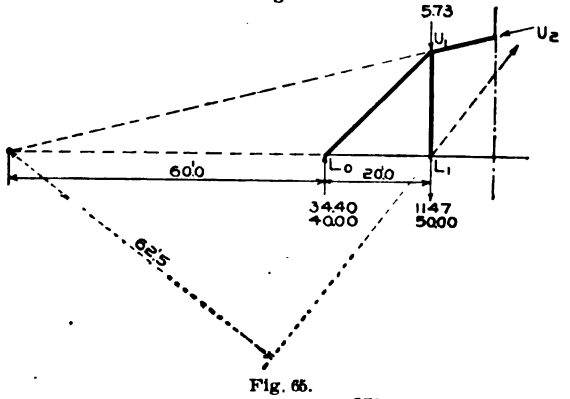


Fig. 65.

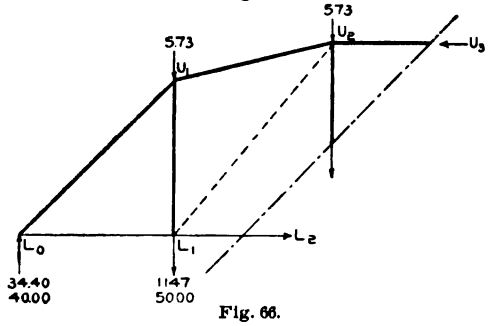


Fig. 66.

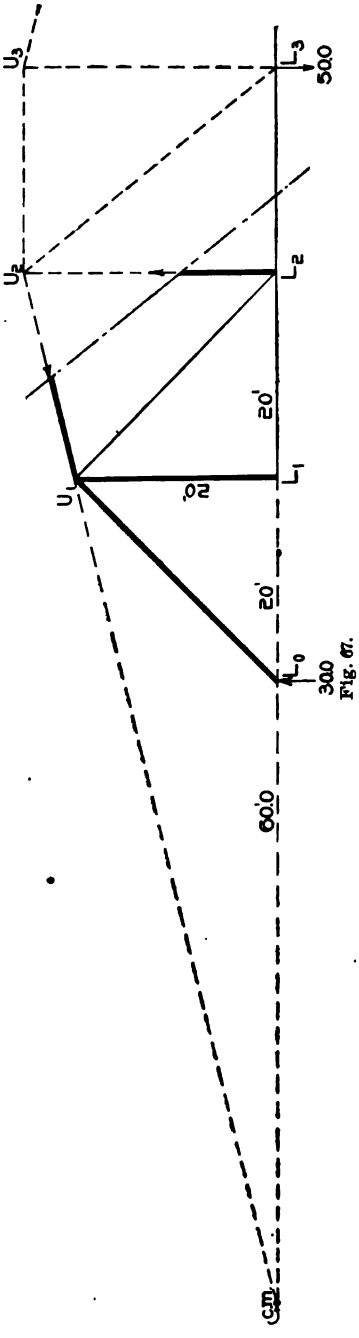


Fig. 67.

Analysis of Stresses in Various Members of the Bowstring Truss of Fig. 54.

ones. Articles 27, 28, and 29 should be carefully reviewed before going further. The shear times the secant method cannot be conveniently employed for the live-load stresses in the members U_1L_2 and L_1U_2, as the section will cut the member U_1U_2, and the vertical component of its stress must be reckoned with in the stress equation The method of moments as illustrated in Fig. 27, Article 29, will be used for these members.

The dead-load reaction is $2 \times 17.2 = +34.4$. The dead-load chord stresses should first be computed.

By resolving the horizontal forces around L_1, it is seen that $L_0L_1 = L_1L_2$ (see Fig. 55). Passing the section $a - a$, taking the center of moments, at U_1, and stating the equation of the moments to the left of the section, there results (see Fig. 56):

$$+34.4 \times 20 - L_1L_2 \times 20 = 0 \qquad \therefore L_1L_2 = +34.4$$

For L_2L_3, the section $b - b$ is passed; the center of moments is at U_2; and the equation of the moments to the left of this section (see Fig. 57) is:

$$+34.4 \times 2 \times 20 - (11.47 + 5.73)\,20 - L_2L_3 \times 25 = 0 \quad \therefore L_2L_3 = +41.26.$$

By passing a vertical section cutting L_0L_1 and L_0U_1, the stress in L_0U_1 can be determined by taking the sum of the vertical forces to the left and equating them to the vertical component of the stress (see Fig. 58). The equation is:

$$+34.4 + L_0U_1 \times 0.707 = 0, \text{ from which } L_0U_1 = -34.4 \times 1.414 = -48.7.$$

A section $a - a$ (Fig. 59) shows that the center of moments for U_1U_2 is at U_2; and stating the moments of the stress, and the forces to the left of the section, there results an equation in which an unknown lever arm enters. This lever arm l is readily computed to be 24.28 feet, and the equation can now be written:

$$+34.4 \times 2 \times 20 - (11.47 + 5.73)\,20 + U_1U_2 \times 24.28 = 0$$
$$\therefore U_1U_2 = -42.51.$$

The stress in U_2U_3 is determined by passing a vertical section in the 3d panel, and taking the sum of the horizontal forces. As there is no dead-load stress in the members L_2U_3 and U_2L_3, their components will be zero. Therefore (see Fig. 60) it is evident that U_2U_3 must be equal and opposite to L_2L_3 and will be equal to -41.26.

By reference to Fig. 55, the stress in L_1U_1 is seen to be tensile and equal to $+11.47$.

Pass a circular section around U_2 and take the sum of the vertical components, assuming that the stress in U_2L_2 acts away from the section. The length of U_1U_2 is $\sqrt{5^2 + 20^2} = 20.6$, and therefore the vertical component of U_1U_2 will be $(42.51 \div 20.6) \times 5 = 10.32$, which acts upward. The stress equation of U_2L_2 (see Fig. 61) is:

$$+10.32 - 5.73 - U_2L_2 = 0 \qquad \therefore U_2L_2 = +4.59,$$

showing that a tensile stress occurs in U_2L_2 when all panel points are loaded.

The simplest method of ascertaining the stress in U_1L_2 is to pass a vertical section cutting members as shown in Fig. 62, and to equate the horizontal forces and stresses. The horizontal component of U_1U_2 is:

$$\frac{42.51}{20.6} \times 20 = 41.30, \text{ which acts toward the left.}$$

The equation of stress is, then:

$$-41.30 + 34.40 + U_1L_2 \sin \phi = 0; \text{ but } \sin \phi = 0.707;$$
$$U_1L_2 = +6.90 \times 1.414$$
$$= +9.76$$

All the dead-load stresses being computed, the next operation will be to determine the live-load chord stresses. These are proportional to the dead-load stresses in the same ratio as the live panel load is to the dead panel load. This ratio is $50 \div 17.2 = 2.907$, and the chord and end-post live-load stresses are:

$$L_0U_1 = -48.71 \times 2.907 = -141.7$$
$$U_1U_2 = -42.51 \times 2.907 = -123.6$$
$$U_2U_3 = -42.26 \times 2.907 = -123.0$$
$$L_0L_2 = +34.40 \times 2.907 = +100.2$$
$$L_2L_3 = +41.26 \times 2.907 = +120.3$$

Also, the stress in U_2L_2 when the live load covers the entire bridge is *not* 2.907×4.59, as it must be remembered that part of the dead load is at the panel points of the upper chord. Taking a circular section around U_2 (see Fig. 61), and noting that there is no load at U_2, it is seen that the stress in U_2L_2 due to live load is simply equal to the vertical component of the live-load stress of U_1U_2 and will be tensile. It is:

$$U_1U_2 = (123.6 \div 20.6) \times 5 = +30.0.$$

The maximum live-load stress in U_1L_1 is tensile, and equal to the live panel load at L_1 (see Fig. 55).

To obtain the maximum stress in U_2L_3, load L_3 and L_4. The shear V_3 will then be $\frac{50}{5}(1 + 2) = +30.0$. The section will cut the members as shown in Fig. 63, and the equation of stress will be:

$$+30.0 - U_2L_3 \cos\phi = 0; \text{ but } \cos\phi = \frac{25}{\sqrt{\overline{20}^2 + \overline{25}^2}} = 0.782;$$

$$\therefore U_2L_3 = +38.4.$$

If panel points L_1 and L_2 were loaded, it is evident that the stress in L_2U_3 would be $+38.4$.

To obtain the maximum live-load stress in U_1L_2, a section is passed cutting U_1U_2, L_1L_2, and U_1L_2 (Fig. 64). The center of moments will be at the intersection of U_1U_2 and L_1L_2, and this point lies some place to the left of the support L_0. The lever arm of U_1L_2 will be the perpendicular distance from this point to the line U_1L_2 extended. The panel points L_2, L_3, and L_4 are loaded. The left reaction is then $(1 + 2 + 3)\frac{50}{5} = +60.0$. The lever arms are easily computed, and these, together with the members cut, are shown in Fig. 64. The equation of stress is:

$$-60.0 \times 60.0 + U_1L_2 \times 70.8 = 0 \qquad \therefore U_1L_2 = +50.80.$$

If a load were put on L_1 only, then the reaction at L_0 would be $\frac{4}{5} \times 50 = 40$; and the equation of stress would then be:

$$-40.0 \times 60.0 + 50 \times (60.0 + 20.0) + U_1L_2 \times 70.8 = 0 \quad \therefore U_1L_2 = -22.6.$$

As this is compression and greater than the dead-load stress, $+9.76$, a counter is required in that panel. In order to get the stress in the counter, it must be inserted, U_1L_2 being left out, and the dead and live load stresses computed and their difference taken. Fig. 65 gives the lever arms, center of moments, and the forces acting in this case.

The dead-load stress is:

$$-34.4 \times 60.0 + (11.47 + 5.73)(60 + 20) - L_1U_2 \times 62.5 = 0$$
$$\therefore L_1U_2 = -11.02;$$

and the live-load stress is:

$$-40 \times 60 + 50(60 + 20) - L_1U_2 \times 62.5 = 0 \qquad \therefore L_1U_2 = +25.60,$$

and the stress in the counter is the algebraic sum of these two, or $-11.02 + 25.60 = +14.58$.

When a live panel load is at L_1, L_1U_2 is acting, as has just been proved. As this load at L_1 causes a negative shear in all panels to the right, this negative shear in the center panel will cause L_2U_3 to act. A section may now be passed as shown in Fig. 66, and the stress equations for U_2L_2 written:

For dead load, $+34.4 - 11.47 - 2 \times 5.73 - U_2L_2 = 0 \quad \therefore U_2L_2 = +11.47$
For live load, $+40.0 - 50.0 - U_2L_2 = 0 \quad \therefore U_2L_2 = -10.00$
Total $= +1.47$

This is evidently not a maximum for U_2L_2, for when a full live load was on the span, the stress was $+30.0$ due to live load and $+4.59$ due to dead load.

It might be well to consider what effect is produced by loading L_3 and L_4. The loading of L_2 and L_1 need not be considered, since it is evident that, as this causes the total shear in panel 2 to be positive and the total shear in panel 3 to be negative, therefore U_1L_2 and L_2U_3

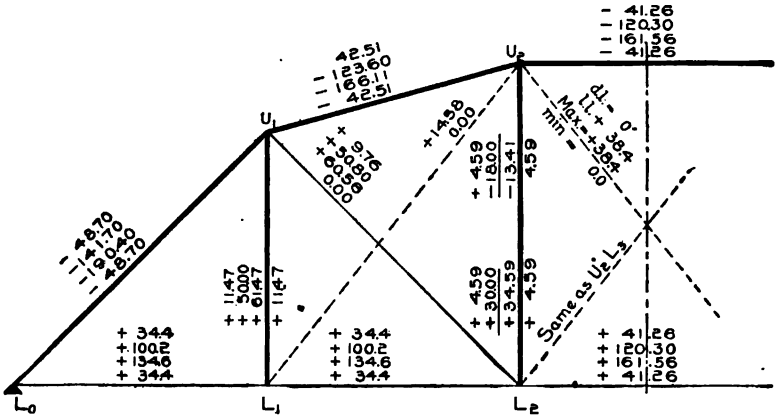


Fig. 68. Stress Diagram of Half-Span of Parabolic Truss of Fig. 54.

will act, and this causes a tensile stress in U_2L_2 equal to the vertical components of the dead and live load stresses in U_1U_2 less the dead panel load at U_2. With a live panel load at L_3 and L_4, the left reaction is $\frac{50}{5}(1 + 2) = +30.0$. The section, the live-load forces, the center of moments, and the members acting are shown in Fig. 67. The dead-load stress in U_2L_2 will be the same as when the truss has no live load on it. The stress equation for the live load is:

$$-60 \times 30 - (60 + 20 + 20) \times L_2U_2 = 0 \qquad \therefore L_2U_2 = -18.0.$$

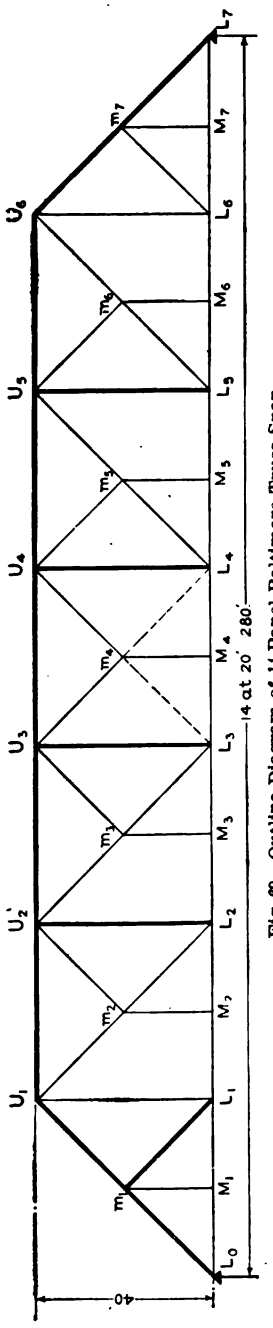


Fig. 69. Outline Diagram of 14-Panel Baltimore Truss Span.

The dead-load stress being +4.59, this stress of −18.0 causes a reversal of stress in the vertical. For this reason the verticals of bowstring trusses are, like web members of Warren trusses, built so as to take either tension or compression. The minimum stresses in the diagonals will be zero, for when one diagonal in a panel is acting, the other is not.

The diagram of half of the truss in Fig. 68 gives all the stresses.

It is to be noted by the student, that in some cases one method for the determination of stresses is preferable to others in that it saves labor of computation. The analysis of the truss of Fig. 68 illustrates this fact.

42. **The Baltimore Truss.** Baltimore trusses are of two classes—those in which the half-diagonals, called *sub-diagonals*, are in compression, and those in which the sub-diagonals are in tension. The latter class is the one most usually built, as it is more economical on account of many of its members being in tension, in which case these members are cheaper and easier to build than if they were compression members. Fig. 14 shows both types of truss. The Baltimore truss does not have a simple system of webbing, and for that reason the analysis is here presented. As the tension sub-diagonal truss is the type in most common use, its analysis will be given.

Let it be required to compute the maximum stresses in the 14-panel 280-foot span of Fig. 69. The height is 40 feet, the dead panel load 24 000 pounds,

and the live panel load 40 000 pounds. One-third of the dead panel load is applied at the upper ends of the long verticals and also of the half-verticals. These half-verticals are designated as *sub-verticals*. Attention is called to the system of notation used for the ends of the sub-verticals. The full lines in Fig. 69 represent the main members, being stressed by dead load only. The heavy lines indicate those members that take compression, the light lines those that take tension, and the broken lines the counter-braces. In this, as in nearly all Baltimore trusses, the diagonals make an angle of 45 degrees with the vertical.

The dead and the positive live-load shears in the various panels should be computed. They are:

Dead-Load V		+ Live-Load V	
V_1	+156.00	$V_1 = (1 + \ldots 13)\frac{40}{14} =$	+260.00
V_2	+132.00	$V_2 = (1 + \ldots 12)\frac{40}{14} =$	+223.00
V_3	+108.00	$V_3 = (1 + \ldots 11)\frac{40}{14} =$	+188.50
V_4	+ 84.00	$V_4 = (1 + \ldots 10)\frac{40}{14} =$	+157.20
V_5	+ 60.00	$V_5 = (1 + \ldots 9)\frac{40}{14} =$	+128.50
V_6	+ 36 00	$V_6 = (1 + \ldots 8)\frac{40}{14} =$	+102.80
V_7	+ 12.00	$V_7 = (1 + \ldots 7)\frac{40}{14} =$	+ 80.00

It is only necessary to determine the negative live-load shear in panels 5 and 7, in order to ascertain if there is a counter required. These shears are:

$$-V_5 = (10 + 11 + 12 + 13)\frac{40}{14} - 4 \times 40 = -28.60$$

$$-V_7 = (8 + 9 + 10 + 11 + 12 + 13)\frac{40}{14} - 6 \times 40 = -60.00$$

From a comparison of these with the dead-load shears, it is seen (see Article 37) that a counter is required in panel 7 only.

The dead-load stresses are first to be computed. The stress in any sub-vertical is found by passing a circular section around its lower end, and equating the sum of the vertical forces, assuming in this, as in all cases, that the unknown stress acts away from the

section. Take M_1m_1, for example. Fig. 70 gives the section, the forces acting, and the members cut. Then,

$$+M_1m_1 - 16.0 = 0 \qquad \therefore M_1m_1 = +16.0$$

As all sub-verticals have the same dead load at their lower end, it follows that the dead-load stress in all sub-verticals is the same, a tensile stress of 16 000 pounds.

The dead-load stresses in the sub-diagonals are determined by resolving the forces around the joint at their lower end. The components perpendicular to the diagonal are taken (see Fig. 71). Take

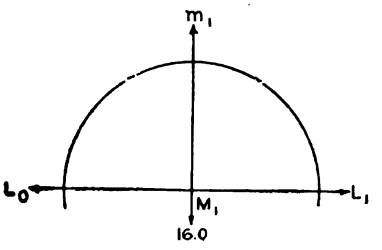


Fig. 70. Diagram for Calculating Stress in Sub-Vertical of Baltimore Truss.

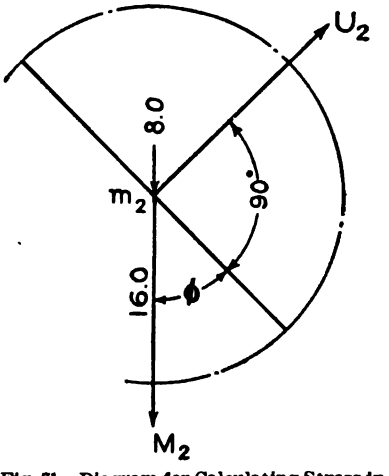


Fig. 71. Diagram for Calculating Stress in Sub-Diagonal of Baltimore Truss.

m_2U_2. The known forces or stresses are the dead panel load of 8.0 and the stress in m_2M_2, which is 16.0 and which being tensile acts away from the section. The stress equation is:

$$+m_2U_2 - 8.0 \text{ sine } \phi - 16.0 \text{ sine } \phi = 0.$$
$$\phi = 45°, \text{ sine } \phi = 0.707, \text{and}$$
$$m_2U_2 - 8.0 \times 0.707 - 16.0 \times 0.707 = 0$$
$$\therefore m_2U_2 = +16.96.$$

This equation may be put in another form by multiplying and dividing the numerical values by 2:

$$m_2U_2 - \frac{(8.0 + 16.0)}{2} \times 1.414 = 0;$$

or,

$$m_2U_2 = + \frac{24}{2} \sec \phi,$$

which proves the well-known saying that *the stress in the sub-diagonals is equal to one-half the panel load, times the secant of the angle* ϕ. It also shows that *the vertical component of the sub-diagonal is equal to*

one-half the panel load. This fact should be remembered, as it will be frequently used further on.

In a similar manner, the stress in all the tension sub-diagonals will be found to be the same, +16.96, and the stress in the compression sub-diagonal m_1L_1 is −16.96.

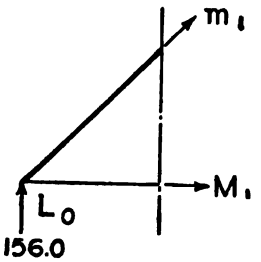


Fig. 72. Stress in Diagonal of Baltimore Truss.

The stress in the member L_0m_1 and in the upper half of any main diagonal (*i. e.*, U_1m_2, U_2m_3, and U_3m_4) is determined as in the diagonals of the Pratt or Howe truss, for the section passed cuts but one member, which has a vertical component. Take L_0m_1 (see Fig. 72). Then $+156.0 + L_0m_1 \cos 45° = 0$, from which $L_0m_1 = -220.5$. For U_1m_2 the section is passed as in Fig. 73, and the equation of stress is $+V_3 - U_1m_2 \cos 45° = 0$, or $+108.0 - U_1m_2 \times 0.707 = 0$, from which $U_1m_2 = +152.9$.

In a similar manner,

$$U_2m_3 = +60.0 \div 0.707 = +84.84;$$
$$U_3m_4 = +12.0 \div 0.707 = +16.96.$$

The stresses in m_1U_1, m_2L_2, and m_3L_3 may be determined by

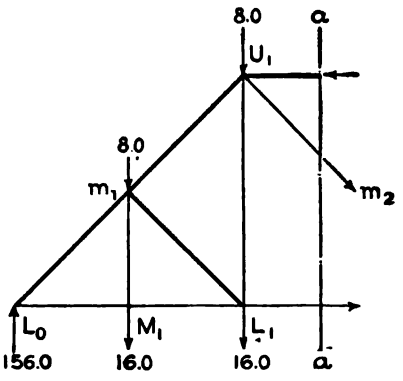


Fig. 73. Stress in Upper Half of Main Diagonal of Baltimore Truss.

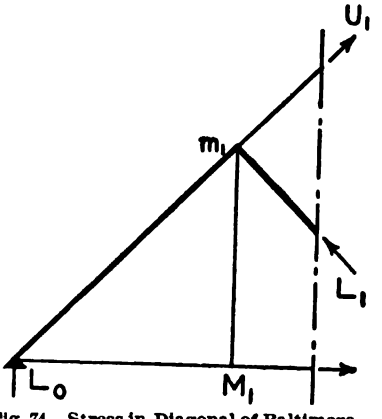


Fig. 74. Stress in Diagonal of Baltimore Truss.

resolving the forces about m_1, m_2, and m_3; but a neater solution is to pass a vertical section cutting the member whose stress is desired, and to equate to zero the shear and the vertical components of all

the members cut (see Fig. 24, Article 28). The section for m_1U_1 is passed as in Fig. 74. The equation of stress is then:

$$m_1U_1 \cos 45° + m_1L_1 \cos 45° + V_2 = 0;$$

but the vertical component of m_1L_1 is $\frac{24}{2} = 12$; and therefore,

$$m_1U_1 \times 0.707 + 12 + 132 = 0$$
$$\therefore m_1U_1 = -203.6.$$

For m_2L_2, the section is as shown in Fig. 75, and the stress equation is:

$$-m_2L_2 \times 0.707 + \text{vert. component } m_2U_2 + V_4 = 0$$
$$-m_2L_2 \times 0.707 + 12 + 84 = 0$$
$$\therefore m_2L_2 = +135.6.$$

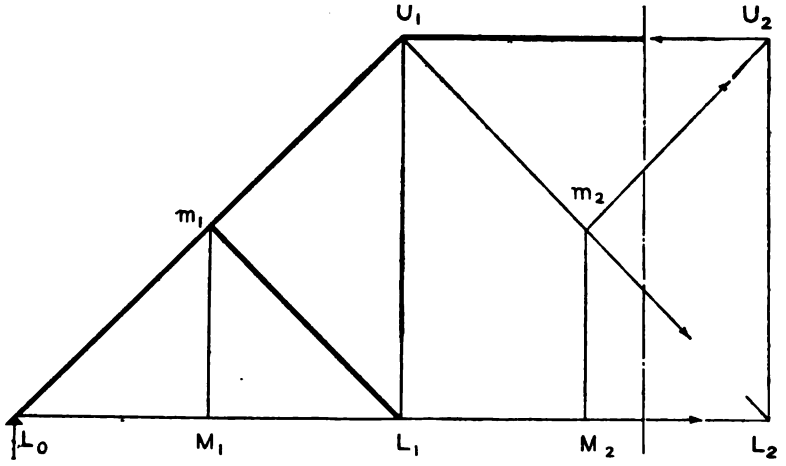


Fig. 75. Calculating Stress in Lower Half-Diagonal of Baltimore Truss.

In a similar manner, passing a section cutting U_2U_3, m_3U_3, m_3L_3, and M_3L_3, the stress equation may be written:

$$-m_3L_3 \times 0.707 + 12 + 36 = 0$$
$$\therefore m_3L_3 = +67.85.$$

The stresses in the verticals are best determined by resolving the vertical forces at their lower end. Referring successively to diagrams *a*, *b*, and *c* of Fig. 76, the stress equations are:

$$+U_1L_1 - 16.0 - 12.0 = 0 \qquad \therefore U_1L_1 = +28.0$$
$$+U_2L_2 - 16.0 + 96.0 = 0 \qquad \therefore U_2L_2 = -80.0$$
$$+U_3L_3 - 16.0 + 48.0 = 0 \qquad \therefore U_3L_3 = -32.0$$

96 and 48 being the vertical components of m_2L_2 and m_3L_3 respectively.

The chord stresses are easiest computed by considering the resolution of horizontal forces at the panel points. As the diagonals make an angle of 45° with the vertical, their horizontal and vertical

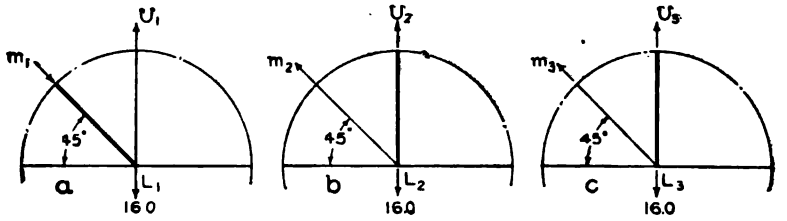


Fig. 76. Calculating Stresses in Verticals of Baltimore Truss of Fig. 69.

components are equal. For instance, the horizontal component of the members L_0m_1, U_1m_2, and U_2m_3 are equal to the shear in that panel, which is their vertical component. At point L_0 (see Fig. 77), there results:

$$+L_0M_1 - \text{horizontal component of } M_1L_0 = 0; \text{ or,}$$
$$+L_0M_1 - 156 = 0$$
$$\therefore L_0M_1 = +156.0;$$

and from Fig. 70 it is evident that $L_0M_1 = M_1L_1$. At point L_1 (see Fig. 78), L_1M_2 is equal to M_1L_1, less the horizontal component of M_1L_1, and the equation is:

$$-156 + 12 + L_1M_2 = 0$$
$$\therefore L_1M_2 = +144.0; \text{ and } M_2L_2 = +144.0.$$

At point L_2 (see Fig. 79), L_2M_3 is equal to the sum of the horizontal components of M_2L_2 and m_2L_2; that is,

$$+L_2M_3 - 144.0 - 96.0 = 0$$
$$\therefore L_2M_3 = M_3L_3 = +240.0.$$

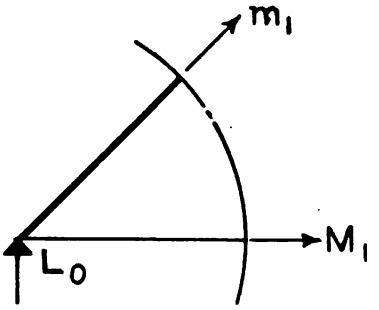


Fig. 77. Chord Stress in Baltimore Truss.

In a similar manner, at point L_3, the stress equation is:

$$+L_3M_4 - 240.0 - 48.0 = 0$$
$$\therefore L_3M_4 = M_4L_4 = +288.0.$$

At the upper panel point U_1 (see Fig. 80), there results the equation:

$$+U_1U_2 + \text{hor. comp. } U_1m_2 + \text{hor. comp. } m_1U_1 = 0;$$
$$U_1U_2 + 108.0 + 144 = 0; \text{ or, } U_1U_2 = -252.0.$$

For the member U_2U_3 (see Fig. 81), the equation is:

$$+U_1U_2 + U_2U_3 - \text{hor. comp. } m_2U_2 + \text{hor. comp. } U_2m_3 = 0$$
$$+252.0 + U_2U_3 - 12 + 60 = 0$$
$$\therefore U_2U_3 = -300.0.$$

In a similar manner, by resolving the horizontal forces at U_3, it will be seen that the action of m_3U_3 will neutralize that of U_3m_4, as

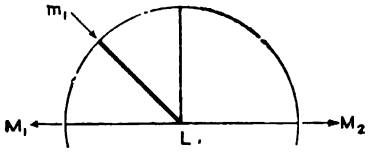


Fig. 78.

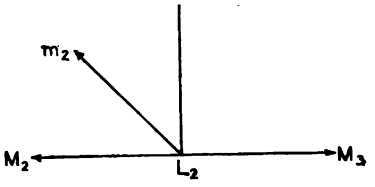


Fig. 79.

Bottom Chord Stresses in Baltimore Truss.

they are equal and pull in opposite directions, and U_3U_4 is equal to $U_2U_3 = -300.0$.

The live-load stresses in the chords, the end-post, and the sub-diagonals are all proportional to the dead-load stresses in the same

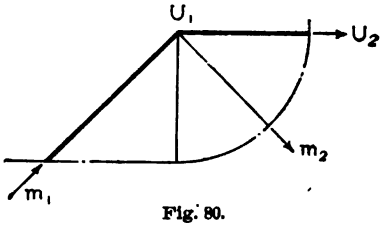


Fig. 80.

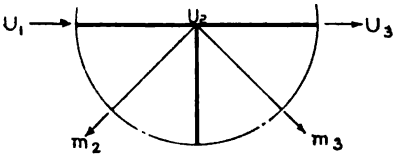


Fig. 81.

Top Chord Stresses in Baltimore Truss.

ratio as the live panel load is to the dead panel load. This ratio is $\frac{40}{24} = 1.667$. By reference to Fig. 70, it will be seen that the live-load stress in the sub-verticals is $+40.0$ for each one. The following stresses can now be determined:

$$L_0m_1 = -220.5 \times 1.667 = -367.5$$
$$m_1U_1 = -203.6 \times 1.667 = -339.5$$
$$U_1U_2 = -252.0 \times 1.667 = -420.0$$
$$U_2U_3 = -300.0 \times 1.667 = -500.0$$
$$U_3U_4 = -300.0 \times 1.667 = -500.0$$
$$L_0L_1 = +156.0 \times 1.667 = +260.0$$
$$L_1L_2 = +144.0 \times 1.667 = +240.0$$
$$L_2L_3 = +240.0 \times 1.667 = +405.0$$
$$L_3L_4 = +288.0 \times 1.667 = +481.0$$

$$m_1L_1 = -16.96 \times 1.667 = -28.28$$
$$m_2U_2 = m_3U_3 = +16.96 \times 1.667 = +28.28$$

The vertical U_1L_1 will have its maximum live-load stress when points M_1 and L_1 are loaded, for these are the only loads which cause a stress in that member (see Fig. 76*a*). The equation is:

$$-\frac{40}{2} - 40 + U_1L_1 = 0,$$

from which,

$U_1L_1 = +60.0.$

The maximum live-load stresses in U_1m_2, U_2m_3, and U_3m_4 are obtained in a manner exactly like that used in obtaining dead-load stress, only the live-load positive shear is used. The stresses are:

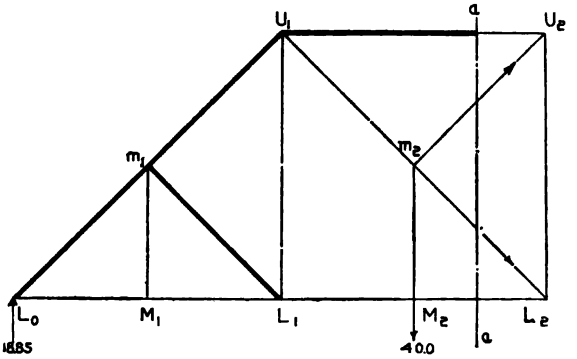


Fig. 82. Stress in Lower Half of Main Diagonal of Baltimore Truss.

$$-U_1m_2 \times 0.707 + 188.5 = 0 \qquad \therefore U_1m_2 = +266.5$$
$$-U_2m_3 \times 0.707 + 128.5 = 0 \qquad \therefore U_2m_3 = +181.5$$
$$-U_3m_4 \times 0.707 + 80.0 = 0 \qquad \therefore U_3m_4 = +113.1$$

In the determination of the maximum live-load stress in the lower halves of the main diagonals, m_2L_2, m_3L_3, and m_4L_4, one of the peculiarities of this truss becomes apparent. A section being passed as in Fig. 82, the panel point ahead of the section, and all between the section and the right support, must be loaded. This of course produces a stress in m_2U_2, and the vertical component of this enters the stress equation. The shear in the section *a — a under this loading* is:

$$V_{a-a} = +188.5 - 40 = +148.5;$$

and the stress equation is:

$$-m_2L_2 \times 0.707 + \frac{40}{2} + 148.5 = 0$$
$$\therefore m_2L_2 = +238.0.$$

If the truss had been loaded from the section to the right, there being no load on M_2, no stress would result in m_2U_2, and the stress in m_2L_2

would have been $m_2L_2 = \frac{157.2}{0.707} = +222.2$. In a similar manner, by loading successively points M_3 and to the right, and M_4 and to the right, the stress equations of m_3L_3 and m_4L_4 are:

$$-m_3L_3 \times 0.707 + \frac{40}{2} + 128.5 - 40 = 0 \qquad \therefore m_3L_3 = +153.3$$

$$-m_4L_4 \times 0.707 + \frac{40}{2} + 80.0 - 40 = 0 \qquad \therefore m_4L_4 = +84.8$$

The maximum live-load stresses in the main verticals occur when the panel points to the right of the section which cuts the member under consideration are loaded. There being no load at the end of the sub-vertical just to the left of the section, there will be no stress in the sub-diagonal which the section cuts. The chords, of course, do not exert a vertical component; and so the only unknown term of the stress equation is the stress in the member itself. Fig. 83 shows how the section should be passed when U_2L_2 is considered. The stress equation is:

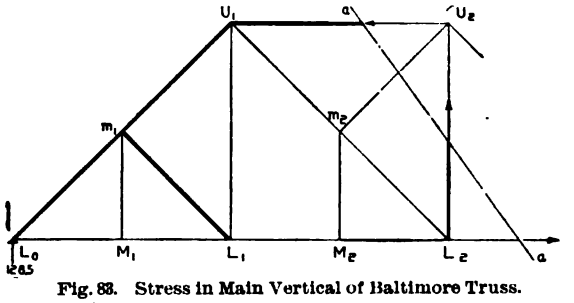


Fig. 83. Stress in Main Vertical of Baltimore Truss.

$$+U_2L_2 + V_{a-a} = 0; \qquad +U_2L_2 + 128.5 = 0; \qquad \therefore U_2L_2 = -128.5.$$

In a similar manner, by passing a section cutting U_2U_3, m_3U_3, U_3L_3, L_3m_4, and loading M_4 and to the right, it is seen that the stress equation for U_3L_3 is:

$$+U_3L_3 + 80.0 = 0 \qquad \therefore U_3L_3 = -80.0$$

The components of m_3U_3 and L_3m_4 are zero, as can readily be proved by solving for them under this loading.

Fig. 84 gives all the stresses, and they are written in order of dead load, live load, and maximum.

43. **Other Trusses.** The analysis of the foregoing trusses will enable one to solve any of the trusses of modern times. For the solution of the Whipple (sometimes called the "double-intersection Pratt") and others which are not mentioned in this text, the student

is referred to the text-books of F. E. Turneaure and Mansfield Merriman.

ENGINE LOADS

44. Use of Engine Loads. It was formerly the custom for railroads to specify that the engine to be used in computing the stresses in their bridges should be one of their own which was in actual use. The engines of different roads were usually different both in regard to the weight on the various wheels and in regard to the number and spacing of the wheels. Of late years, considerable progress has been made towards the adoption of a typical engine loading as standard. These typical engines (see Fig. 17, Article 25) vary only in regard to the weights on the wheels, the number and spacing of wheels being the same in all engines.

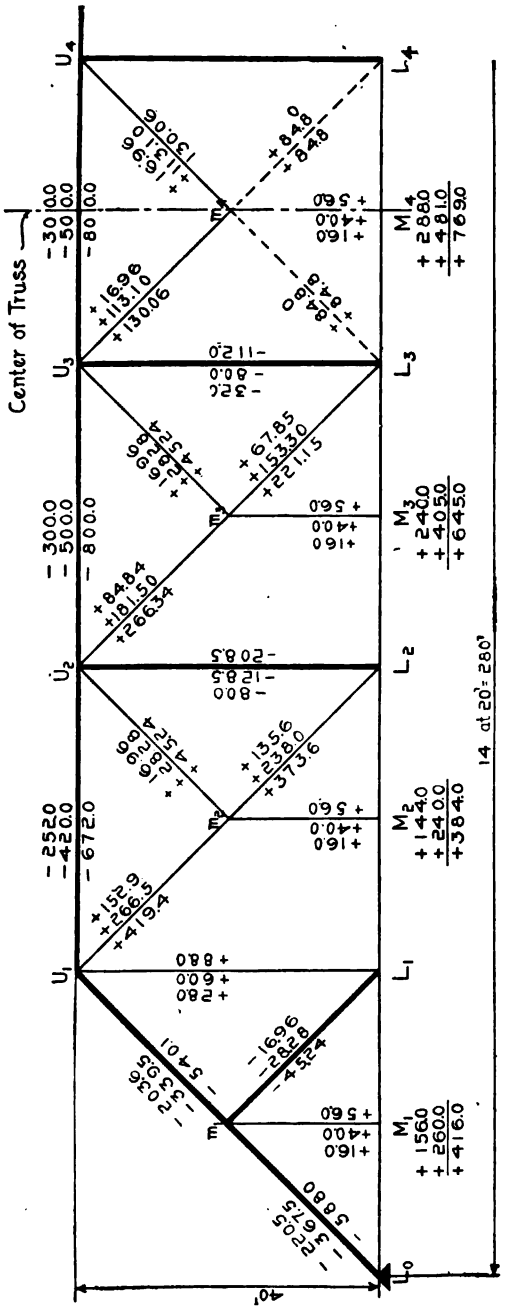


Fig. 84. Stress Diagram of Baltimore Truss of Fig. 69. Stresses indicated in order of Dead-Load, Live-Load, and Maximum.

Fig. 85. Diagram for Computing Stresses under Engine Loads.

The distance between wheels is an even number of feet, instead of an odd number of feet and inches and fractions thereof. For examples of loadings which are in almost universal use, consult the specifications of Cooper or Waddell.

The labor of computation of stresses when engine loads are used is considerably lessened by the use of the so-called *engine diagrams*. Fig. 85 gives a diagram which has been found very convenient. The first line at the top represents the bending moment of all the loads about the point to the right of it. All the loads are given in thousands of pounds, and all the moments are in thousands of pound-feet. The practice of writing results in thousands of pounds—or, in case of moments, in thousands of pound-feet or pound-inches—is to be recommended, as it saves the unnecessary labor of writing ciphers. Throughout this text this practice has been extensively followed, the stresses being written to the nearest ten pounds or one-hundred pounds as the case may be. For example, 6 433 may be written 6.43 or 6.4, the few pounds which are neglected making no appreciable difference in the design. The distances are in feet.

As an example of the use of the first line at the top, suppose that it is desired to find the moment of all the loads to the left of a certain point when wheel 6 (the numbers of the wheels are placed inside of the circles

representing the wheels) is just over the point. The moment will be 1 640 000 pound-feet, which is obtained by reading off the 1 640 just to the right of the line through wheel 6.

When using the first line for values at sections in the uniform load, the values given represent the moment of all wheel and uniform loads about the points in the line or section to the left of the value given. For example, if it is desired to find the moment about a point in line 2, it will be 19 304 000 pound-feet, the value 19 304 appearing to the right of the line.

The line of figures below the wheels indicates the distances between any two wheels.

The third line of figures indicates the distance from the first wheel to the wheel to the right. For instance, 37 is the distance from wheel 1 to wheel 7.

The values in the fourth line indicate the sum total of all the loads to the left of the value given. For example, 245 signifies that the loads 1 to 15 inclusive weigh 245 000 pounds.

The values in lines 5 and 6 are similar to those of lines 3 and 4, except that the starting point is at the head of the uniform load. For example, 40 in line 5, and 112 in line 6, indicate that it is 40 feet from the head of the uniform load to the wheel 12, and that wheels 18 to 13 inclusive weigh 112 000 pounds.

The values in lines 7 to 16 indicate the value of the moment of all the wheels from the zigzag line up to and including the one to the left or the right, according as the value is to the left or the right of the zigzag line. For example, 2 745, line 11, indicates that the moments of wheels 8 to 14 inclusive about the zigzag line just under wheel 15, is 2 745 000 pound-feet; or the value 1 704, line 14, shows that the moments of wheels 13 to 18 about the zigzag line just under wheel 12 is 1 704 000 pound-feet.

When line 4 of figures is under the uniform load, the values refer to the vertical line to the right; thus 324 is the value of all loads to the left of line 3 about that line.

For values of moments at points which fall in between wheels, or at positions in the uniform load where the value of the moment is not given, a very important principle of applied mechanics is used. It is:

$$M_x = M' + Wx + \frac{wx^2}{2}.$$

in which,

M_a = Moment at section desired;
M' = Value of moment at preceding vertical line;
W = Sum total of all loads to the left of and at the point where M' is taken;
x = Distance from section under consideration to vertical line to which M' is referred;
w = Uniform load on the distance x.

Let it be desired, for example, to determine the moment at a point c, 3 feet to the right of wheel 13. The position of the loads is given in Fig. 86. The moment is:

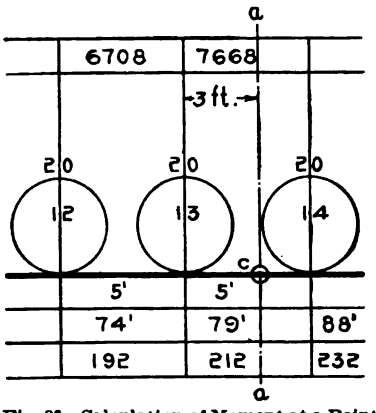


Fig. 86. Calculation of Moment at a Point under Engine Load.

$$M_{a-a} = 7\,668 + 212 \times 3$$
$$= 7\,668 + 636$$
$$= 8\,304 = 8\,304\,000 \text{ pound-feet, there being no uniform load.}$$

To illustrate the method when applied to points in the uniform load, assume the point to be 7 feet to the right of line 2. The position is illustrated in Fig. 87. The moment is:

$$M_{b-b} = 19\,304 + 304 \times 7 + \frac{7^2 \times 2}{2},$$
$$= 21\,481 = 21\,481\,000 \text{ pound-feet.}$$

The use of the moment diagram is now apparent. Reactions due to any position of the engines may be determined by dividing the span into the value obtained for the moment at the right end of the span. Likewise, if the moment of the reaction about any panel point is determined and from it the moment of the wheel loads about that same panel point are subtracted, then the result, divided by the height of the truss, will give the chord stress. For example, if the right end of an 8-panel 196-foot span truss, height 25 feet, came 7 feet to the right of the vertical line 2, then the moment at this point (see Fig. 87) would be 21 481 000, and the reaction would be 21 481 000 ÷ 196 = 109 600. This position of the loads would cause the panel point L_6 to come 3 feet to the right of wheel 13. The moment of the reaction about L_6 is 109 600 × 6 × 24.5 = 16 111 200;

and the chord stress U_6L_6 for this position of the engine is:

$$\frac{16\,111\,200 - 8\,304\,000}{25} = -312\,000 \text{ pounds.}$$

In using the engine to determine the shear in any particular panel, it must be remembered that the shear is *not* the left reaction less all the loads to the left of the panel point on the right of the section, as the loads in the panel under consideration are carried on stringers, and these stringers transfer a portion of the loads to the

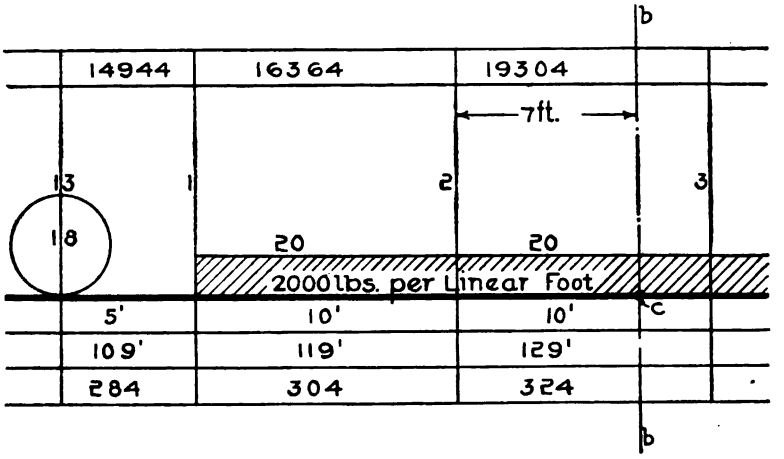


Fig. 87. Calculation of Moment at Point under Uniform Load.

panel point on the left of the panel, and a portion to the panel point on the right of the panel. Only that portion of the loads in the panel which is transferred to the left panel point should be subtracted from the reaction, as should all of the loads to the left of the panel under consideration. If, in a 6-panel 120-foot span Pratt truss, the wheel 6 comes at L_2, the left reaction will be:

$$R_1 = \frac{1}{120}\left(16\,364 + 3 \times 284 + \frac{3^2 \times 2.0}{2}\right) = 143.6;$$

and the loads in the first two panels will be in position as indicated by Fig. 88, the wheel 3 being 1 foot to the right of point L_1. Let it be required to determine the shear in the panel L_1L_2 when the loads are in this position. It will be the reaction 143.6 minus loads 1 and 2 and also that portion of the loads 3, 4, and 5 which will be transferred by the stringers to the point L_1. As the stringers are simple

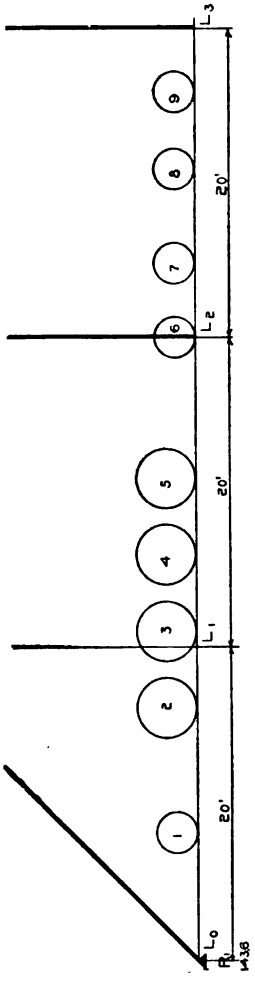


Fig. 88. Determination of Shear in Panel under Engine Load.

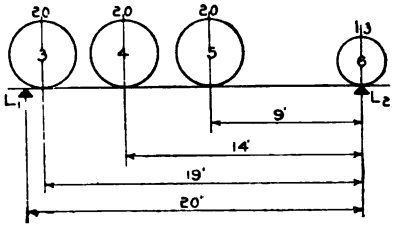


Fig. 89. Shear Diagram for Panel under Engine Load.

beams, the amount transferred to L_1 will be the reaction of the stringer L_1L_2. Referring to Fig. 89, the reaction is:

$$R_{L_1} = (20 \times 9 + 20 \times 14 + 20 \times 19) \div 20 = 42.0$$

The shear in the second panel is now found to be:

$$V_2 = 143.6 - (10 + 20 + 42.0) = +71.6$$

In the majority of cases where it is necessary to determine the shear in a panel, none of the loads will be in the panel to the left of the one under consideration. In this case the operation is somewhat simplified, as the engine diagram can be used directly. If the engine be placed so that the third wheel is at L_2, wheel 16 will be just over the right support, and the left reaction will be:

$$R_1 = 12\,041 \div 120 = 100.3.$$

As there are no wheel loads in the first panel, the amount to be subtracted from the reaction will be that proportion of the loads 1 and 2 which is transferred to L_1; and this (see Fig. 90) is $230 \div 20 = 11.5$. The shear in the second panel is then $100.3 - 11.5 = +88.8$.

From inspection of the resulting shear in the second panel when wheel 6 is at L_2 and when wheel 3 is at L_2, it is seen that different wheels at L_2 will give different shears in the panel to the left. Evidently there is some wheel which will give the greatest shear possible. The same is true of the relation between wheels and moments. The

next two articles are devoted to subject-matter which will enable one to tell which of several wheels is the correct wheel at the point, without the necessity of solving for the shear each time every wheel is at the point.

45. **Position of Wheel Loads for Maximum Shear.** By methods of differential calculus, it can be proved that, for any system, either of wheel loads or wheel loads followed by a uniform load (see Fig. 91), the correct wheel that should be at the panel point b in order to

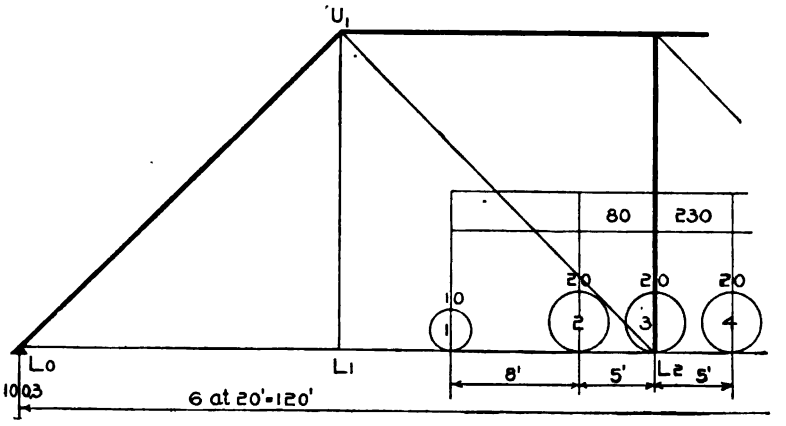


Fig. 90. Determination of Shear in Panel under Engine Load.

give a very great or maximum shear in the panel $a - b$, is such a wheel that the quantity $Q = \frac{W}{m} - G$ is positive when $q = \frac{W}{m} - (G + P)$ is negative. In these equations,

W = Total load on the truss;
m = Number of panels in the truss;
G = Load in panel under consideration; and
P = Load at panel points on right of panel.

If a load is directly over the panel point a, it is not to be included in the weight G; neither is P included in the weight G. If a wheel load should come directly over the right end of the truss, it should not be considered in the quantity W.

The only way to determine which wheel is the correct one, is to try wheel 1, then wheel 2, and so on, until the wheel or wheels are reached that will give the Q and q signs of an opposite character.

The process should not be stopped there, but the next succeeding wheels should be tried until Q and q again have the same sign.

As an example, let it be required to determine the position of the wheel loads to produce the maximum positive shears in a 6-panel 120-foot Pratt truss. This work should be arranged in tabular form, and Table V is found to be convenient.

TABLE V

Determination of Position of Wheel Loads for Maximum Shear

($m = 6$)

Panel Point	Wheel at Point	G	$\frac{W}{m}$	P	$G+P$	Q	q	Remarks
L_1	2	10	$\frac{284}{6} = 47.33$	20	30	+	+	
L_1	3	30	$\frac{292}{6} = 48.67$	20	50	+	−	gives a maximum
L_1	4	50	$\frac{302}{6} = 50.03$	20	70	+	−	gives a maximum
L_1	5	60	$\frac{302}{6} = 50.03$	20	100	−	−	
L_2	2	10	$\frac{232}{6} = 38.67$	20	30	+	+	
L_2	3	30	$\frac{245}{6} = 40.83$	20	50	+	−	gives a maximum
L_2	4	50	$\frac{258}{6} = 43.00$	20	70	−	−	
L_3	1	0	$\frac{152}{6} = 25.67$	10	10	+	+	
L_3	2	10	$\frac{172}{6} = 28.67$	20	30	+	−	gives a maximum
L_3	3	30	$\frac{192}{6} = 34.0$	20	50	+	−	gives a maximum
L_3	4	50	$\frac{212}{6} = 37.0$	20	70	−	−	
L_4	1	0	$\frac{116}{6} = 19.33$	10	10	+	+	
L_4	2	10	$\frac{129}{6} = 21.50$	20	30	+	−	gives a maximum
L_4	3	30	$\frac{142}{6} = 23.67$	20	50	−	−	
L_5	1	0	$\frac{70}{6} = 11.67$	10	10	+	+	
L_5	2	10	$\frac{90}{6} = 15.00$	20	30	+	−	gives a maximum
L_5	3	30	$\frac{103}{6} = 17.17$	20	50	−	−	

A study of Table V shows the fact that wheel 1 can never produce a maximum. It also shows that there are in some cases two positions which will give large values of the moment. In these cases the shears for each position of the engines must be determined in order to tell which wheel at the panel point in reality gives the greatest. In practical work it is customary to use the first position found, as the difference in the shears resulting from the use of the two positions is not large enough to affect the final design.

Fig. 92 shows the engine diagram on the truss in the correct position to give the maximum shear in the second panel. The weight of wheel 16 is not included in the weight W, as it is directly over the right support.

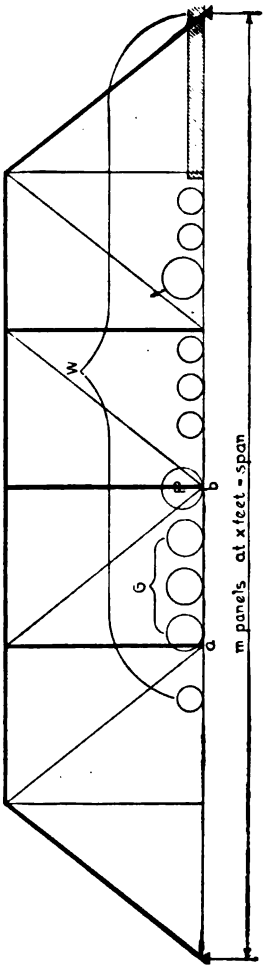


Fig. 91. Engine Diagram for Determining Position of Wheel Loads to Give Maximum Shear in Panel.

46. **Position of Wheel Loads for Maximum Moments.** In this case the methods of differential calculus are employed to determine which wheels will, if placed at a point, give a maximum moment at that point. For any system, either of wheel loads or of wheel loads followed by a uniform load, that wheel which will cause $K = \frac{Wn}{m} - L$ to be positive, and $k = \frac{Wn}{m} - (L + P)$ to be negative, is the wheel. Here n is the number of the panel under consideration, and is to be reckoned from the left end; L = the load to the left of the point under consideration; and the remainder of the letters signify the same as they do in Article 45. In some cases there will be more than one position of the loads which will satisfy the above condition. It is then necessary to work out the actual moments created by the loads in each position, in order to find out which is the largest. The

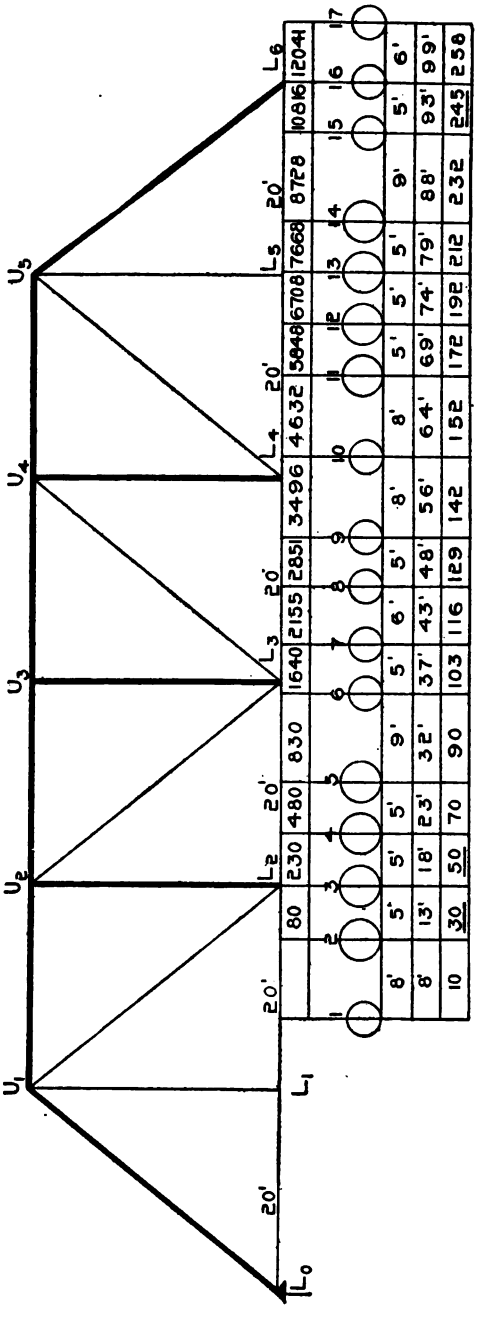


Fig. 92. Diagram of 6-Panel Pratt Truss Loaded with Engine in Position to Give Maximum Shear in Second Panel.

position of the loads for the greatest moments should be determined for all panel points except the one on the extreme right, as the greatest moment possible may be caused by wheels of the rear engine being on the point on the right-hand side of the truss, instead of the wheels of the front engine being at the corresponding point on the left-hand side.

In general, it may be said that there will be a number of wheels which, if placed at the panel point in the center of the span, will satisfy the given conditions. In this particular case, it is not necessary to determine all of the moments. The greatest moment possible will occur when that one of the heaviest wheels of the second locomotive which gives the heaviest load upon the truss is at the point. In case several of the heavy wheels give the

same maximum load W, use the first wheel which gives this maximum W.

Let it be required to determine the position of the wheel loads for maximum moments at the lower panel points of the 6-panel 120-foot Pratt truss of Article 45. The necessary work can be conveniently arranged in the form of a table, as is done in Table VI.

TABLE VI

Determination of Position of Wheel Loads for Maximum Moments

Panel Point	Wheel at Point	L	$\frac{Wn}{m}$	P	$L+P$	K	k	Remarks
L_1	2	10	284 ÷ 6 = 47.3	20	30	+	+	
L_1	3	30	292 ÷ 6 = 48.7	20	50	+	−	gives a maximum
L_1	4	50	302 ÷ 6 = 50.3	20	70	+	−	gives a maximum
L_1	5	60	302 ÷ 6 = 50.3	20	80	−	−	wheel 1 is off bridge
L_2	5	70	(271 ÷ 6) × 2 = 90.3	20	90	+	+	
L_2	6	90	(290 ÷ 6) × 2 = 96.7	13	103	+	−	gives a maximum
L_2	7	103	(300 ÷ 6) × 2 = 100.0	13	116	−	−	
L_3	8	116	(271 ÷ 6) × 3 = 135.5	13	129	+	+	
L_3	9	129	(284 ÷ 6) × 3 = 142.0	13	142	+	−	gives a maximum
L_3	10	142	(298 ÷ 6) × 3 = 149.0	10	152	+	−	gives a maximum
L_3	11	142	(304 ÷ 6) × 3 = 151.3	20	162	+	−	gives a maximum
L_3	12	142	(294 ÷ 6) × 3 = 147.0	20	162	+	−	gives a maximum
L_3	13	142	(284 ÷ 6) × 3 = 142.0	20	162	0	−	gives a maximum
L_3	14	142	(274 ÷ 6) × 3 = 137.0	20	162	−	−	
L_4	11	152	(271 ÷ 6) × 4 = 180.6	20	162	+	+	Note wheel 18 not included.
L_4	12	172	(284 ÷ 6) × 4 = 189.4	20	192	+	−	gives a maximum
L_4	13	192	(294 ÷ 6) × 4 = 196.0	20	212	+	−	gives a maximum
L_4	14	212	(304 ÷ 6) × 4 = 202.6	20	232	−	−	

One should carefully note that in certain positions, as when wheels 11, 12, 13, and 14 are at L_3, some wheels are to the left of the left support; that is, they are not upon the bridge. In all such cases they are counted neither in the quantity L nor in W.

In the case of L_3, wheel 11, being the first large driver of the second engine, will give the greatest moment, as it is the first driver to come at the point when the maximum load of 304 000 pounds is on the truss. Fig. 93 shows the engine diagram on the truss in correct position to give the maximum moment at point L_2.

47. **Pratt Truss under Engine Loads.** In order to exemplify the use of the engine-load diagram, let it be required to determine the stresses in the Pratt truss of Article 45 due to E 40 loading, the

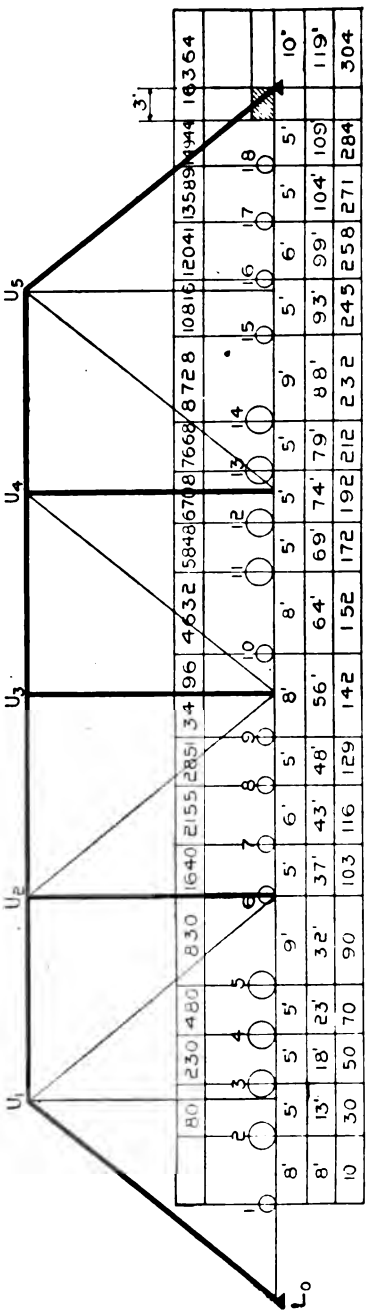


Fig. 93. Diagram of 6-Panel Pratt Truss Loaded with Engine in Position to Give Maximum Moment at Panel Point.

height being 25 feet. The secant is $(25^2 + 20^2)^{\frac{1}{2}} \div 25 = 1.28$.

The maximum positive shears in the various panels should first be computed. These, written in reverse order, will be the maximum negative or minimum shears. Table V should now be referred to, and an outline diagram drawn to the same scale as the engine used, on which to place the engine diagram in the correct position. The various values can then be read off the diagram at the right-hand end of the truss. It will be found convenient to lay off to scale the first ten feet of the lower chord of the truss from the right support, making the divisions one foot apart. This will enable one to ascertain the distance of the last wheel load from the right support, or the amount of uniform load upon the bridge, without scaling or further computation. In case it is desired to have the wheel loads appear on the lower chord, as in Fig. 93, the outline of the truss should be on tracing cloth or transparent paper. This is not to be advised, however, as errors are likely to occur because of failure to distinguish clearly

the various numerical values. It is far better to place the diagram as in Fig. 92, in which case the outline of both the truss and the diagram can be drawn on good stiff paper.

For wheel 3 at point L_1 (see Articles 44 and 45), the left reaction is as follows, there being four feet of uniform load upon the truss:

$$R_1 = \left(16\,364 + 284 \times 4 + \frac{4^2 \times 2.0}{2}\right) \div 120 = 146.0;$$

and the proportion of loads in the panel which is transferred to the point L_0 by the stringers is $230 \div 20 = 11.5$. The shear is therefore $V_1 = +146.0 - 11.5 = +134.5$. The computation for the shear when wheel 4 is at the point, will not be made; for, as has been noted before, the result will not be much different from the above.

For wheel 3 at L_2, wheel 16 comes over the right support. The left reaction is:

$$R_1 = 12\,041 \div 120 = 100.3;$$

the proportional part of the loads which is transferred to L_1 is 11.5; and the shear is:

$$V_2 = (+100.3 - 11.5) = +88.8.$$

For wheel 2 at L_3, wheel 11 comes four feet from the right support. The left reaction is:

$$R_1 = (5\,848 + 172 \times 4) \div 120 = 54.5.$$

That part of wheel 1 which is transferred to L_2 is $80 \div 20 = 4.0$, and the shear is therefore:

$$V_3 = (+54.5 - 4) = +50.5.$$

For wheel 2 at L_4, wheel 9 comes over the right support. The left reaction is:

$$R_1 = 3\,496 \div 120 = 29.1.$$

That part of wheel 1 which is transferred to L_3 is 4.0, and the shear is therefore:

$$V_4 = (+29.1 - 4.0) = +25.1.$$

For wheel 2 at L_5, wheel 5 is five feet from the right support, and the left reaction is:

$$R_1 = (830 + 90 \times 5) \div 120 = 10.7$$

and the shear is:

$$V_5 = (+10.7 - 4.0) = +6.7.$$

If the dead panel load is 20 000 pounds, all the shears may now be written as follows:

Dead-Load V	+ Live-Load V	− Live-Load V
$V_1 = +50.0$	$+134.5$	$\pm\ 0.0$
$V_2 = +30.0$	$+\ 88.8$	$-\ 6.7$
$V_3 = +10.0$	$+\ 50.5$	-25.1
$V_4 = -10.0$	$+\ 25.1$	-50.5
$V_5 = -30.0$	$+\ \ 6.7$	-88.8
$V_6 = -50.0$	$\pm\ \ 0.0$	-134.5

A comparison of the shears in the third and fourth panels shows that counters are required. The stress in these counters is:

$$U_3L_2 = U_3L_4 = +1.28 \times (25.1 - 10.0) = +19.2$$

As it is known that positive shears cause a compressive stress in L_0U_1 and tensile stresses in the diagonals, and that negative shears produce compressive stresses in the intermediate posts, the left half of the bridge being considered, the web stresses for dead and live load can be determined without in all cases writing the stress equations in order to determine the sign. It should be remembered that one-third of the dead panel load, or 6 700 pounds, is applied at the panel points of the top chord.

Dead-Load Stresses in the Diagonals—

$$L_0U_1 = -1.28 \times 50 = -64.0$$
$$U_1L_2 = +1.28 \times 30 = +38.4$$
$$U_2L_3 = +1.28 \times 10 = +12.8$$

Dead-Load Stresses in the Verticals. For U_2L_2, the section passed will cut U_1U_2, U_2L_2, and L_2L_3, and the shear on this section will be $50 - 2 \times 13.3 - 6.7 = +16.7$. The stress equation is $+16.7 + U_2L_2 = 0$, from which $U_2L_2 = -16.7$.

The dead-load stress in U_3L_3 is found by passing a circular section around U_3. Then $-U_3L_3 - 6.7 = 0$, from which $U_3L_3 = -6.7$. In a similar manner, by passing a section around L_1, the stress is found to be $+13.3$.

Live-Load Stresses in the Diagonals—

	Maximum	Minimum
$L_0U_1 =$	$-1.28 \times 134.5 = -172.2$	0
$U_1L_2 =$	$+1.28 \times\ 88.8 = +113.6$	$-1.28 \times 6.7 = -8.6$
$U_2L_3 =$	$+1.28 \times\ 50.5 = +\ 64.7$	0

Live-Load Stresses in the Verticals. The maximum stress in U_1L_1 occurs when one of the large drivers is at L_1, and the loads in

the first panel are as near as possible one-half the sum of the loads in panels 1 and 2 and the load at L_1. This can be established as a fact by use of the differential calculus. In the present case, this condition is satisfied when wheel 4 is at L_1. Then the weight of the wheels in panel 1 is 50 000 pounds, and the sum total is 116 000 pounds. If wheel 13 be placed at L_1, the result will be the same, and then the engine diagram can be used. Fig. 94 represents the engine diagram in place, ready to use. According to Article 44, the value 480 is the moment of wheels 10 to 12 about L_1. Therefore 480 ÷ 20 (20 is the panel length) = 24.00, is that amount of wheels 10 to 12 which is transferred to L_0. In like manner, 529 ÷ 20 = 26.45 is the amount of wheels 14 to 16 transferred to L_2. As the total weight of the loads in the two panels is 116 000 pounds, the amount transferred to L_1 must be 116.0 − (24.00 + 26.45) = 65.55, and the stress in U_1L_1 is therefore +65.55.

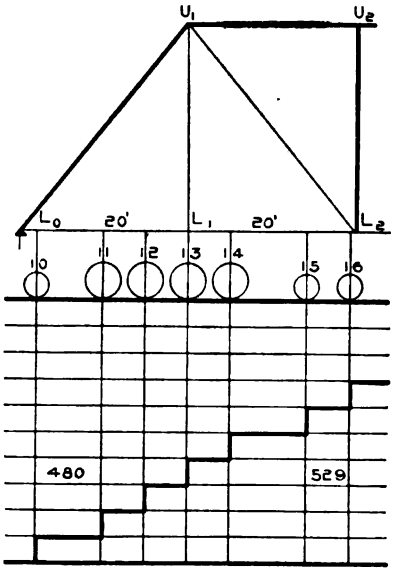


Fig. 94. Engine Diagram for Determination of Live-Load Stress in Vertical of Pratt Truss.

The maximum live-load stress in L_2U_2 occurs when the loading is in a position to give the maximum shear in the third panel, as the shear at a section cutting U_1U_2, U_2L_2, and L_2L_3 is the same as that at a vertical section in the panel. The stress equation is $+U_2L_2 + 50.5 = 0$, from which $U_2L_2 = -50.5$. In a similar manner, the stress equation for the maximum live-load stress in U_3L_3 is $+U_3L_3 + 25.1 = 0$, as U_3L_4 is working, and therefore $U_3L_3 = -25.1$. As in the case of the analysis of the Pratt truss under uniform load (see Article 39), the dead-load stress of −6.7 cannot be added to this stress of −25.1 to obtain the maximum; but the dead-load stress in U_3L_3 must be obtained when diagonals U_3L_3 and U_3L_4 are in action. In the manner explained in Article 39, this is found to be +3.30.

It will be found that as the engines come on the bridge from the

left, the counters come into action in the case of U_2L_2; and in the case of U_3L_3, both U_2L_3 and L_3U_4 act, thus causing the live-load stress in these verticals to be zero; and when this is the case, the dead-load stress is $-$ 6.7, which is the minimum.

Dead-Load Chord Stresses. The dead-load chord stresses can be found by any of the methods previously given; but they will be found by the tangent method as indicated below, the tangent being $20 \div 25 = 0.8$:

$$L_0L_1 = +0.8 \times 50 = +40.0 = L_1L_2$$
$$U_1U_2 = -(50 + 30) \times 0.8 = -64.0$$
$$U_2U_3 = -(50 + 30 + 10) \times 0.8 = -72.0$$
$$L_2L_3 = -U_1U_2 = -(-64.0) = +64.0$$

Live-Load Chord Stresses. On account of the wheel loading, no ratio can be established between these stresses and the dead-load chord stresses. The maximum moments at each point must be determined, and these divided by the height of the truss will give the chord stresses. For all points to the left of the center of the bridge, the main diagonal will act. For points to the right of the center, an uncertainty exists. The shear in the panels on either side of the point under consideration should be determined when the loading is in position to give the maximum moment at that point. This will indicate which diagonals act, which fact will indicate for what chord member that point is the center of moments.

When wheel 3 is at L_1, four feet of uniform load are on the truss, and the left reaction is:

$$R_1 = (16\,364 + 384 \times 4 + \frac{4^2 \times 2}{2}) \div 120 = 146.0.$$

The moment of this reaction about L_1, less the moment of wheels 1 and 2 about L_1, will be the moment at L_1 due to this loading. The moment of wheels 1 and 2 about L_1 is taken from the diagram, where it occurs in the first line of values just to the right of the vertical line through wheel 3, and therefore:

$$M_1 = 146.0 \times 20 - 230 = 2\,690\,000 \text{ pound-feet.}$$

When wheel 4 is at L_1, there are nine feet of uniform load on the truss, and the left reaction is:

$$R_1 = (16\,364 + 284 \times 9 + \frac{9^2 \times 2}{2}) \div 120 = 158.0;$$

and in this case,

$$M_1 = 158.0 \times 20 - 480 = 2\,680\,000 \text{ pound-feet.}$$

As this is less than when wheel 3 is at the point, wheel 3 gives the greatest moment.

When considering the point L_2 with wheel 6, the left reaction is:

$$R_1 = (16\,364 + 284 \times 3 + \frac{3^2 \times 2}{2}) \div 120 = 143.5$$
$$M_2 = 143.5 \times 2 \times 20 - 1\,640$$
$$= 4\,100\,000 \text{ pound-feet.}$$

The conditions at L_3 indicate that there are several wheels which give large moments; but according to Article 46, wheel 11 gives the maximum moment. When this wheel is at L_3, wheel 1 is off the truss, and 15 feet of uniform load are on the truss. The moment of *all* the loads about the *right support* is $19\,304 + 304 \times 5 + \frac{5^2 \times 2}{2}$ $= 20\,849$, from which should be subtracted the moment of wheel 1 about the right support. This moment of wheel 1 is $10 \times 124 = 1\,240$, and the moment about L_6 of all loads on the truss is $20\,849 - 1\,240 = 19\,609$. The left reaction is:

$$R_1 = 19\,609 \div 120 = 163.4$$
$$M_3 = 163.4 \times 3 \times 20 - (5\,848 - 10 \times 64)$$
$$= 4\,596\,000 \text{ pound-feet.}$$

In the case of L_4, the reactions and the moments for the two positions are:

For wheel 12, $R_1 = 16\,364 \div 120 = 136.4$
$$M_4 = 136.4 \times 4 \times 20 - 6\,708$$
$$= 4\,204\,000 \text{ pound-feet.}$$

For wheel 13, $R_1 = (16\,364 + 5 \times 284 + \frac{5^2 \times 2}{2}) \div 120 = 148.4$
$$M_4 = 148.4 \times 4 \times 20 - 7\,668$$
$$= 4\,204\,000 \text{ pound-feet,}$$

which shows that each wheel gives the same moment, and also that the moment is greater than that at L_2, the corresponding point on the left-hand side of the center of the truss. As L_2 is the center of moments of U_1U_2, then, if the center of moments for U_4U_5 falls at L_4 (that is, if L_4U_5 acts), the stress in U_4U_5 will be greater than the stress in U_1U_2 when wheel 6 is at L_2. Of course, if the engine came on the truss from the left, U_1U_2 would receive the same stress that U_4U_5 now receives. According to the shears, L_4U_5 always acts, and therefore the center of moments for U_4U_5 does fall at L_4.

TABLE VII

Stresses in a Pratt Truss

Kind of Stress	End-Post	Verticals			Diagonals			Upper Chord		Lower Chord		
	L_0U_1	U_1L_1	U_2L_2	U_3L_3	U_1L_2	U_2L_3	L_2U_3	U_1U_2	U_2U_3	L_0L_1	L_1L_2	L_2L_3
Dead-Load	− 64.0	+ 13.3	− 16.7	− 6.7	+ 38.4	+ 12.8	− 12.8	− 64.0	− 72.0	+ 40.0	+ 40.0	+ 64.0
Live-Load	− 172.2	+ 65.6	− 50.5	+ 3.3	+ 118.6	+ 64.7	0.0	− 168.2	− 183.8	+ 107.5	+ 107.5	+ 168.2
				− 25.1	− 8.6	0.0	+ 32.0					
	0.0	0.0	0.0	0.0								
Maximum	− 236.2	+ 78.9	− 67.2	− 21.8	+ 157.0	+ 77.5	+ 19.2	− 232.2	− 255.8	+ 147.5	+ 147.5	+ 232.2
Minimum	− 64.0	+ 13.3	− 16.7	− 6.7	+ 29.8	0.0	0.0	− 64.0	− 72.0	+ 40.0	+ 40.0	+ 64.0

The various moments are written in order, as such action will facilitate the remainder of the computations.

$$M_1 = 2\,690$$
$$M_2 = 4\,100$$
$$M_3 = 4\,596$$
$$M_4 = 4\,204$$

The chord stresses are now found to be:

$$L_0L_1 = L_1L_2 = +\frac{2\,690}{25} = +107.5$$

$$U_1U_2 = -\frac{4\,100}{25} = -164.0$$

$$U_2U_3 = -\frac{4\,596}{25} = -183.8$$

$$U_4U_5 = -\frac{4\,204}{25} = -168.2$$

$$L_2L_3 = -U_1L_2 = -(-164.2) = +164.2$$
$$L_3L_4 = -U_4U_5 = -(-168.2) = +168.2$$

When the load comes on from the left, the stresses in U_1U_2 and L_2L_3 will be -168.2 and $+168.2$ respectively, which are the maximum live-load stresses for these members.

Instead of placing the values of the stresses on a truss outline, they are sometimes put in tabular form, as in Table VII.

48. **Impact Stresses.** When an engine is at rest on a bridge, the stresses in the members are the same as those computed for that loading. When the loads move across the bridge at any speed, the vibrations and the shocks produced by the counterweights in the drivers and by other causes create stresses in the various members in excess of those computed by aid of the engine diagram. The excess stresses

are designated as *impact stresses*. This term, however, is misleading to a certain extent, as causes other than the impact or pounding of the engine wheels help to produce the stresses referred to.

It is a well-known fact capable of mathematical demonstration, that a load, if suddenly applied, will produce a stress equal to twice that which it will produce as a static load; also, that as the ratio of the weight of the load to the weight of the structure increases, the vibrations produced by the impact will be less. These two facts are the basis of most of the empirical formulæ for impact stresses; and empirical formulæ are used to obtain these stresses, as the existing conditions and producing causes are not such as to make them susceptible of mathematical treatment. The result of experiments on actual bridges under the effect of passing engines and trains, have been the basis of many formulæ. One of these is:

$$I = S\left(\frac{300}{L + 300}\right),$$

where I = Impact stress in the member;
S = Live-load stress in the member caused by the engine load when at rest;
L = Length of that part of the bridge which is loaded when the stress S is produced; and
300 = A constant value derived from experiments.

This formula was proposed by C. C. Schneider in 1887, and is given in the "Transactions" of the American Society of Civil Engineers, Vol. 34, page 331. While it does not take into consideration the relative weights of the bridge and the live-load loads, this formula does make allowance for the time it takes to produce the stress, by introducing L, the distance over which the engine passes before causing the stress S. It is seen that the smaller the distance L, the greater will be the impact stress for any given value of S. When L becomes exceedingly small, the effect would be that of a suddenly applied load, and the impact stress would equal the stress S. Table VIII gives the values of $\frac{300}{L + 300}$, which is called the *impact coefficient*, for different values of L. Values not given may be interpolated.

For example, by consulting Fig. 92, which gives the position of the engines for the maximum live-load stress in U_1L_2, it is seen that 93 feet (the distance from wheel 1 to the right support) is the loaded

TABLE VIII

Values of the Impact Coefficient

L	$\frac{300}{L+300}$	L	$\frac{300}{L+300}$	L	$\frac{300}{L+300}$	L	$\frac{300}{L+300}$	L	$\frac{300}{L+300}$
5	0.984	31	0.906	57	0.840	83	0.783	145	0.674
6	0.980	32	0.904	58	0.838	84	0.781	150	0.667
7	0.977	33	0.901	59	0.836	85	0.779		
8	0.974	34	0.898	60	0.833	86	0.777	155	0.659
9	0.971	35	0.896			87	0.775	160	0.652
10	0.968	36	0.893	61	0.831	88	0.773	165	0.645
		37	0.890	62	0.829	89	0.771	170	0.638
11	0.965	38	0.888	63	0.826	90	0.769	175	0.632
12	0.962	39	0.885	64	0.824			180	0.625
13	0.958	40	0.882	65	0.822	91	0.767	185	0.619
14	0.955			66	0.820	92	0.765	190	0.612
15	0.952	41	0.880	67	0.817	93	0.763	195	0.606
16	0.949	42	0.877	68	0.815	94	0.761	200	0.600
17	0.946	43	0.875	69	0.813	95	0.759		
18	0.943	44	0.872	70	0.811	96	0.758	210	0.588
19	0.940	45	0.870			97	0.756	220	0.577
20	0.937	46	0.867	71	0.809	98	0.754	230	0.566
		47	0.865	72	0.806	99	0.752	240	0.556
		48	0.862	73	0.804	100	0.750	250	0.546
21	0.935	49	0.860	74	0.802			260	0.536
22	0.932	50	0.857	75	0.800	105	0.741	270	0.526
23	0.929			76	0.798	110	0.732	280	0.517
24	0.926			77	0.796	115	0.725	290	0.508
25	0.923	51	0.855	78	0.794	120	0.714	300	0.500
26	0.920	52	0.852	79	0.792	125	0.706		
27	0.917	53	0.850	80	0.789	130	0.698	400	0.429
28	0.915	54	0.847			135	0.690	500	0.375
29	0.912	55	0.845	81	0.787	140	0.682	600	0.333
30	0.909	56	0.843	82	0.785				

length. From Table VII, it is seen that the stress in U_1L_2 produced by this loading is +118.6; and from Table VIII, the impact coefficient for 93 feet is found to be 0.763. The impact stress is now computed:

$$I = 0.763 \times 118.6 = +90.6.$$

The maximum stress in U_1L_2 is now:

Dead-load	=	+ 38.4
Live-load	=	+118.6
Impact	=	+ 90.6
Maximum	=	+247.6

Table IX gives the necessary information for computing the impact stresses, and also gives the impact stresses corresponding to the maximum live-load stresses in the members of the truss of Article 47.

TABLE IX

Impact Stresses in a Pratt Truss

Member	S	L	$\frac{300}{L+300}$	I	Remarks
L_0U_1	−172.2	113	0.727	−125.2	4 ft. of uniform load on truss
U_1L_1	+ 65.6	37	0.890	+ 58.4	See succeeding text.
U_2L_2	− 50.5	68	0.815	− 41.3	Same as for U_2L_3
U_3L_3	− 25.1	48	0.862	− 21.6	Same as for U_3L_4
U_1L_2	+118.6	93	0.763	+ 90.6	Wheel 16 at L_6
U_2L_3	+ 64.7	68	0.815	+ 52.7	Wheel 11 is 4 ft. from L_6
U_3L_4	+ 32.0	48	0.862	+ 27.6	Wheel 9 at L_6.
U_1U_2	−168.2	114	0.724	−121.8	Wheel 13 at L_4. 5 ft. of uniform load on bridge.
U_2U_3	−183.8	114	0.724	−133.2	15 ft. of uniform load on bridge. Wheel 1 off bridge.
L_0L_2	+107.5	113	0.727	+ 78.2	Same loading as for L_0U_1
L_2L_3	+168.2	114	0.724	+121.8	Same loading as for U_2U_3

In the case of U_1L_1, it should be noted that only the wheels 10 to 16 inclusive cause the stress (see Fig. 94), and that the loaded length is the distance from wheel 10 to wheel 16.

Some specifications do not call for impact stresses. The unit-stresses in these specifications are made low, and the sections designed are large enough to withstand the additional stresses due to impact. In cases where the impact stresses are required, they must be considered in computing the maximum and minimum stresses.

49. **Snow-Load Stresses.** In some localities the snowfall is considerable, and its weight should be taken into account in computing stresses. This should be done by considering it as an additional dead load of 15 pounds per square foot of floor surface for every foot of snowfall. As it covers the entire floor surface, the stresses will be proportional to the dead-load stresses. Also it is evident that the snow load should not be taken into account in railroad bridges unless they have solid floors, as most of it falls through the open spaces between ties and stringers.

As an example, let it be required to determine the snow-load stresses in a member of a highway bridge, the dead-load stress in the member being +84.0, the dead panel load being 12 000 pounds, and

the snow being $1\frac{1}{2}$ feet deep on the roadway, which is 14 feet wide. The snow panel load is:

$$\frac{1}{2}(14 \times 15 \times 1\tfrac{1}{2} \times 20) = 3\,150 \text{ pounds.}$$

In the above equation, 14 is the width of roadway; 15 is the weight in pounds of one square foot of snow one foot deep; and 20 is the length of one panel. One-half of the weight of snow must be taken, as half is carried by each truss. The snow-load stress is then:

$$\frac{3\,150}{12\,000} \times 84.0 = +22.1.$$

In like manner, all snow-load stresses can be computed.

Most of the standard specifications which have been published do not specify snow loads; and in fact it is not customary to include the snow load in any designs except those for bridges in extreme northern latitudes. It is hardly probable that the greatest load will come upon a country bridge when it is covered with snow. Also, in cities, the sidewalks are cleaned of snow; and so is the roadway if the city is of large size.

WIND-LOAD EFFECTS

50. **Top Lateral System Through-Bridges.** The unit-loads for this system are given in Article 26. Common practice is to take 150 pounds per linear foot of top chord, the end-post being considered part of the top chord in this computation.

In many of the longer-span modern bridges, the diagonals of this system are designed to take either tension or compression; but in the majority of the shorter spans, 200 feet and under, while generally consisting of angles or other stiff shapes, they are designed to take tension only. The verticals or top lateral struts take compression. This combination of tension diagonals and compression verticals makes the so-called *Pratt system of webbing;* and indeed the lateral systems, both top and bottom, are Pratt trusses in a horizontal position. Fig. 95 shows the side elevation of the truss of Article 47, and also the top and bottom laterals. The diagonals shown in full lines act when the wind is *right*, and those shown by dotted lines act when the wind is *left*. Wind *right* indicates that the wind is blowing from the right hand when a person stands facing the right end of

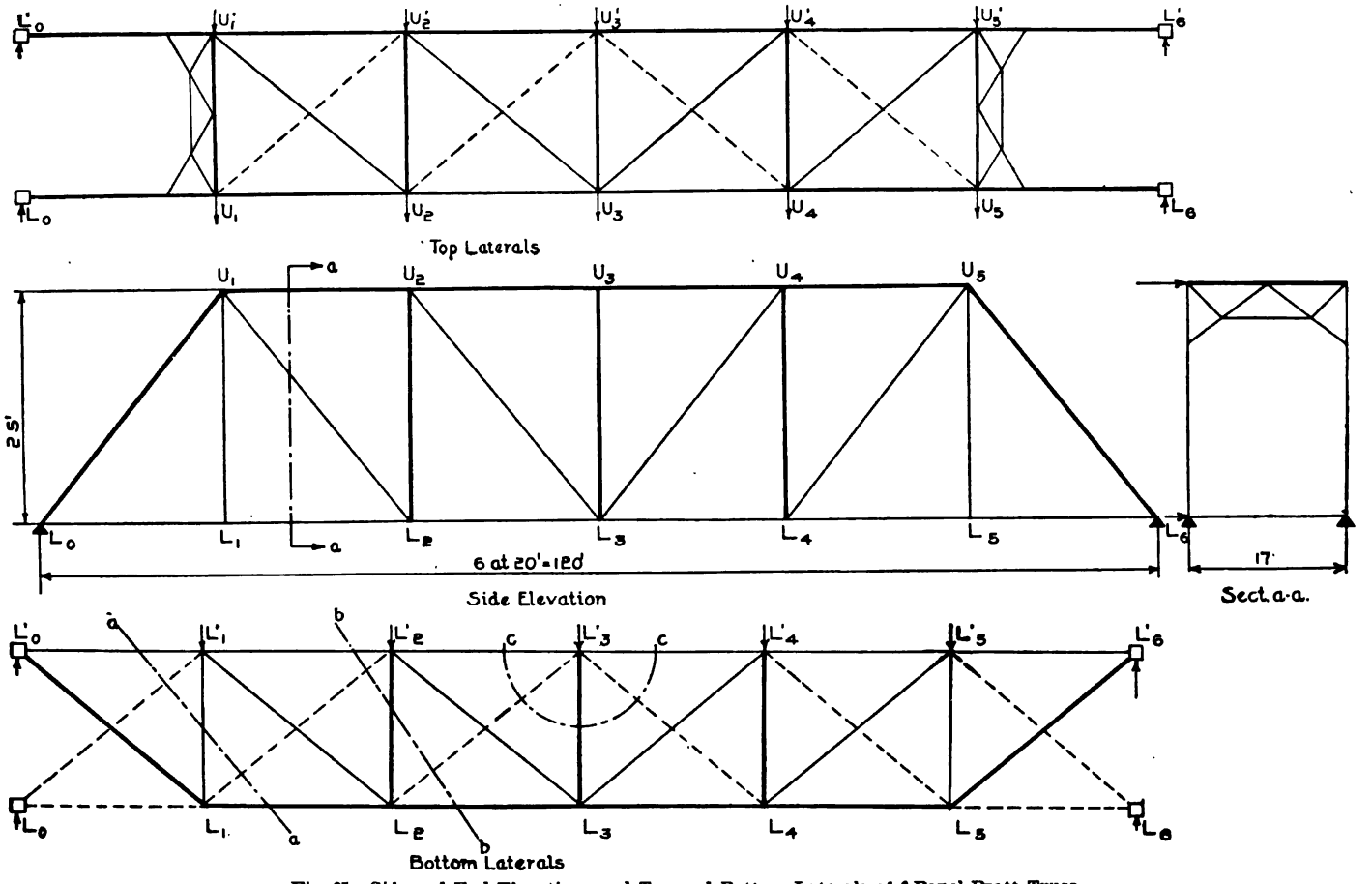


Fig. 95. Side and End Elevations and Top and Bottom Laterals of 6-Panel Pratt Truss.

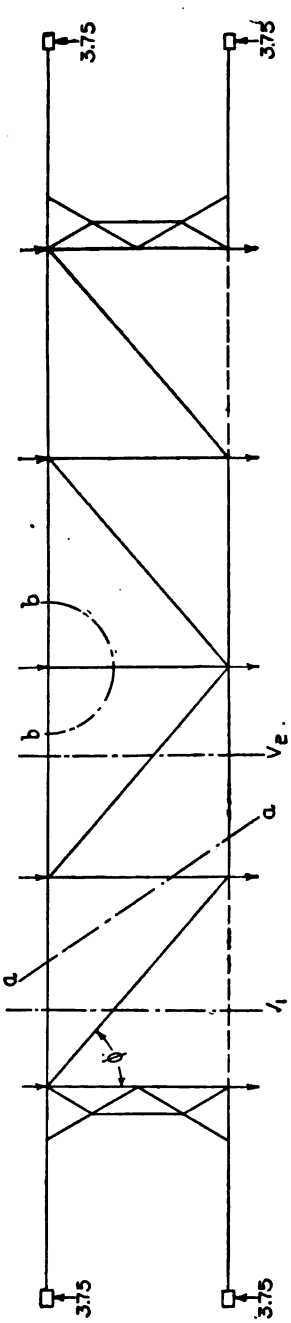


Fig. 96. Diagram Showing Distribution of Loads and Reaction in Pratt Truss of Fig. 95.

the bridge. Wind *left* indicates that the wind blows from a person's left when standing as above described.

The wind load of 150 pounds is divided between the two trusses, this being exact enough for practical purposes; for, by actual experiment, the difference between the readings of wind-pressure gauges placed at points opposite each other in the top chords of a through-bridge was only from 8 to 10 per cent.

The problem, then, is one of a deck Pratt truss with a dead panel load of $150 \times 20 = 3.0$ divided between the two chords. Fig. 96 shows the distribution of loads and the reaction, it being considered that the portal bracings and the end-posts (see Fig. 95) are stiff enough to distribute the reaction equally between the bearing points L_0, L_0', L_6, L_6'. Each panel load is indicated by an arrow, and is equal to $3.0 \div 2 = 1.5$. The reaction at each of the points L_0, L_0', L_6, and L_6' is $10 \times 1.5 \div 4 = 3.75$. The truss being symmetrical, the stresses in like members on each side of the center will be the same. The shears in the top system are:

$$V_1 = +2 \times 3.75 - 2 \times 1.5 = +4.5$$
$$V_{a-a} = +2 \times 3.75 - 3 \times 1.5 = +3.0$$
$$V_2 = +2 \times 3.75 - 4 \times 1.5 = +1.5$$

and the secant ϕ_1 is $(17^2 + 20^2)^{\frac{1}{2}} \div 17 = 1.544$. The stresses in the diagonals are:

$$U_1'U_2 = +1.544 \times 4.5 = +6.95$$
$$U_2'U_3 = +1.544 \times 1.5 = +2.32.$$

The vertical $U_2'U_2 = -3.0$; and by passing a section $b - b$ around U_3', the stress in $U_3'U_3$ is found to be -1.5.

In obtaining the chord stresses in this system, the case is the same as if the reactions were applied at U_1' and U_5', as the portal and end-posts are not in the same plane as the lateral system. The tangent method is the simplest to use in this case. The tangent is $20 \div 17 = 1.176$, and the stresses (see Fig. 95) are:

$$U_1'U_2' = -4.5 \times 1.176 = -5.29$$
$$U_2'U_3' = -(4.5 + 1.5) \times 1.176 = -7.06$$
$$U_1U_2 = 0$$
$$U_2U_3 = -U_1'U_2' = +5.29$$

Fig. 97 is a diagram with the stresses caused by wind right and

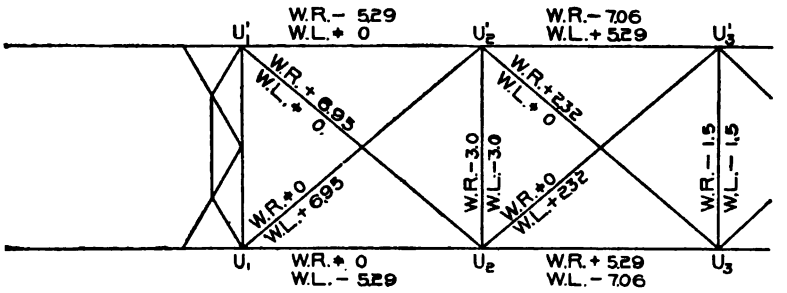


Fig. 97. Wind Stress Diagram of Pratt Truss of Fig. 95.

wind left indicated thereon. The stresses for wind left can easily be written by inspection.

51. **Bottom Lateral Bracing, Through-Bridges.** Fig. 95 shows the lower lateral system with the panel points loaded with the fixed or dead wind load. In this case it is all taken as acting on one side, it being assumed that the floor system protects the leeward truss. The problem then becomes that of determining the stresses in a deck Pratt truss of 6 panels of 20 feet each, the height being 17 feet. When wind is *right*, the members shown by broken lines in Fig. 95 do not act.

The fixed wind load (Article 26) is 150 pounds per linear foot of chord. The panel load will be the same as before, 3.0, but all will be on one chord. The shears are:

$$V_1 = 2\tfrac{1}{2} \times 3.0 = +7.5$$
$$V_{a\text{-}a} = +7.5$$
$$V_2 = +7.5 - 3.0 = +4.5$$
$$V_{b\text{-}b} = +4.5$$
$$V_3 = +7.5 - 2 \times 3.0 = +1.5$$

The secant being 1.544, as previously computed, the web stresses are:

$$L_0'L_1 = +7.5 \times 1.544 = +11.60 \qquad L_1'L_1 = -7.5$$
$$L_1'L_2 = +4.5 \times 1.544 = +\ 6.95 \qquad L_2'L_2 = -4.5$$
$$L_2'L_3 = +1.5 \times 1.544 = +\ 2.32 \qquad L_3'L_3 = -3.0$$

The stress in $L_3'L_3$ is determined by passing section $c - c$ and resolving the vertical forces at L_3' (see Fig. 95).

By using the tangent method, the chord stresses are computed as follows:

$$L_0'L_1' = -7.5 \times 1.176 = -8.82$$
$$L_1'L_2' = -(7.5 + 4.5) \times 1.176 = -14.12$$
$$L_2'L_3' = -(7.5 + 4.5 + 1.5) \times 1.176 = -15.88$$
$$L_1L_2 = -L_0'L_1' = -(-8.82) = +8.82$$
$$L_2L_3 = -L_1'L_2' = -(-14.12) = +14.12$$

The wind load acting on the train is 450 pounds per linear foot. It is evident that the train may cover the span either partially or entirely, and therefore its action on the lower lateral system is the same as if it were stressed by a live load of 450 pounds per linear foot of truss.

The live panel load is $450 \times 20 = 9.0$. The maximum live-load reaction is $5 \times 9.0 \div 2 = 22.5$, and the positive live-load shears are:

$$V_1 = +22.5$$
$$V_2 = (1 + 2 + 3 + 4)\frac{9.0}{6} = +15.0$$
$$V_3 = (1 + 2 + 3)\frac{9.0}{6} = +9.0$$

It is unnecessary to go further than the center, as only the maximum stresses are required in the members. The web stresses are computed as given below:

$$L_0'L_1 = +22.5 \times 1.544 = +34.75 \qquad L_1'L_1 = -22.5$$
$$L_1'L_2 = +15.0 \times 1.544 = +23.15 \qquad L_2'L_2 = -15.0$$
$$L_2'L_3 = +\ 9.0 \times 1.544 = +13.91 \qquad L_3'L_3 = -\ 9.0$$

The maximum chord stresses due to this load of 450 pounds per linear foot of train, occur when the train covers the entire span; and they are directly proportional to the stresses produced by the fixed load, in the same ratio as the live panel load is to the fixed panel load. This ratio is $\frac{9.0}{3.0} = 3.0$. The chord stresses, therefore, are:

$L_0'L_1' = -8.82 \times 3 = -26.46$

$L_1'L_2' = -14.12 \times 3 = -42.36$

$L_2'L_3' = -15.88 \times 3 = -47.64$

$L_1L_2 = +8.82 \times 3 = +26.46$

$L_2L_3 = +14.12 \times 3 = +42.36$

Table X, Article 53, gives the stresses in the top and bottom lateral systems for wind right and wind left.

52. Overturning Effect of Wind on Truss. When the wind blows on the top chord, it tends to overturn the truss. As the truss is held down by its own weight, the action of the wind does not overturn it, but causes the dead-load reaction on the windward side to be less and that on the leeward side to increase by a like amount. The amount is $\pm V = \frac{\Sigma w}{2} \times \frac{h}{b}$, where Σw = the sum of all the wind panel loads, h = the height of the truss, and b = the distance center to center of trusses. The effect upon the leeward truss is the same as if two ver-

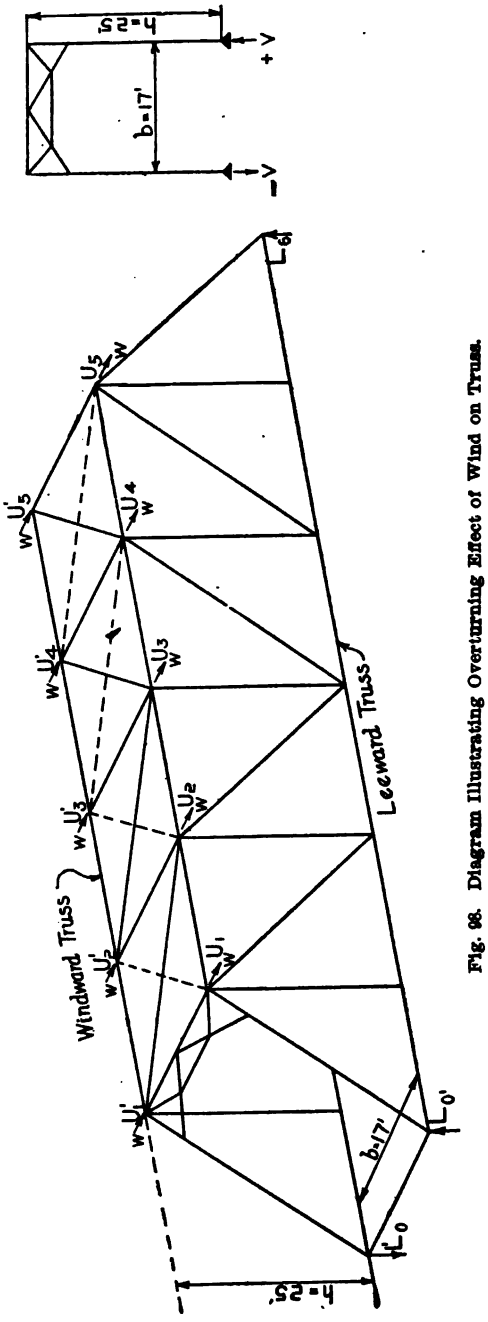


Fig. 98. Diagram Illustrating Overturning Effect of Wind on Truss.

tical loads, each equal to V and acting downward, were placed at the hips U_1 and U_5 (see Fig. 98). The effect on the windward truss is the same as if two vertical loads, each equal to V and acting upward, were placed at the hips U_1' and U_5'.

The stresses in the leeward truss will now be worked out. The stresses in the windward truss are the same, but with opposite signs. The truss is that of Article 47. Here $V = \frac{10 \times 1.5}{2} \times \frac{25}{17} = 11\ 000$. Fig. 99 shows the truss with the loads in the correct position, the

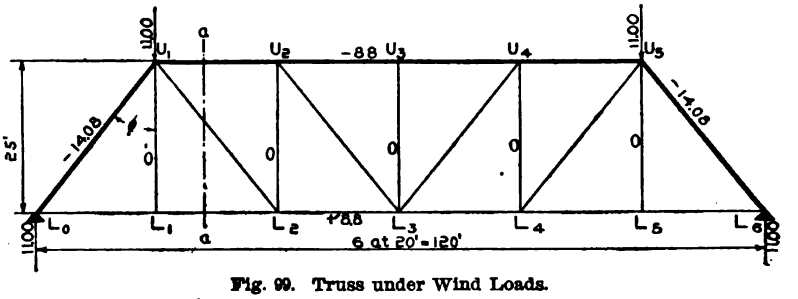


Fig. 99. Truss under Wind Loads.

reactions each being 11.00. $V_1 = +11.00$, and $V_2 = +11.00 - 11.00 = 0$. The shears in the 2d, 3d, 4th, and 5th panels are also zero. As the shear in these panels is zero, the stress in the diagonals and vertical posts is zero $\times$ secant ϕ = zero. The stress in the hip verticals U_1L_1 and U_5L_5 is zero, as there are no loads at L_1 and L_5. The stress in the end-post is $-11.00 \times 1.28 = -14.08$. Taking the center of moments at U_1, the stress equation of $L_0L_1 = L_1L_2$ is: $-L_1L_2 \times 25 + 11.00 \times 20 = 0$; whence $L_1L_2 = +8.8$. The stress in all the lower chord members will be found to be $+8.8$. By summing the horizontal forces at the section $a - a$, noting that, as U_1L_2 is zero, its component is also zero, there results: $+L_1L_2 + U_1U_2 = 0$; whence $U_1U_2 = -L_1L_2 = -(+8.80) = -8.80$. This is also the stress in all members of the top chord.

It is now seen that the overturning effect of the wind on the truss causes stresses only in the end-posts and chords. The wind on the lower chord causes no overturning effect, as it is transferred directly to the abutments.

53. **Overturning Effect of Wind on Train.** The wind blowing upon the train tends to overturn it, and in so doing the pressure on

the leeward stringer is increased and that on the windward stringer decreased by the same amount. This difference in pressures is transferred to the floor-beam and then to the panel points (see Fig. 100), where its value is:

$$\pm L = \frac{W \times (8.5 + a)}{b},$$

where W = Panel load due to wind on train;

8.5 = A constant established by the Specifications (see Article 26, p. 15);

a = Distance from base of rail to center line of lower chord. It may be taken as 3 feet in most cases, as this is approximately the usual depth of floor).

b = Distance center to center of trusses.

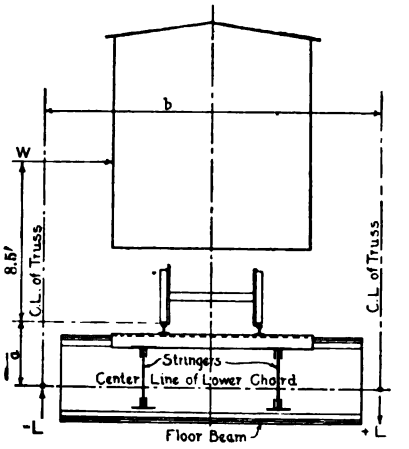


Fig. 100. Illustrating Overturning Effect of Wind on Train.

For the case in hand, $W = 20 \times 450 = 9\,000$. Therefore,

$$L = \pm \frac{9\,000 \times (8.5 + 3)}{17} = \pm 6\,090 \text{ pounds.}$$

The action of the wind in tending to overturn the train is the same as if the truss were under a live panel loading of L, the panel load L acting upward on the windward and downward on the leeward truss.

The chord stresses due to this will be proportional to the dead-load stresses in the same ratio as this panel load L is to the dead panel load. For the truss of Article 47, this ratio is $\frac{6\,090}{20\,000} = 0.303$, and the chord stresses caused by the overturning effect of the wind on the train (see Table VII) are:

$$U_1U_2 = -64.0 \times 0.303 = -19.39$$
$$U_2U_3 = -72.0 \times 0.303 = -21.82$$
$$L_0L_1 = L_1L_2 = +40.0 \times 0.303 = +12.12$$
$$L_2L_3 = +64.0 \times 0.303 = +19.39$$

The stress in U_1L_1 is $+6.09$, the panel load at L_1.

The maximum positive shears are:

$$V_1 = \frac{6.09}{6}(1 + 2 + 3 + 4 + 5) = +15.22$$

$$V_2 = \frac{6.09}{6}(1 + 2 + 3 + 4) = +10.15$$

$$V_3 = \frac{6.09}{6}(1 + 2 + 3) = +6.09$$

$$V_4 = \frac{6.09}{6}(1 + 2) = +3.05$$

and the maximum web stresses are found to be:

$$L_0U_1 = -15.22 \times 1.28 = -19.50$$
$$U_1L_2 = +10.15 \times 1.28 = +13.00$$
$$U_2L_3 = +\ 6.09 \times 1.28 = +\ 7.80$$
$$U_3L_4 = +\ 3.05 \times 1.28 = +\ 3.91$$

$$U_2L_2 = -6.09$$
$$U_3L_3 = -3.05$$

It is unnecessary to compute the shears further than one panel past the middle of the span, as only the maximum stresses are usually required.

The wind stresses from various causes are grouped together and given in Table X.

From Table X it is seen that large wind stresses occur in some of the members. Most specifications require that the stresses due to wind shall be neglected in the design unless they exceed 25 per cent of the sum of the dead-load and live-load stresses.

The subject of wind stresses does not ordinarily receive the consideration it should have; in fact, it appears to be common practice, in the case of spans up to 200 feet, to neglect the action of the wind in all members of the bridge except the top and bottom lateral diagonals, the top struts, the portal, and the bending effect in the end-post. For the last two effects mentioned, see the next succeeding article.

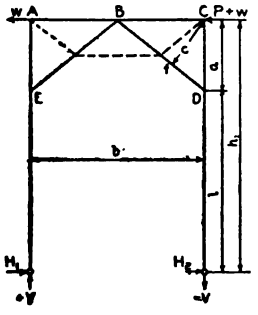


Fig. 101. Style of Portal Bracing in Common Use on Spans up to 250 Feet.

54. **Portals and Sway Bracing.** One-half of the wind on the top chord is transferred to the hips $U_1'U_1$ and $U_6'U_6$. From there it is carried to the abutments by means of the portal bracing and the end-posts. Various styles of portal bracing are in use, but few are so easily analyzed and constructed as that of Fig. 101. This form

TABLE X
Wind Stresses in Pratt Truss
WEB MEMBERS

Overturning	L_0U_1	U_1L_2	U_2L_2	U_2L_3	U_1L_1	U_2L_3	U_3L_3
Wind Right							
on Truss	−14.08	0	0	0	0	0	0
on Train	−19.50	−13.00	+7.80	+3.91	+6.09	−6.09	−3.05
Wind Left							
on Truss	+14.08	0	0	0	0	0	0
on Train	+19.50	−13.00	−7.80	−3.91	−6.09	+6.09	+3.05
Maximum +Stress	+33.58	+13.00	+7.80	+3.91	+6.09	+6.09	+3.05
Maximum −Stress	−33.58	−13.00	−7.80	−3.91	−6.09	−6.09	−3.05

CHORDS

Member	L_0L_1	L_1L_2	L_2L_3	U_1U_2	U_2U_3
Direct					
Wind Right	0	+ 8.82	+14.12	0	+ 5.29
	0	+26.46	+42.36		
Wind Left	− 8.82	−14.12	−15.88	− 5.29	− 7.06
	−26.46	−42.36	−47.64		
Overturning Truss					
Wind Right	+ 8.80	+ 8.80	+ 8.80	− 8.80	− 8.80
Wind Left	− 8.80	− 8.80	− 8.80	+ 8.80	+ 8.80
Overturning Train					
Wind Right	+12.12	+12.12	+19.39	−19.39	−21.82
Wind Left	−12.12	−12.12	−19.39	+19.39	+21.82
Maximum +Stress	+20.92	+56.20	+84.67	+28.19	+35.91
Maximum −Stress	−56.20	−77.40	−91.71	−33.48	−37.68

LATERAL SYSTEMS

Member	$U_1'U_2$	$U_2'U_3$	$U_2'U_3$	$U_3'U_3$	$L_0'L_1$	$L_1'L_2$	$L_2'L_3$
Wind Right							
on Truss	+6.95	+2.32	−3.0	−1.5	+11.60	+ 6.95	+ 2.32
on Train					+34.75	+23.15	+13.91
Wind Left							
on Truss	0	0	−3.0	−1.5	0	0	0
on Train					0	0	0
Maximum	+6.95	+2.32	−3.0	−1.5	+46.35	+30.10	+16.23

The stresses in $L_1'L_1$, $L_2'L_2$, and $L_3'L_3$ are not given in the above table. These members are the floor-beams, and the small stress due to wind is neglected in their design.

of portal is at present being used almost universally on all spans up to 250 feet.

Let it be required to analyze a portal of this form, all the distances being as indicated in Fig. 101; and let:

w = Wind panel load of upper chord;
m' = Number of panels in upper chord;

Then,

$$P = (m' - 1)\,w;$$

and,

$$V = \pm \left\{ (P + w) + w \right\} \frac{h_1}{b};$$

also,

$$H_1 = H_2 = \left\{ (P + w) + w \right\} \div 2.$$

The stress in BC, the center of moments being at D, is:

$$S_{BC} = -\frac{(P + w)\,a + H_2 l}{a} = -\left\{ (P + w) + H_2 \frac{l}{a} \right\}.$$

The stress in AB, the center of moments being at E, is:

$$S_{AB} = +\frac{wa + H_1 l}{a} = +w + H_1 \frac{l}{a}.$$

For the stress in BD, the center of moments is taken at C, and the perpendicular distance c to BD is determined. The stress in BD, then, is:

$$S_{BD} = +H_2 \frac{h_1}{c}.$$

The stress in BE is:

$$S_{BE} = -H_1 \frac{h_1}{c}.$$

It must be remembered that h_1 is not the height of the truss, but is the length of the end-post from L_0 to U_1.

For the truss of Article 47, $w = 1.5$; $m' = 4$; and $P = 4.5$. The value $h_1 = (20^2 + 25^2)^{\frac{1}{2}} = 32.0$ feet. The distance a must be so chosen that BD will not interfere with engines or other traffic which passes through the bridge. It will be assumed as 5 feet in this case. Then $V = (4.5 + 1.5 + 1.5)\,\frac{32}{17} = 14.08$; and $H_1 = H_2 = \frac{7.5}{2} = 3.75$; whence,

$$S_{BC} = -\left(6.0 + 3.75 \times \frac{27}{5}\right) = -26.25$$

$$S_{AB} = 1.5 + 3.75 \times \frac{27}{5} = +21.75$$

The distance $BD = \sqrt{\overline{BC}^2 + \overline{CD}^2} = \sqrt{\overline{8.5}^2 + \overline{5.0}^2} = 9.85$. Then, from similar triangles DCB and DFC, is obtained the proportion:

$$\frac{CF}{CD} = \frac{BC}{BD};$$

$$C = \frac{5 \times 8.5}{9.85} = 4.3 \text{ feet; and}$$

$$S_{BD} = +3.75 \times \frac{32.0}{4.3} = +27.90$$

$$S_{BE} = -3.75 \times \frac{32.0}{4.3} = -27.90$$

When the wind blows from the other side, the stresses in the diagonals are reversed, and those in the top are transposed. The members shown by broken lines take no stress. When the wind blows, the end-posts tend to bend as shown in Fig. 102. This is with-

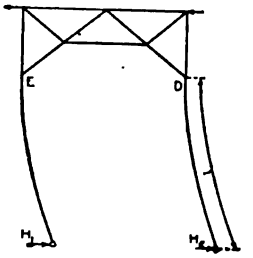


Fig. 102. Illustrating Tendency of End-Posts to Bend under Wind Load.

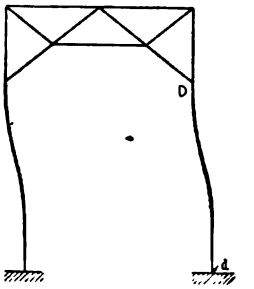


Fig. 103. Bending Tendency when End-Posts are Fixed at Lower End.

stood by the cross-section of the post at the points E and D. The bending moment caused at these points by the wind is $H_1 \times l$ and $H_2 \times l$. For the truss under consideration,

$$M_D = M_E = 3.75 \times 27 \times 12 = 1\,215\,000 \text{ lb.-ins.}$$

If the posts are fixed at the lower end, then they will tend to bend as shown in Fig. 103, the post resisting the bending at two points D and d. The section at each point withstands in this case only half of the moment just computed, or $1\,215\,000 \div 2 = 607\,500$ lb.-ins. A further discussion of this will be given in Part II, on "Bridge Design."

Various forms of sway bracing are used to connect the intermediate posts and thus stiffen the cross-section of the bridge at those points. The form of portal just given is often used, as is also the form shown in Fig. 104. Here h is the height of the truss. The braces BD are called *knee-braces*. Here w is the wind panel load of the top chord, and

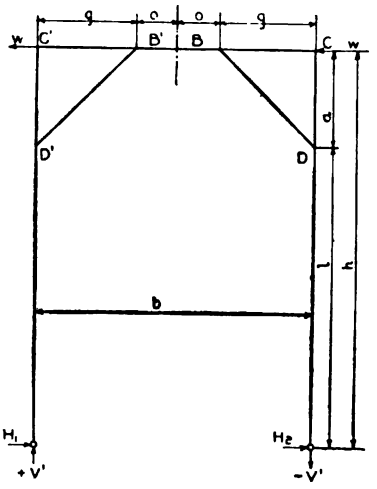


Fig. 104. A Type of Portal and Sway Bracing in Frequent Use.

$$V' = \pm \frac{2wh}{b}$$

$$H_1 = H_2 = \frac{2w}{2}$$

$$S_{BC} = -(wa + H_2 l) \div a$$

$$= -(w + H_2 \frac{l}{a})$$

$$S_{B'C'} = +(w + H_1 \frac{l}{a})$$

$$S_{BD} = +H_2 \frac{h}{c}$$

$$S_{B'D'} = -H_1 \frac{h}{c}$$

The stress in $B'B$ is the direct compression due to wind right or left, and differs in accordance with the position of the top strut. There is also a bending moment at B' and B, which is:

$$M_{B'} = M_B = -V'g + H_2 h.$$

The bending moment at D and D' is equal to $H_2 l$ or $H_2 l \div 2$, according to whether or not the lower ends of the posts are fixed.

The determination of the stresses for the truss of Article 47 is left to the student.

When the wind is from the other side of the truss, the signs of the stresses in the knee-braces and the members $C'B'$ and CB are reversed.

55. **Final Stresses.** The class of stresses which go to make up the maximum or minimum for which the member is designed, is determined by the specifications used. The dead-load and live-load stresses are always included, and then those due to impact and wind should be added if required. In computing the maximum stresses, the *algebraic* sum should always be used. In a large majority of cases, all stresses which go to make up the maximum have the

same sign, but some exceptions have been noted, as in the middle vertical of a Pratt or Howe truss. The minimum stresses are, with rare exceptions, obtained by combining stresses with signs of opposite character.

GIRDER SPANS

56. **Moments and Shears in Floor-Beams.** In any bridge the floor-beam acts as a support for either the joists or stringers, and the moments and shears occurring in it are due to the loads which come on the joists or stringers. In a highway bridge the joists are spaced so closely that the load which they transmit to the floor-beams may be considered as uniformly distributed, providing the live load is a uniform load, in which case,

$$M = \frac{(2P_L + P_D) \times \text{panel length in inches}}{8}$$

$$V = (2P_L + P_D) \div 2,$$

where M = Maximum moment in pound-inches;
V = Maximum shear;
P_L = Live panel load;
P_D = Weight of stringers and floor material in one panel.

It will be seen that these formulæ are those for the maximum moment and shear in a uniformly loaded beam, the total load being $2P_L + P_D$.

As an example, let it be required to determine the maximum moment and shear in the floor-beam of a highway bridge whose panels are 20 feet long, and trusses 16 feet center to center, the live load being 100 pounds per square foot of floor surface, the flooring weighing 10 pounds per square foot, and there being 5 lines of joists weighing 15 pounds per linear foot, and 2 lines of joists weighing 8 pounds per linear foot.

$$P_L = \frac{16}{2} \times 20 \times 100 = 16\,000 \text{ pounds.}$$

$$P_D = 5 \times 20 \times 15 + 2 \times 20 \times 8 + 16 \times 20 \times 10 = 5\,020 \text{ pounds.}$$

Therefore,

$$M = \frac{(2 \times 16\,000 + 5\,020) \times 20 \times 12}{8}$$

$$= 1\,110\,600 \text{ pound-inches at center of floor-beam.}$$

$$V = (2 \times 16\,000 + 5\,020) \div 2$$

$$= 18\,510 \text{ pounds at ends of floor-beam.}$$

In the case of a single-track railroad bridge, there are only two stringers upon which the weight of the track, the engine, and the train is supported. These join the floor-beam at points equally distant from the center of same. The weight of the ties, rails, and fastenings is usually taken at 400 pounds per linear foot of *one track.* As regards the live load, the proposition reduces itself to placing the wheel loads so that the sum of the reactions of stringers in the adjacent panels will be a maximum on the floor-beam under consideration. This is discussed in Article 47, page 87 (see Fig. 94).

In determining the values of the maximum moment and shear in the floor-beam, the case is that of a beam symmetrically loaded with two equal concentrated loads. Each load is equal to the dead weight of one stringer, one-half the track weight in one panel, and the maximum sum total of the reactions due to the wheel loads on the stringers in adjacent panels which meet at that point. This latter quantity is called the *floor-beam reaction.* For a general arrangement of the loads, see Fig. 105. The distance a has become standard for single-track spans, and is 6 feet 6 inches.

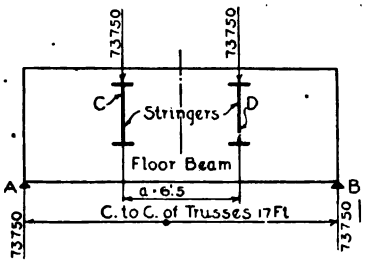


Fig. 105. Arrangement of Loads for Calculating Moments and Shears in Floor-Beams.

Let it be required to determine the maximum shears and moments in the floor-beam of the truss of Article 47.

The weight of the stringer may be obtained by the formula of Table II, and is:

$$\text{Stringer} = 20\,(123.5 + 10 \times 20) \div 2 = 3\,200 \text{ pounds.}$$

The weight of one-half of the ties, rails, etc., in one panel is:

$$\tfrac{1}{2}\ \text{Track} = (400 \times 20) \div 2 = 4\,000 \text{ pounds.}$$

The weight that comes from the engine wheels is given in Article 47, page 87 (see Fig. 94), and is 65.55. Each load is therefore the sum of all the above weights, as follows:

$$3\,200 + 4\,000 + 65\,550 = 73\,750.$$

The maximum shear (see Fig. 105) is seen to be 73 750 pounds; and the maximum moment occurs at C and D, and is:

$$M = 73\,750 \times \left(\frac{17}{2} - \frac{6.5}{2}\right) \times 12 = 4\,646\,250 \text{ pound-inches.}$$

For any particular engine the floor-beam reactions for different length panels are easily tabulated for future reference. Table XI gives the floor-beam reactions for panel lengths from 10 to 24 feet inclusive.

TABLE XI

Floor-Beam Reactions

E 40 Loading

Panel Length	Maximum Floor-Bm. Reaction	Panel Length	Maximum Floor-Bm. Reaction	Panel Length	Maximum Floor-Bm. Reaction
10	41 000	15	55 000	20	65.55
11	43 800	16	57 000	21	67.10
12	46 600	17	59 000	22	69.20
13	44 400	18	61 000	23	71.30
14	52 200	19	63 000	24	73.40

In many cases it is desirable to keep the dead-load shears and moments separate from those of the live load; and this can easily be done.

In neither of the above cases has the weight of the beam itself been taken into account. This should be done in the final design. The method of procedure is to compute the moment and shears as above; then make a provisional design of the beam. Next, compute the weight of the beam thus designed, and add the moments and shears caused by this weight to the other dead-load moments and shears; then re-design the beam and compute its weight. If this last weight varies 10 per cent from the previous weight, another re-design should be made. The above proceeding belongs to Bridge Design, Part II, and will there be treated.

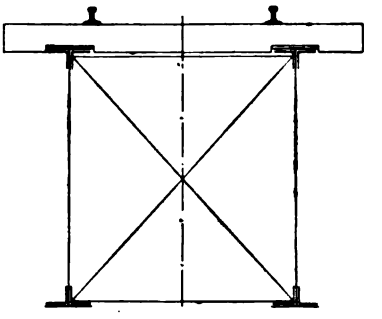


Fig. 106. Cross-Section of Deck Plate-Girder Railway Bridge.

57. **Moments in Plate-Girders.** Plate girders are of two classes —namely (1) those which have the ties or floor laid directly upon the upper flanges of the girders; these are called *deck plate-girder bridges*;

and (2) those in which the webs of the girders are connected with each other at intervals by floor-beams which in turn carry stringers or joists in exactly the same manner as in the floor system of a railroad or highway truss-bridge; this latter type is called a *through plate-girder bridge.* Figs. 106 and 107 show cross-sections of deck and through plate-girder bridges respectively, for railway service. Fig. 108 is a side view of a deck plate-girder bridge. Fig. 109 is a longitudinal section of a through plate-girder railroad bridge. The section is taken down the middle of the track. The bridge shown has 5 panels. An odd number of panels should be chosen, as this does not bring a floor-beam at the center of the span, and hence the great moment which would then be caused is avoided.

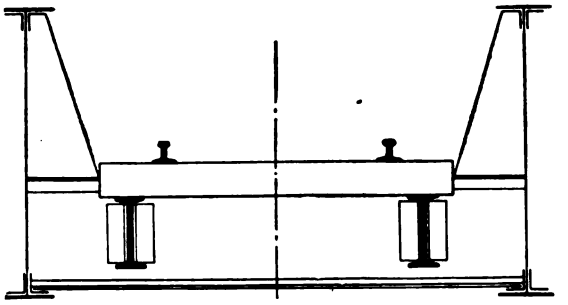


Fig. 107. Cross-Section of Through Plate-Girder Railway Bridge.

The analysis of the shears and moments of a through plate-girder is precisely the same as that for a truss bridge. The shear is

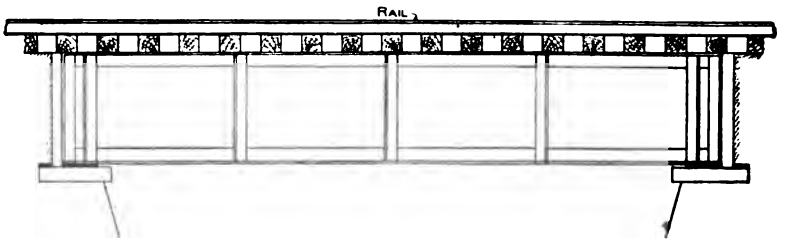


Fig. 108. Side View of Deck Plate-Girder Railway Bridge.

constant between any two panel points as 0-1 or 1-2, etc., and the moments are computed for the points 1, 2, 3, and 4.

If wheel loads are used for moments, the relation that $K = \frac{Wn}{m} - L$ must be $+$, and that $k = \frac{Wn}{m} - (L + P)$ must be $-$, holds true when the loads are in correct position for maximum moments. Here $m =$ the number of panels, and $n =$ the panel under considera-

tion and is to be reckoned from the left end; in fact, all terms have the same value as mentioned in Article 46. A careful review of Articles 44 and 46 should enable the student to follow the example which will now be given.

EXAMPLE. It is required to determine the moments at the points of floor-beam support for a 5-panel through plate-girder of 75-foot span. The live loading is Cooper's E 40.

Dead-Load Moments. Through plate-girders, on account of the heavy floor system and the fact that the floor system transfers its own weight and that of the live load to the girders as concentrated loads, are about 40 per cent heavier than deck plate-girder bridges of the same span. The weight of the entire span, therefore, is;

$$1.4 \times 75\,(123.5 + 10 \times 75) = 91\,700 \text{ pounds.}$$

Part of this 91 700 pounds (the weight of the girders themselves) acts as a uniform load; the remainder (the weight of the floor-beams and stringers) acts as concentrated loads at the points where the floor-beams join the web. Experience has shown that the weight of the floor for a single-track railroad system is about 400 pounds

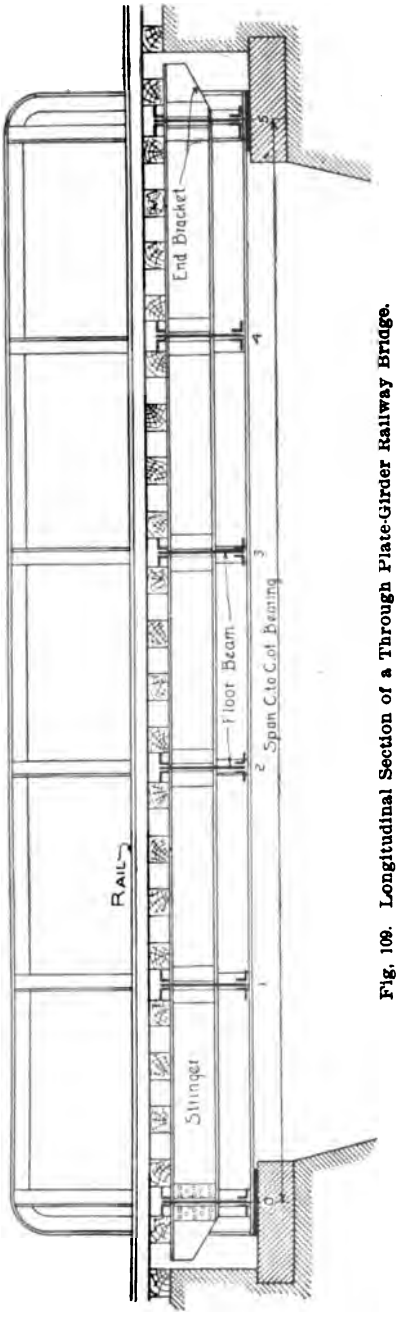


Fig. 109. Longitudinal Section of a Through Plate-Girder Railway Bridge.

TABLE XII

Wheel Position, Moments in a Through Plate-Girder

Panel Point	Wheel at Point	L	$\frac{Wn}{m}$	P	$L+P$	K	k	Remarks
1	2	10	$\frac{172}{5} = 34.4$	20	30	+	+	
1	3	30	$\frac{192}{5} = 38.4$	20	50	+	−	Maximum
1	4	40	$\frac{202}{5} = 40.4$	20	60	+	−	Maximum
1	5	40	$\frac{202}{5} = 40.4$	20	60	+	−	Maximum
1	6	40	$\frac{195}{5} = 39.0$	13	53	−	−	
2	4	50	$\frac{152 \times 2}{5} = 60.8$	20	70	+	−	Maximum
2	5	70	$\frac{172 \times 2}{5} = 68.8$	20	90	−	−	
3	6	90	$\frac{152 \times 3}{5} = 91.2$	13	103	+	−	Maximum
3	7	103	$\frac{172 \times 3}{5} = 103.2$	13	116	+	−	Maximum
3	8	116	$\frac{192 \times 3}{5} = 115.2$	13	129	−	−	
4	6	90	$\frac{129 \times 4}{5} = 103.2$	13	103	+	−	Maximum
4	7	103	$\frac{142 \times 4}{5} = 113.6$	13	116	+	−	Maximum
4	8	116	$\frac{152 \times 4}{5} = 121.6$	13	142	+	−	Maximum
4	9	129	$\frac{152 \times 4}{5} = 121.6$	13	142	−	−	

per linear foot. The weight of the stringers and floor-beams for this bridge is therefore $75 \times 400 = 30\,000$ pounds, and $91\,700 - 30\,000 = 61\,700$ pounds acts as a uniform load. This 61 700 pounds is distributed over two girders, and so gives $61\,700 \div (2 \times 75) =$ say, 412 pounds per linear foot of one girder.

The dead load which is concentrated at each panel point is that due to the weight of the steel floor and the weight of ties, rails, and fastenings. It is, for one girder,

$$15 \times (400 + 400) \div 2 = 6\,000 \text{ pounds.}$$

The dead-load moments are now computed by the methods of Strength of Materials, and are found to be:

$M_0 = M_5 = 0;$
$M_1 = M_4 = +4\,390\,000$ pound-inches;
$M_2 = M_3 = +6\,580\,000$ pound-inches.

Live-Load Moments. The positions of the wheels for maximum moments are now determined (see Table XII).

The computations for the reactions are best arranged in tabular form. Table XIII gives the values.

TABLE XIII

Reactions for a Through Plate-Girder

Point	Wheel at Point	Equation for Reaction	Reaction
1	3	$R = (6\,708 + 192 \times 4) \div 75$	99.7
1	4	$R = (7\,668 + 212 \times 4 - 10 \times 78) \div 75$	103.2
1	5	$R = (8\,728 + 232 \times 4 - 10 \times 83 - 20 \times 75) \div 75$	97.8
2	4	$R = (4\,632 + 152 \times 7) \div 75$	76.0
3	6	$R = (4\,632 + 152 \times 6) \div 75$	73.9
3	7	$R = (5\,848 + 172 \times 3) \div 75$	84.9
4	6	$R = (2\,851 + 129 \times 4) \div 75$	44.9
4	7	$R = (3\,496 + 142 \times 4) \div 75$	54.2
4	8	$R = (4\,632 + 152 \times 2) \div 75$	65.4

The live-load moments are computed as follows:

Point 1:
- Wheel 3, $M = 99.7 \times 15 - 230 = 1\,265\,000$ pound-feet.
- Wheel 4, $M = 103.2 \times 15 - 20 \times 5 - 20 \times 10 = 1\,247\,000$ pound-feet.
- Wheel 5, $M = 97.8 \times 15 - 20 \times 5 - 20 \times 10 = 1\,167\,000$ pound-feet.

Point 2 Wheel 4, $M = 76.0 \times 30 - 480 = 1\,800\,000$ pound-feet.

Point 3:
- Wheel 6, $M = 73.9 \times 45 - 1\,640 = 1\,785\,000$ pound-feet.
- Wheel 7, $M = 84.9 \times 45 - 2\,155 = 1\,665\,000$ pound-feet.

Point 4:
- Wheel 6, $M = 44.9 \times 60 - 1\,640 = 1\,054\,000$ pound-feet.
- Wheel 7, $M = 54.2 \times 60 - 2\,155 = 1\,097\,000$ pound-feet.
- Wheel 8, $M = 65.4 \times 60 - 2\,851 = 1\,073\,000$ pound-feet.

The above values show that the greatest live-load moments are:

M at Point 1, by wheel 3 = 1 265 000 pound-feet
M at " 2, " " 4 = 1 800 000 " "
M at " 3, " " 4 = 1 800 000 " "
M at " 4, " " 3 = 1 265 000 " "

The last two values are obtained when the load comes on the bridge from the left. Inspection of the results obtained at points 3 and 4 when the load comes on from the right, shows that they are considerably smaller than the results obtained at their symmetrical points 1 and 2, and therefore it was not necessary to determine the moments for any points to the right of the center. This is true of all girder spans, deck or through.

The method of procedure when the girder is a deck plate-girder is the same as that just illustrated, except that in the computation of the dead-load moments there is no concentration of certain portions of the dead load, the weight of the girders themselves being a uniform load, as is also the weight of the ties and rails or, if it be a highway bridge, the floor-joists which run transversely. Highway spans are seldom built of deck plate-girders, it being preferable to use the through girders, as then the girders themselves serve as a railing and keep the traffic confined to the roadway. The girder span is usually divided into ten equal divisions, the points of division being called the *tenth-points*. The shears and moments are computed for the center point and those points which lie to the left of the center. After the values are computed, they are laid off as ordinates, with the corresponding tenth-points as abscissæ. A curve is then drawn through their upper ends, and the *curve of maximum shears or moments* is the result. To get the maximum shear or moment at any point other than a tenth-point, the ordinate is scaled at the desired point.

EXAMPLE. Let it be required to determine the maximum moments at the tenth-points of a 100-foot-span deck plate-girder.

Dead-Load Moments. The weight of steel in the span is 100 $(123.5 + 10 \times 100) = 112\,350$ pounds, and the weight of the track is $400 \times 100 = 40\,000$ pounds, making a total of 152 350 pounds, or $152\,350 \div (2 \times 100) =$ say, 762 pounds per linear foot per girder. The dead-load moments are now determined according to the methods of Strength of Materials, and are:

$M_1 = 342\,800$ pound-feet
$M_2 = 609\,400$ " "
$M_3 = 799\,840$ " "
$M_4 = 914\,100$ " "
$M_5 = 952\,200$ " "

Live-Load Moments. The determination of the wheel load

positions is made by the use of the formulæ $K = (\frac{Wn}{m} - L)$ and $k = \frac{Wn}{m} - (L + P)$; only, in this case, n is the number of divisions from the left support to the section, and m is the number of divisions into which the girder is divided.

The determination of the wheel positions is given in Table XIV.

TABLE XIV

Wheel Positions, Moments in Deck Plate-Girder

Point	Wheel at Point	L	$\frac{Wn}{m}$	P	$L+P$	K	k	Remarks
1	2	10	258 × 0.1 = 25.8	20	30	+	−	Maximum
1	3	20	261 × 0.1 = 26.1	20	40	+	−	"
1	4	20	254 × 0.1 = 25.4	20	40	+	−	"
1	5	20	242 × 0.1 = 24.2	20	40	+	−	"
1	6	20	240 × 0.1 = 24.0	13	33	+	−	"
1	7	13	230 × 0.1 = 23.0	13	26	+	−	"
2	2	10	232 × 0.2 = 46.4	30	30	+	+	
2	3	30	245 × 0.2 = 49.0	20	50	+	−	Maximum
2	4	50	258 × 0.2 = 51.6	20	70	+	−	"
2	5	60	261 × 0.2 = 52.2	20	80	−	−	
3	3	30	232 × 0.3 = 69.6	20	50	+	+	
3	4	50	232 × 0.3 = 69.6	20	70	+	−	Maximum
3	5	70	245 × 0.3 = 73.5	20	90	+	−	"
3	6	80	261 × 0.3 = 78.3	13	93	−	−	
4	4	50	212 × 0.4 = 84.8	20	70	+	+	
4	5	70	232 × 0.4 = 92.8	20	90	+	−	Maximum
4	6	90	245 × 0.4 = 98.0	13	103	+	−	"
4	7	103	258 × 0.4 = 103.2	13	116	+	−	"
4	8	106	261 × 0.4 = 104.4	13	119	−	−	
5	6	90	232 × 0.5 = 116.0	13	103	+	+	
5	7	103	232 × 0.5 = 116.0	13	116	+	0	Maximum
5	8	116	245 × 0.5 = 127.5	13	129	+	−	"
5	9	129	258 × 0.5 = 129.0	13	142	0	−	"
5	10	132	274 × 0.5 = 137.0	10	142	+	−	"
5	11	102	244 × 0.5 = 122.0	20	122	+	0	"
5	12	102	234 × 0.5 = 117.0	20	122	+	−	"
5	13	102	224 × 0.5 = 112.0	20	122	+	−	"
5	14	122	234 × 0.5 = 117.0	20	132	−	−	

While many wheels on point 1 satisfy the condition, the greatest moment will occur when one of the large drivers is at the point, and it is therefore unnecessary to examine the point for other wheels. The same is true at the center point, 5, the maximum occurring under one of the heavy driver wheels. The reactions and the computations for the same are given in Table XV.

TABLE XV

Reactions for a Deck Plate-Girder

Point	Wheel at Point	Equation for Reaction	Reaction
1	2	$R = (12\,041 + 5 \times 258) \div 100$	133.31
1	3	$R = (12\,599 + 4 \times 261) \div 100$	136.43
1	4	$R = (11\,984 + 4 \times 254) \div 100$	130.00
1	5	$R = (11\,334 + 4 \times 234 \div 2 \times 4^2 \div 2) \div 100$	122.86
2	3	$R = 12\,041 \div 100$	120.41
2	4	$R = (12\,041 + 5 \times 258) \div 100$	133.31
3	4	$R = 10\,816 \div 100$	108.16
3	5	$R = 12\,041 \div 100$	120.41
4	5	$R = (8\,728 + 4 \times 232) \div 100$	96.56
4	6	$R = (10\,816 + 4 \times 245) \div 100$	117.96
4	7	$R = (12\,041 + 4 \times 258) \div 100$	130.73
5	7	$R = (8\,728 + 8 \times 232) \div 100$	105.84
5	8	$R = 12\,041 \div 100$	120.41
5	9	$R = (12\,041 + 5 \times 258) \div 100$	133.31
5	10	$R = (13\,904 + 2 \times 274) \div 100$	144.52
5	11	$R = (11\,334 + 5 \times 234 + 2 \times 5^2 \div 2) \div 100$	125.29
5	12	$R = (9\,514 + 10 \times 214 + 2 \times 10^2 \div 2) \div 100$	117.54
5	13	$R = (7\,794 + 15 \times 194 + 2 \times 15^2 \div 2) \div 100$	109.29

TABLE XVI

Maximum Moments in a Deck Plate-Girder

Point	Wheel at Point	Equation for Moment	Moment in Pound-Feet
1	2	$M = 133.31 \times 10 - 80$	1 253 100
1	3	$M = 136.43 \times 10 - 5 \times 20$	1 264 000
1	4	$M = 130.00 \times 10 - 5 \times 20$	1 200 000
1	5	$M = 122.86 \times 10 - 5 \times 20$	1 128 000
2	3	$M = 120.41 \times 20 - 230$	2 178 200
2	4	$M = 133.31 \times 20 - 480$	2 186 200
3	4	$M = 108.16 \times 30 - 480$	2 764 800
3	5	$M = 120.41 \times 30 - 830$	2 782 300
4	5	$M = 96.56 \times 40 - 830$	3 032 400
4	6	$M = 117.96 \times 40 - 11\,640$	3 078 400
4	7	$M = 130.73 \times 40 - 2\,155$	3 074 200
5	7	$M = 105.84 \times 50 - 2\,155$	3 137 000
5	8	$M = 120.41 \times 50 - 2\,851$	3 169 500
5	9	$M = 133.31 \times 50 - 3\,496$	3 169 500
5	10	$M = 144.52 \times 50 - 4\,072$	3 154 000
5	11	$M = 125.29 \times 50 - 3\,068$	3 196 500
5	12	$M = 117.54 \times 50 - 2\,658$	3 219 000
5	13	$M = 109.29 \times 50 - 2\,248$	3 216 500

Table XVI gives the computations of the live-load moments at the tenth-points, the final results being in pound-feet.

Whenever any loads were off the left end of the bridge, the lines 7 to 16 of the engine diagram were used (Fig. 85). For example, with wheel 10 at 5, wheel 1 would be off the left end. By looking in the second space of line 8, there is found the quantity 13 904, which is the moment of wheels 2 to 18 inclusive about a point directly under wheel 18. Just to the right of the vertical line through wheel 18, is the value 284, which is the weight of wheels 1 to 18 inclusive; but this must be decreased by 10, the weight of wheel 1, as that wheel is off the span. As wheel 18 is 2 feet from the right end of the girder, the moment about the point is $13\,904 + 274 \times 2$. By looking in the second space of line 16, the value 4 072 is found. This is the value of the moment of loads 2 to 9 inclusive about a point directly under wheel 10, and must be subtracted from the moment of the reaction in order to get the moment at 5 for this loading. See Articles 21 and 47 for further information regarding the use of the values in lines 7 to 16 of the engine diagram.

By the help of differential calculus it can be proved that *the greatest possible moment does not occur at the middle of a beam loaded either with concentrated loads or with concentrated loads followed by a uniform load, but it occurs under the load nearest the middle of the beam when the loads are so placed that the middle of the beam is half way between the center of gravity of all the loads and the nearest load.*

The wheel which produces this greatest moment is the same one which produces the maximum moment at the middle of the beam. The exact solution of this problem involves the use of quadratic equations, but for all practical purposes the following rule will suffice:

> Place the loading so that the wheel which produces the maximum moment at the middle of the beam is at that point. Find the distance of the center of gravity of all the loads from the right end. Move the loads so that the middle of the beam is half way between the center of gravity as found above and the load which produced the maximum moment at the middle of the beam. Find the moment under that load, with the loads in the position just mentioned.

For the case in hand, wheel 12 at 5 gives the maximum moment. The moment at the right end of the span, wheel 12 being at 5, is:

$$9\,514 + 10 \times 214 + 2 \times \overline{10}^2 \div 2 = 11\,754\,000 \text{ pound-feet.}$$

The center of gravity is $\frac{11\ 754\ 000}{234} = 50.2$ feet from the right support, or 0.2 foot to the left of the center of the girder. Now place wheel 12 one-tenth of a foot to the right of the center, and determine the moment under it. The reaction will be:

$$R = (9\ 514 + 9.9 \times 214 + 2 \times \overline{9.9}^2 \div 2) \div 100 = 117.306;$$

and,

$$M = 117.306 \times 50.1 - 2\ 658 = 3\ 219\ 306 \text{ pound-feet.}$$

In this particular case the difference between the greatest moment possible and the greatest moment at the middle is not sufficient to warrant the extra labor involved in computing it. In general it may be said that *if the greatest moment possible occurs within six inches of the middle of the beam, it is not necessary to compute it, the moment at point* 5 *being taken.*

58. **Shears in Plate-Girders.** In the case of through plate-girders, the maximum live-load shears are determined by placing the wheels in such a position that $Q = \frac{W}{m} - G$ is $+$, and $q = \frac{W}{m} - (G + P)$ is $-$.

In these equations, m is the number of panels into which the span is divided; and the other quantities are the same as given in Article 45, which should now be reviewed.

For example, let it be required to determine the dead and live load shears in the through plate-girder of Article 57, p. 111.

The weight of one girder = 61 700 ÷ 2 =	30 850 lbs.
" " " ½ the floor = 5 × 6 000 =	30 000 lbs.
Total weight on one girder =	60 850 lbs.

The dead-load shears are then computed by the methods given in Strength of Materials, and are given as follows, it being remembered that the concentrated load which comes at the end is one-half a panel load, or 3 000 pounds:

$$V_0 = 60\ 850 \div 2 = 30\ 475 \text{ lbs.} = \text{end shear};$$

$$V_1 = 30\ 475 - 3\ 000 - \frac{30\ 850}{5} = 21\ 305 \text{ lbs.};$$

$$V_2 = 30\ 475 - 3\ 000 - 6\ 000 - 2 \times \frac{30\ 850}{5} = 9\ 135 \text{ lbs.}$$

$$V_3 = 30\ 475 - 3\ 000 - 2 \times 6\ 000 - 2 \times \frac{30\ 850}{5} = 3\ 135 \text{ lbs.}$$

$$V_c = 0,$$

where V_0 = the shear at the end; V_1 = the shear just to the left of point 1; V_2 = the shear just to the left of point 2; V_3 = the shear just to the right of point 2; and V_C = the shear at the middle of the girder.

The determination of the wheel load position for maximum live-load shears is given in Table XVII. By comparing the formulæ Q and K, it will be seen that for the first panel $Q = K$, and $q = k$, as $n = 1$. The position of wheel loads for maximum moments at point 1 is the same as for maximum shear in the first panel. According to Table XII, wheels 3, 4, and 5 at point 1 all give maximum shears in the first panel. In this case, as in previous ones, only the shear for the first position of the loading found for any particular point will be determined, as the difference between this and the other cases is too small to warrant the additional labor necessary in computing them. It is evidently unnecessary to go past panel 3, as only the maximum shears are required.

TABLE XVII

Wheel Positions, Shears in a Through Plate-Girder

($m = 5$)

Point	Wheel at Point	G	$\frac{W}{m}$	P	$P+G$	Q	q	Remarks
1								See Table XII, and text above this table
2	2	10	142 ÷ 5 = 28.4	20	30	+	−	Maximum
2	3	30	152 ÷ 5 = 30.4	20	50	+	−	Maximum
2	4	40	152 ÷ 5 = 30.4	20	60	−	−	
3	2	10	116 ÷ 5 = 23.2	20	30	+	−	Maximum
3	3	30	116 ÷ 5 = 23.2	20	50	−	−	

For wheel 3 at point 1, the left reaction (see Table XIII) is 99.7. That portion of wheels 1 and 2 which is transferred to point 0 is 230 ÷ 15 = 15.33; and the shear in the first panel, therefore, is:

$$V_1 = 99.70 - 15.33 = +84.37.$$

For wheel 2 at point 2, the left reaction is:

$$R = (3\,496 + 142 \times 5) \div 75 = 56.10;$$

therefore,

$$V_2 = 56.10 - \frac{80}{15} = +\,50.67.$$

A computation with wheel 3 at point 2 will give a shear only 560 pounds greater, which difference would not influence the design to any appreciable extent.

For wheel 2 at point 3, the left reaction is:

$$R = (2\,155 + 116 \times 1) \div 75 = 30.30;$$

therefore,

$$V_3 = 30.3 - \frac{80}{15} = +24.97.$$

When the girder is a deck one, the computation of the dead shears is very much simplified, as all of the load is uniform.

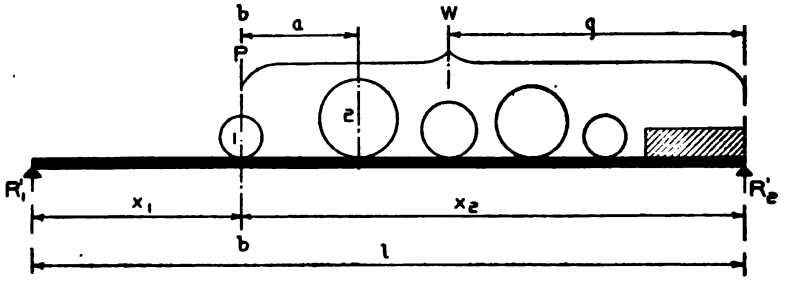


Fig. 110. Beam under Wheel Loads Followed by Uniform Load.

Let it be required to determine the dead and live load shears at the tenth-points of the deck plate-girder of Article 57.

The total weight of one girder and track is 76 175 lbs.

$$V_0 = 76\,175 \div 2 = +38\,088 \text{ pounds.}$$

$$V_1 = 38\,088 - \frac{76\,175}{10} = +30\,470 \text{ pounds.}$$

$$V_2 = 38\,088 - \frac{2}{10} \times 76\,175 = +22\,450 \text{ pounds.}$$

$$V_3 = 38\,088 - \frac{3}{10} \times 76\,175 = +15\,230 \text{ pounds.}$$

$$V_4 = 38\,088 - \frac{4}{10} \times 76\,175 = +\ 7\,618 \text{ pounds.}$$

$$V_5 = 0.$$

The position of the wheel loads to produce the maximum shear cannot be determined by the same relation as that used in structures which have a system of floor-beams and stringers, for here not a portion, but all of the load to the left of the section, must be subtracted from the left reaction in order to give the shear.

The correct relation for the wheel load position will now be deduced.

Let Fig. 110 represent a beam of span l loaded with a series of wheel loads followed by a uniform load. Let P equal the weight of the first wheel, W equal the weight of all the loads, and g the distance from the center of gravity of all of the loads to the right abutment. The distance between the first and second wheel centers is a, and the first wheel is at the section $b-b$ at a distance x_1 from the left support. Then,

$$R_1' = \frac{Wg}{l};$$

and

$$V'_{b-b} = R_1' - (\text{loads to left of section}) = \frac{Wg}{l}.$$

Now, assume that the loads move forward the distance a. The wheel 2 will be at section $b-b$, and Fig. 111 will represent the position of the loads. Then,

$$R_1'' = \frac{W(g+a)}{l};$$

and

$$\begin{aligned} V''_{b-b} &= R_1'' - P \\ &= \frac{W(g+a)}{l} - P \\ &= \frac{Wg}{l} + \frac{Wa}{l} - P. \end{aligned}$$

It is now evident that in order to get the greatest shear at section $b-b$, wheel 2 must be placed at the section whenever V''_{b-b} is greater than V'_{b-b}. Then,

$$V''_{b-b} > V'_{b-b};$$

$$\frac{Wg}{l} + \frac{Wa}{l} - P > \frac{Wg}{l}.$$

Now, canceling out the term $\frac{Wg}{l}$, which appears on both sides of the equation, there results:

$$\frac{Wa}{l} > P.$$

For the engine under consideration, $a = 8$ feet, and $P = 10\,000$ pounds, and the equation reduces to:

$$W > \frac{10.0}{8} l;$$

$$W > 1\tfrac{1}{4} l,$$

which is to say that when the load on the girder is greater than $1\frac{1}{4}$ times the span, then wheel 2 should be placed at the section in order to give the maximum shear.

For loading E 40, the following is true:

For all sections up to and including the center of all spans, place wheel 2 at the section to give the maximum shear.

In Fig. 111 it is immaterial whether or not any additional loads come on the span at the right end when the loads move forward the

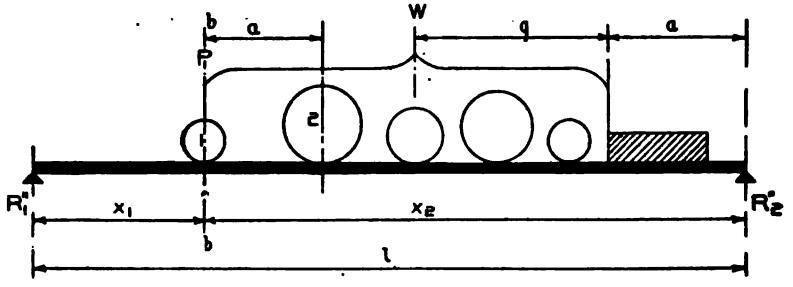


Fig. 111. Beam of Fig. 110 with Loads Moved Forward.

distance a, as they would only tend to increase the left reaction and therefore the shear V''_{b-b}. If the relation deduced is true for the case when no extra loads come on at the right end, it will be true when they do.

The live-load shears at the left end and at the tenth-points, wheel 2 being at the section in all cases, are computed from the general formula, which is:

$$V = R - \Sigma P,$$

in which,

R = Left reaction;

ΣP = All loads to left of section, and is equal to 10 000 pounds for all sections except the end of the girder.

The computations and results can be conveniently placed in tabular form, and are given in Table XVIII.

In order to illustrate the use of the relation $W > 1\frac{1}{4}\,l$, let point 3 in the above span be taken. Place wheel 2 at point 3; then, as wheels 1 to 13 are on the girder, the total weight W is 212. As l = 100, $1\frac{1}{4}l$ = 125. Therefore, as 212 is greater than 125, wheel 2 is the correct wheel.

TABLE XVIII

Maximum Shears in a Deck Plate-Girder

Point	Reaction Equation	R	ΣP	V	Remarks
0	$(13\,904+4\times274)\div100$	150.00	0	150.00	Wheel 18, 4 ft. from rt. end
1	$(12\,041+5\times258)\div100$	133.31	10	123.31	Wheel 16, 5 ft. from rt. end
2	$10\,816\div100$	108.16	10	98.16	Wheel 15 at right end
3	$(7\,668+4\times212)\div100$	85.16	10	75.16	Wheel 13, 4 ft. from rt. end
4	$(5\,848+4\times172)\div100$	65.36	10	55.36	Wheel 11, 4 ft. from rt. end
5	$(4\,632+2\times152)\div100$	49.36	10	39.36	Wheel 10, 2 ft. from rt. end

The curves of maximum live-load moments and shears are shown in Fig. 112. They should always be drawn. From them the shear or moment at any desired section can be determined. For example, let it be desired to determine the maximum live-load shear and moment at a point 24 feet from the left end of the girder. By drawing the ordinate, shown by a broken line in Fig. 112, and scaling, the following values are found:

V_{24} = 88 000 pounds;
M_{24} = 2 440 000 pound-feet.

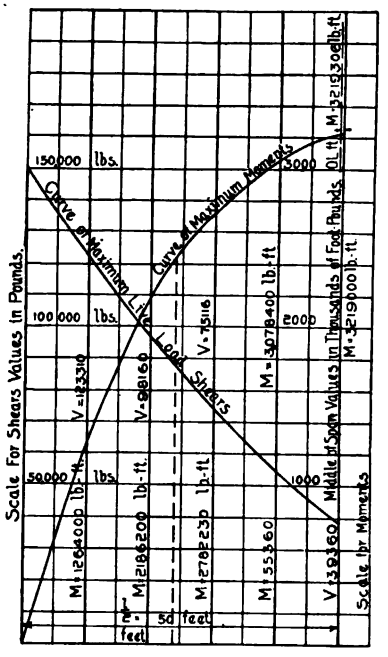


Fig. 112. Curves of Maximum Live-Load Moments and Shears.

A similar set of curves for the dead-load shears and moments should be made. The set for the deck plate-girder in hand is shown in Fig. 113. These are easily constructed by laying off the maximum values of the shear at the end, and the maximum value of the moment at the center. The shear curve is a straight line from the end to the center, while the moment curve is a parabola from the center to the end.

The stresses in the lateral systems of plate-girders are computed in a manner the same as that employed for the lateral systems of trusses, the unit-load being taken according to the specifications used.

59. **Stresses in Plate-Girders.** The stresses in plate-girders are treated in the Instruction Paper on Steel Construction, Part IV, pages 251 to 263, and the student is referred to this treatise for information regarding this subject.

The stress in the flange is seen to depend upon the distance from center of gravity to center of gravity. This distance, in turn, depends

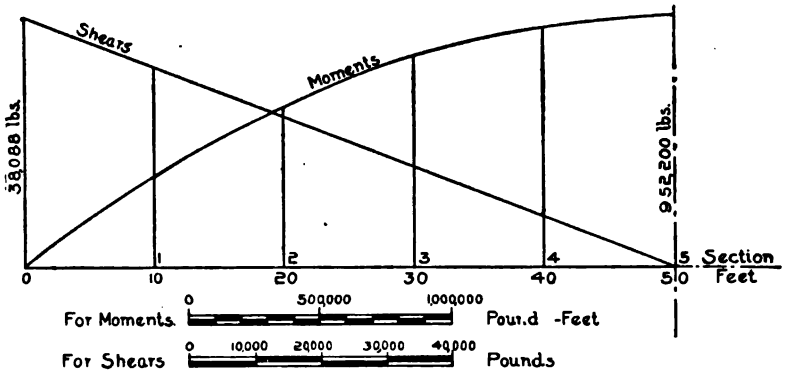


Fig. 113. Curves of Dead-Load Shears and Moments in a 100-Foot Span Deck Plate-Girder.

upon the depth of the girder. Certain approximate rules have been proposed in order to determine this, but the following formula will give the width of the web plate in accordance with best modern practice:

$$d = \frac{l}{0.005\,l + 0.543},$$

in which

d = Width of the web plate, in the even inch;
l = Span, in feet.

For example, let it be required to determine the width of the web plate of a plate-girder of 80-foot span center to center of end bearings.

$$d = \frac{80}{0.005 \times 80 + 0.543} = \frac{80}{0.94} = 85.2 \text{ (say 86) inches.}$$

If the resultant value had been 85 inches, the width would have been taken as either 84 or 86. The reason for this is that the wide plates kept in stock at the mills are usually the even inch in width and can therefore be procured more quickly than if odd-inch widths were ordered, in which case the purchaser would be forced to wait until

they were rolled—often a period of several months. The distance back to back of flange angles, the so-called *depth* of girder, is one-half inch more than the width of the web. This is due to the fact that each pair of flange angles extend one-fourth inch beyond the edge of the web plate, so as to keep any small irregularities caused on the edge of the web plate by the rolling, from extending beyond the backs of the angles.

EXERCISES AND PROBLEMS

1. Determine the maximum positive shears in the first six panels of a 9-panel 114-foot Pratt truss, the live panel load being 8.0. Use the exact and also the conventional method.

ANSWER:

	x	EXACT SHEARS	CONVENTIONAL SHEARS
V_1	16	+32.00	+32.00
V_2	14	+24.54	+24.90
V_3	12	+18.05	+18.70
V_4	10	+12.48	+13.35
V_5	8	+ 7.85	+ 8.90
V_6	6	+ 4.50	+ 5.34

2. Find the maximum and minimum stresses in L_1U_2 and U_3L_3 of an 8-panel 160-foot through Warren truss. Height 20 ft.; dead panel load 10.00, all on lower chord; live panel load 12.00.

ANSWER: In L_1U_2: d. l., -28.00; l. l., -35.30 and $+1.68$; max., -63.30; min., -26.32. In U_3L_3: d. l., $+16.80$; l.l., $+25.20$ and -5.04; max., $+42.00$; min., $+11.76$.

3. In the truss of Problem 2, determine the maximum stress in L_2L_3 by the method of moments, and also by the tangent method.

ANSWER: d. l. $= +67.50$; l. l. $= +81.00$; max. $= +148.50$.

4. Determine the dead-load stresses in the members U_2L_2 and L_4U_5 of a 9-panel 180-foot through Warren truss. Height is 24 feet; dead panel load is 10.0, one-third being at each panel point of the upper chord, and two-thirds being at each panel point of the lower chord.

ANSWER: $U_2L_2 = +30.60$; $L_4U_5 = -1.80$.

5. Determine the stress in the counter of a through Howe truss of 8 panels and 160-foot span. Height is 30 ft.; dead panel load, 9.6; live panel load, 11.5.

ANSWER: -4.59.

6. In the truss of Problem 5, determine the maximum and minimum stress in U_2L_2, L_2U_3, and L_3U_4.

ANSWER:

	U_2L_2	L_2U_3	L_3U_4
d. l.	+20.80	−17.30	− 5.76
l. l.	+30.30	−25.90	−17.30
l. l.	−14.40	+ 5.18	0.00
Max.	+15.10	−43.20	−23.06
Min.	+19.36	−21.20	± 0.00

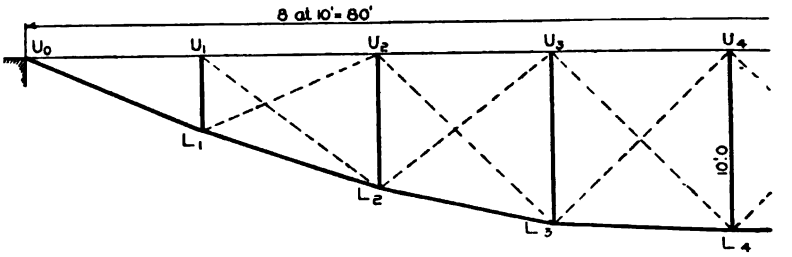


Fig. 114. Deck Parabolic Bowstring Truss.

7. In the deck parabolic bowstring truss of Fig. 114, determine the maximum stress in L_1L_2, L_1U_2, and U_3L_3. The dead panel load is 4.0, all on upper chord; and the live panel load, 20.0.

ANSWER: $L_1L_2 = +201.9$; $L_1U_2 = +21.8$; $U_3L_3 = -33.6$.

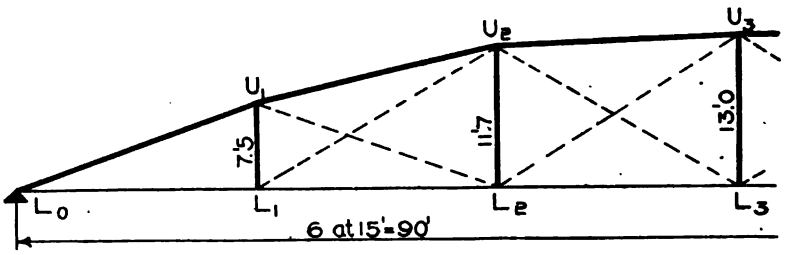


Fig. 115. Through Bowstring Truss.

8. In the through bowstring truss of Fig. 115, determine the maximum stress in U_1L_2 and L_1U_2, the dead panel load being 5.0, and the live panel load 15.0.

ANSWER: $U_1L_2 = +33.50$; $L_1U_2 = +38.0$.

9. Determine the maximum and minimum stresses in the members U_1L_1, U_1L_2, U_2L_2, and U_3L_3 of a 7-panel 175-foot through Pratt truss 30 feet high. Dead panel load is 10.0, all on lower chord; live panel load is 15.0.

ANSWER:

	U_1L_1	U_1L_2	U_2L_2	U_3L_3
d. l.	+10.0	+26.00	−10.00	0.00
l. l.	+15.0	+41.70	−21.40	−12.85
l. l.	0.0	− 2.78	+ 6.42	0.00
Max.	+25.0	+67.70	−31.40	−12.85
Min.	+10.0	+23.22	− 3.58	0.00

10. Determine the maximum and minimum stresses in the members U_1m_2, m_3L_3, U_2L_2, and m_2U_2 of the deck Baltimore truss shown in Fig. 116. Dead panel load, 30 000 lbs.; live panel load, 50 000 lbs. One-third of dead panel load is applied at the lower ends of all the verticals.

ANSWER:

	U_1m_2	m_3L_3	U_2L_2	m_2U_2
d. l.	+190.8	+ 84.8	−110.0	+21.2
l. l.	+333.5	+191.5	−211.0	+56.6
l. l.	− 15.1	− 50.5	+ 10.7	0.0
Max.	+524.3	+276.3	−321.0	+56.6
Min.	+175.7	+ 34.3	− 99.3	+21.2

11. In the truss of Problem 10, determine the maximum stress in M_2U_2 and L_3L_4.

ANSWER: $M_2U_2 = -840.0$; $L_3L_4 = +960.0$.

12. Determine the position of the wheel loads of Cooper's E 40 loading to produce the maximum positive live-load shears in the panels of a 7-panel 175-foot Pratt truss.

ANSWER: L_1, wheel 4; L_2, wheels 3 and 4; L_3, wheel 3; L_4, wheel 3; L_5, wheel 2; L_6, wheel 2.

13 Determine the maximum positive live-load shears for the truss of Problem 12.

ANSWER: $V_1 = 192.8$; $V_2 = 137.8$; $V_3 = 90.8$; $V_4 = 52.6$; $V_5 = 25.0$; $V_6 = 6.8$.

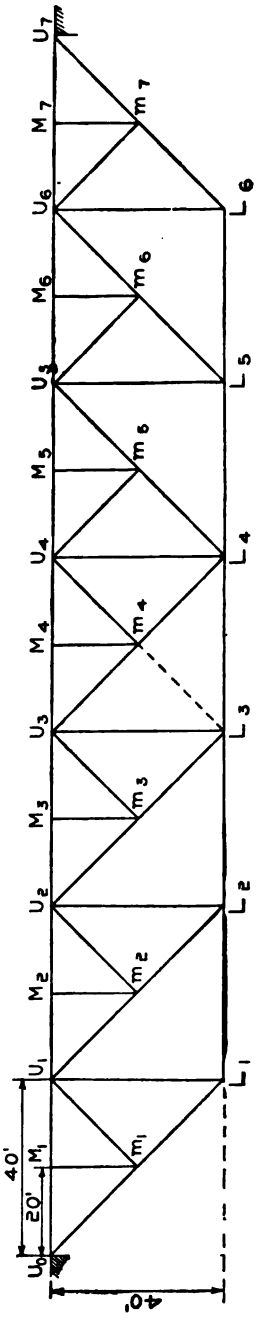


Fig. 116. Deck Baltimore Truss.

14. Determine the position of the wheel loads of Cooper's E 40 loading to produce maximum moments at the panel points of the truss of Problem 12.

ANSWER: L_1, wheel 4; L_2, wheel 7; L_3, wheels 11 and 12; L_4, wheels 13 and 14.

15. Determine the maximum moments at the panel points of the truss of Problem 12. Loading, Cooper's E 40.

ANSWER: $M_1 = 4\,820\,000$; $M_2 = 7\,745\,000$; $M_3 = 9\,192\,000$; $M_4 = 9\,082\,000$, all in pound-feet.

16. Compute the maximum live-load web stresses in the truss of Problem 12, the height being 32 feet. Loading, E. 40.

17. Compute the maximum live-load chord stresses in the truss of Problem 12, the height being 32 feet. Loading, E 40.

18. Compute the impact stresses for all members of the truss of Problem 12.

19. Determine the maximum live-load shears at the tenth-points of a 65-foot span deck plate-girder. Loading, E 40.

ANSWER: $V_0 = 103.0$; $V_1 = 86$; $V_2 = 69.7$; $V_3 = 54.5$; $V_4 = 40.8$; $V_5 = 28.4$.

20. Compute the shear due to impact in the girder of Problem 19.

ANSWER: $V_0 = 84.7$; $V_1 = 71.5$; $V_2 = 58.8$; $V_3 = 47.0$; $V_4 = 35.6$; $V_5 = 25.4$.

21. Compute the maximum live-load moments at the tenth-points of the girder of Problem 19. Loading, Cooper's E 40.

ANSWER:

POINT	WHEEL	MOMENT	IMPACT MOMENT
1	2	6 540	5 520
2	2	11 320	9 520
3	3	14 860	12 500
4	4	16 850	14 050
5	4	16 860	14 530
1.45′ from center	4	16 920	14 650

All moments are in thousands of pound-inches.

WILLIAMSBURG BRIDGE OVER EAST RIVER BETWEEN MANHATTAN AND BROOKLYN, NEW YORK, UNDER CONSTRUCTION

Main towers, 310 ft. high; clear span, 1,600 ft. Built in 1901-03.

Courtesy of American Bridge Company.

BRIDGE ENGINEERING

PART II

BRIDGE DESIGN

60. **General Economic Considerations.** The prime consideration which influences the decision to build is *cost*. After the decision to build has been made, the problem is one of a purely engineering character, whereas in the first case it was one of either a political or an engineering character, or both. The engineering problem is an economic one, in which maximum benefits must be obtained at a minimum cost.

A map of the proposed bridge site and the approaches, as well as of the country for a considerable distance up and down stream, should be made. This map should show the contours, the soundings, the borings, the high and low water-mark elevations, and the exceptional flood line. On this map the bridge should be plotted in its proposed location and also in various others. In the case of each of these locations, various schemes taking into account different numbers of piers and spans should be considered.

Several authors have attempted to present formulæ having a more or less theoretical derivation and purporting to indicate the correct number of piers and spans for a minimum cost. The use of these formulæ should not be encouraged, since they do not in any case give results close enough to serve for anything but a rough guide.

The cost of abutments will vary somewhat with the location and the character of the approach. This variation is usually small, and ordinarily an approximate location of the abutments can be quickly made. As the number of abutments is in all cases constant, their effect upon the problem of the location of the bridge is small, the main proposition being that of the cost and the number of piers and spans.

The cost of the piers will usually not be constant, those closer to the middle of the stream costing more on account of the depth of the water and the more difficult character of the foundation. Piers

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should not be placed on a skew; neither should they be placed directly in the maximum line of action of the current. If a skew is unavoidable, it should be as small as possible. The cost of piers should be ascertained by the most careful estimates. In the case of small bridges where there are only one or two piers, the matter is very simple, but with a considerable number of piers the problem becomes very complicated and requires weeks and sometimes months or years for its solution.

The determination of the cost of the superstructure is a comparatively simple matter. In certain instances the class of bridge is limited to some extent by the specifications. Cooper, in Article 2 of his "Specifications for Steel Railroad Bridges and Viaducts" (edition of 1906), gives the following:

Types of Bridges for Various Spans

Spans	Kind of Bridge
Up to 20 feet	Rolled beams
20 to 75 "	Riveted plate-girders
75 to 120 "	Riveted plate- or lattice-girders
120 to 150 "	Lattice or pin-connected trusses
Over 150 "	Pin-connected trusses

One railroad expresses a preference for plate-girders for all spans from 20 to 115 feet; and for spans from there to 150 feet, riveted trusses.

The question as to whether the bridge will be deck or through is one which is decided by the controlling influences of *water-way, false work, time of erection,* and *extra cost of masonry.* If the clear height required for the water-way is sufficiently small, the deck bridge should be chosen, as in this class the cost of false work is less, the time of erection is less, and the cost of masonry is less by an amount equal to the cross-section of the piers times the depth of the truss. Deck bridges also cost less than through bridges of equal span.

The conditions permitting, girders should be used in preference to trusses. While for equal spans girders are heavier and therefore cost more, the steel work alone being considered, little or no false work is required, and the time of erection is much less than in the

case of trusses. This makes the total cost of girder bridges less than those in which trusses are used. Another item in favor of girders is their great stiffness.

While pin-connected bridges cost less and are easier to erect, their stiffness is not so great as that of riveted bridges, which cost more. The time required for the erection of riveted bridges is also greater than that for pin-connected bridges. This is on account of the great amount of time required to make the riveted connections. For long spans, say over 200 feet, it is necessary to use pin-connected bridges, as the extreme size of the connection plates prohibits the use of the riveted type. Also, it is unnecessary to use riveted long-span trusses to obtain stiffness, as the weight of the pin-connected bridges is so great when compared with the live load that sufficient stiffness is obtained.

The cost of spans of different lengths and character may be obtained directly from the bridge companies; or their weights may be computed from the formulæ given in Article 20, p. 9 (Part I, "Bridge Analysis"), and multiplied by the unit price which your experience indicates is correct, thus giving the total cost.

Evidently the solution of problems of this nature cannot be made within the limits of this text, but the following example will tend to indicate somewhat the manner of procedure in a problem of this kind. For example, if the length between abutments is 1 400 ft., the cost of each abutment is $12 000, and the cost of each pier is $15 000, then, if we have fourteen 100-foot plate-girder spans, each costing $4 300, and thirteen piers, the total cost will be $279 200. On the other hand, if nine piers and ten 140-foot truss spans, each costing $9 200, are used, the cost will be $251 000, showing a balance of $28 200 in favor of the truss scheme. The live loading is E 50.

61. **Economic Proportions.** The depth of girders is given in Article 59, Part I.

In the case of trusses, the effect of an increase in the height is to increase the stresses in the web members and to decrease the stresses in the chord members. This variation does not affect the weights to any considerable extent; in fact, a variation of 20 per cent in the height will not affect the weight more than 2 or 3 per cent.

The height of the bridge is usually fixed by some considerations which are in turn determined by the specifications. The height must

be sufficient to clear whatever traffic will pass through. It should also be sufficient to prevent overturning on account of the wind pressure on the truss or on the traffic. In addition, the height of the bridge is influenced by the depth of the portal bracing. A deep portal bracing is desirable, in that it stiffens the trusses under the action of the wind and the vibration due to the passing traffic; but a deep portal bracing increases the height of the truss and therefore the bending in the end-posts due to the wind. Judgment on the part of the engineer should be used in order to determine the limiting height for securing a maximum amount of benefit as regards stiffness and a minimum amount of bad effect due to the bending in the end-posts. Fig. 117, which gives the height for any given length of span, may be said to represent the best modern practice (1908). Variations of a foot or more from those given do not affect the weight to any appreciable extent.

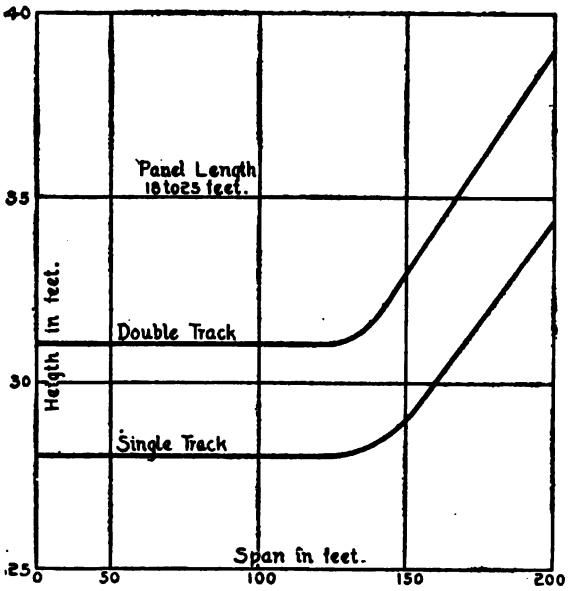


Fig. 117. Curves Showing Relation between Height of Trusses and Length of Span in Double- and Single-Track Railway Bridges.

The distance from center to center of trusses for highway bridges depends upon the width of the street or, if in the country, the width of the roadway. Streets, of course, vary in width in different localities, but country highway bridges usually have a roadway of from 14 to 16 feet in the clear.

In the case of railroad bridges, the distance from center to center of trusses depends upon whether the track is straight or on a curve, and also upon whether the bridge is a deck or a through bridge.

The actual amount varies in most cases, and is fixed by specification. Some specifications require that when the track is straight, the distance from center to center of trusses shall be 17 feet; or that, in case one-twentieth of the span exceeds the 17 feet, then one-twentieth of the span shall be used.

For deck plate-girders the common practice appears to be to space them as given below:

Width of Plate-Girder Bridges for Various Spans

Spans	Distance Center to Center of Plate-Girders
Up to 65 feet	6 feet 6 inches
65 to 80 "	7 feet 0 inches
80 to 115 "	7 feet 6 inches

For through plate-girders the spacing should be such that no part of the clearance diagram will touch any part of the girder. In case of double-track plate-girders with one center girder, great care should be exercised in order that the center girder shall not be so deep nor have so wide a flange as to interfere with the clearance diagram (see Fig. 126).

On account of the wind on a train which runs on track placed at the elevation of the top chord of deck bridges, the overturning effect is exceedingly great, and special care should be taken that the height and width are such as to prevent overturning.

In through bridges the clearance must be such as to allow the clearance diagram to pass. Special attention should be paid to the knee-braces and also to the portal braces. When the bridge is on a tangent, the spacing of the trusses is a comparatively simple matter, being just sufficient for the clearance diagram; but on curves, allowance must be made for the tilt of the diagram due to the superelevation of the outer rail, and also allowance must be made for the fact that the length of the cars between trucks forms a chord to the curve, and as such the middle ordinance must be taken into account. It is also necessary to allow for that part of the car which projects over the trucks, as this will extend beyond the outer rail by an amount greater than one-half the width of the clearance diagram. (See Figs. 119 and 120.)

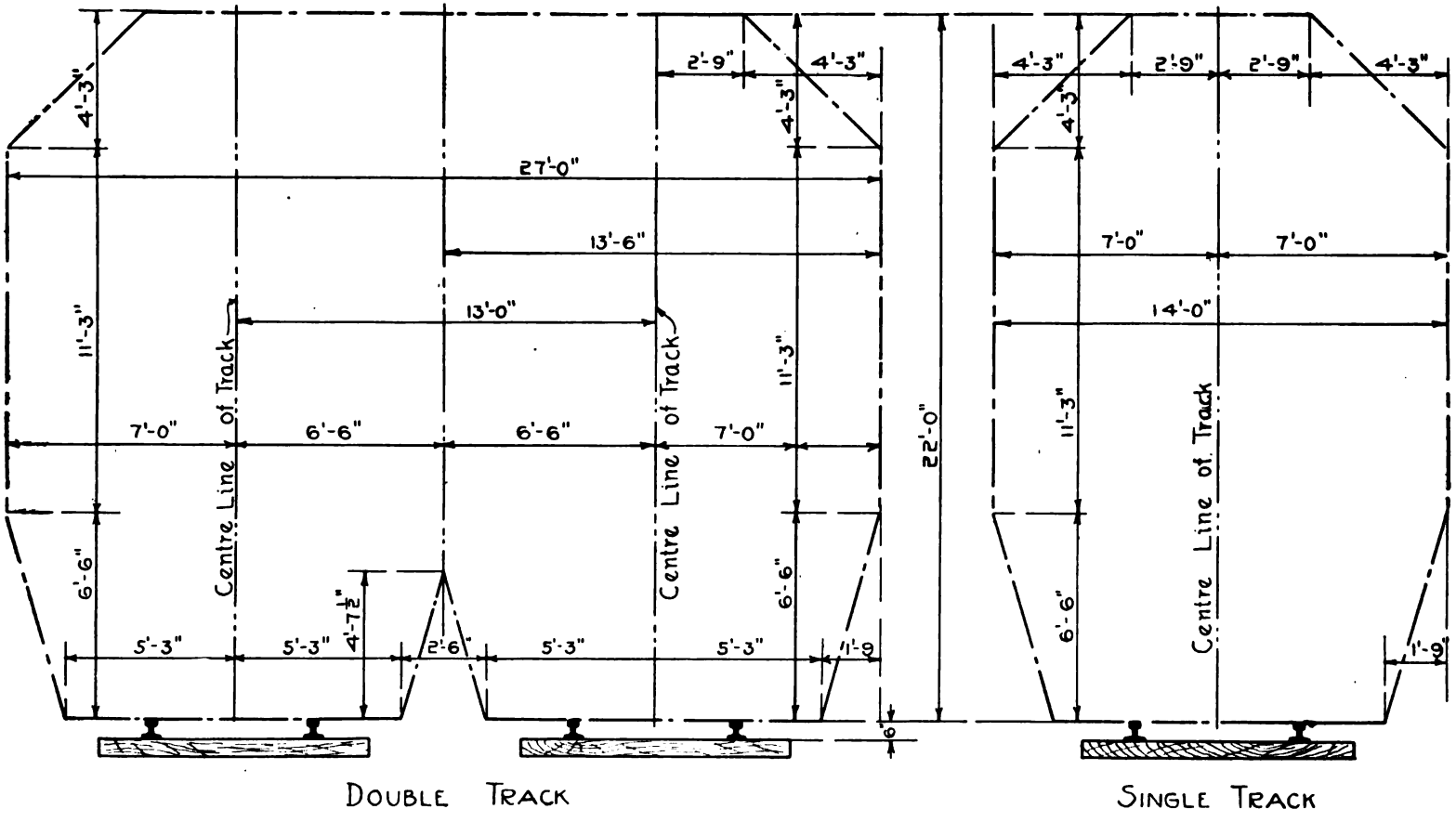


Fig. 118. Standard Clearance Diagram (Straight Track) Used by Lehigh Valley Railroad. Courtesy of F. E. Schall, Bridge Engineer.

Dimensions given show standard clearance for straight tracks. When tracks are on a curve, equivalent clearance is to be provided, assuming length of cars 75 feet over all and 54 feet center to center of trucks. Also, on account of tipping of cars, for structures extending over 13 feet 6 inches above top of rail, an additional clearance of 2½ inches is to be provided on the inside of curves for each inch of elevation of outer rail. For structures of lesser height, proportionate allowance is to be made.

62. **The Clearance Diagram.** The clearance diagram is not supposed to represent the outline of the largest engine or car which may run over the line, but represents the maximum amount of space which may be taken up by objects which are to be shipped over the line. For instance, the lower part of the clearance diagram may allow for snow-plow or ballast distributors, and the upper part may take into account the passage of such material as carloads of lumber, piles, or telegraph poles. The standard clearance diagram of the Lehigh Valley Railroad is given in Fig. 118. This diagram is for the clearance on straight track only. On curves, the diagram tilts as shown in Fig. 119, and to allow for this tilting the Lehigh Valley Railroad requires 2½ inches additional clearance on the inside of curves for each inch of elevation of the outer rail. In addition to this tilting effect, the clearance should also be increased on account of the

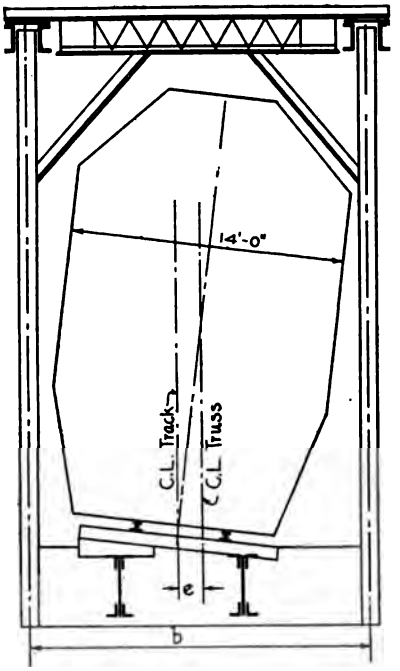


Fig. 119. Clearance Diagram on Curves, Showing Tilting.

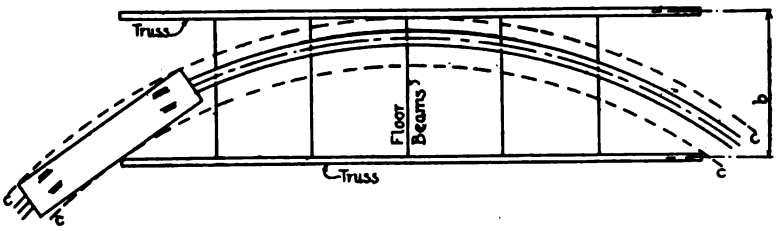


Fig. 120. Standard Car on Curve, Showing Necessity for Wider Spacing of Trusses.

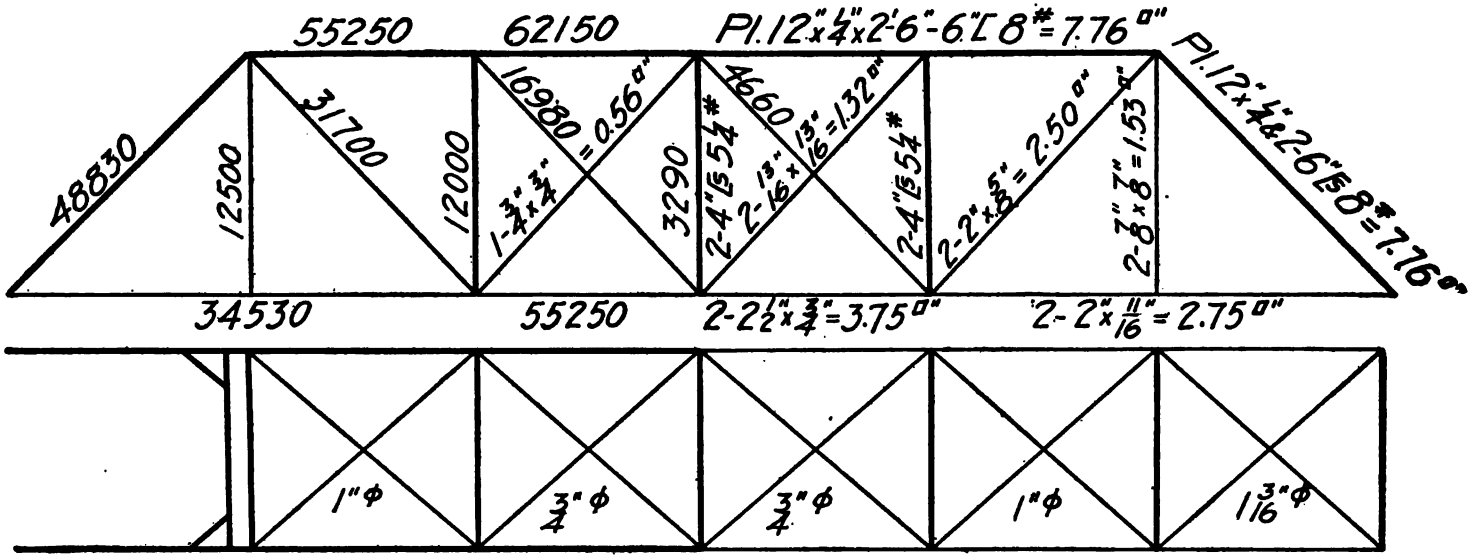
length of the cars and their projection over the outer and inner rails. Fig. 120 shows a standard car according to the specifications of the American Railway Engineering & Maintenance of Way Association, in such a position on a single-track span as to show the effect of the curve upon the widening of the spacing, center to center of trusses.

This car is 80 feet long, 60 feet between centers of trucks, and is as wide as the clearance diagram, 14 feet for single track. It is evident that the trusses cannot be spaced so as to interfere with the clearance line of the body of the car and its projecting ends. These clearance lines are represented as broken lines in Fig. 120, and are marked *c–c*. Note that the center of the track is seldom in the center of the floor-beam. Also, it is evident that the sharper the curve, the greater the required distance between trusses, and accordingly the greater the floor-beams in length. This varies the moment in the different floor-beams and therefore makes them more costly. The stringers, also, are more costly, on account of the fact that their ends are skewed. On account of the eccentricity of the track, one truss takes more of the load than the other, and therefore the trusses are not the same—a fact which further increases the cost.

From the above it is seen that almost all conditions incident to the building of a bridge on a curve tend to increase the cost; and hence a fundamental principle of bridge engineering: *Avoid building bridges on curves.*

63. **Weights and Loadings.** For the weight of steel in any particular span, and for the loading required for any particular class of bridge, see Articles 20 to 23, Part I. The weight of the ties and the rails and their fastenings is usually set by the specifications at 400 pounds per linear foot of track. For highway bridges the weight of the wooden floor is usually taken at 4½ pounds per square foot of roadway for every inch in thickness of floor.

Highway bridges are divided into different classes according to their loadings (see Cooper's Specifications). The decision as to the class to be employed depends somewhat upon the distance to the nearest bridge across the same stream. In case the nearest bridge is only a few miles away and is of heavy construction, it is not actually necessary to construct a heavy bridge at the proposed site, the heavier traffic being required to pass over the other bridge. In case a heavy bridge is not in the neighborhood, then one should be constructed at the proposed site. If the proposed site is on a road connecting adjacent towns of large size, then a heavy bridge should be constructed and provision made for future interurban traffic, even if none is at that time in view, since it will be more economical to do this than to erect a new bridge in the future.



Length of Span; C. to C. 102'-0"
Clear Width of Roadway, 12'-0
6 Panels, 17'-0" Each
Height of Truss, C. to C. of Pins, 17'-0"
Portal Bracing, 4 Ls 2 1/2 x 2" x 3.7#
Top Lateral Struts, 4 Ls 2" x 2" x 3.2#
Sway Struts – Sway Rods –
Shoe Struts, 4 I 7 1/2# Knee Braces, 3" x 3" x 1/4" L

Bottom Lateral Ties, 1 3/16"φ, 1"φ, 3/4"φ
Top Lateral Ties, 1"φ, 3/4"φ
Floor Beam, 18" x 1/4" & 4 Ls 3" x 3" x 4.5#
Sidewalk Brackets, —
Joists, 6 Lines 3" x 12"
Flooring, 2 1/2" Plank, Joist Bearers, 4x4
Wheel Guards, 2 1/2" x 8" Spiking Pieces, —
Hub Guards, 2 Lines 1 1/4" Gaspipe

Live Load 100 Lbs. per Sq. Ft. of Floor

Plate I. Stress Sheet of a Highway Bridge.

In the case of railroad bridges, new ones are nearly always constructed to carry the heaviest main line engines. These are usually of a class corresponding to Cooper's E 40 or E 50. In some localities branch-line bridges are built for the same live loadings; but in the majority of cases the branch-line bridges consist of the old bridges from the main line.

64. **Specifications.** For any particular bridge the specifications are either written by the engineer in charge, or some of the very excellent general specifications which are on the market in printed form are used. Some railroads use these general specifications with the addition of certain clauses which are desired by the chief or bridge engineer. The principal differences in these general specifications are in regard to the allowance for impact.

Whenever highway design is mentioned in this text, it is to be in accordance with Cooper's Highway Specifications (edition of 1901). Wherever plate-girder design is given, it is in accordance with Cooper's Railway Specifications (edition of 1906); and wherever truss design is given, it is in accordance with the general specifications of the American Railway Engineering & Maintenance of Way Association (second edition, 1906).

65. **Stress Sheet.** Before the sections are designed, the computer makes a skeleton outline of the truss, and on this places the dead-load and live-load stresses, and, in case the wind should be considered, the wind-load stresses. This is sent to the designer. The designer determines the various sections, and also the moments and shears in the stringers and floor-beams. These are placed on a sheet usually 17 by 23 inches. This is called a *stress sheet.* This sheet is now given to the draftsman, who makes a shop drawing. The stress sheets for railroad bridges are usually more elaborate than those for highway bridges. Plate I is the stress sheet of a highway bridge; and Plate II (Article 78) and Plate III (Article 93) are examples of the best modern practice in the making of plate-girder and truss-bridge stress sheets.

66. **Floor System.** Perhaps no part of bridge design is better standardized than the construction of the open steel floors for railroad bridges. The stringers are usually placed 6 feet 6 inches apart, and consist of small plate-girders, or, if the panel length is short, of one or more I-beams. I-beams are economical in regard to

BLACKWELL'S ISLAND BRIDGE, NEW YORK, UNDER CONSTRUCTION

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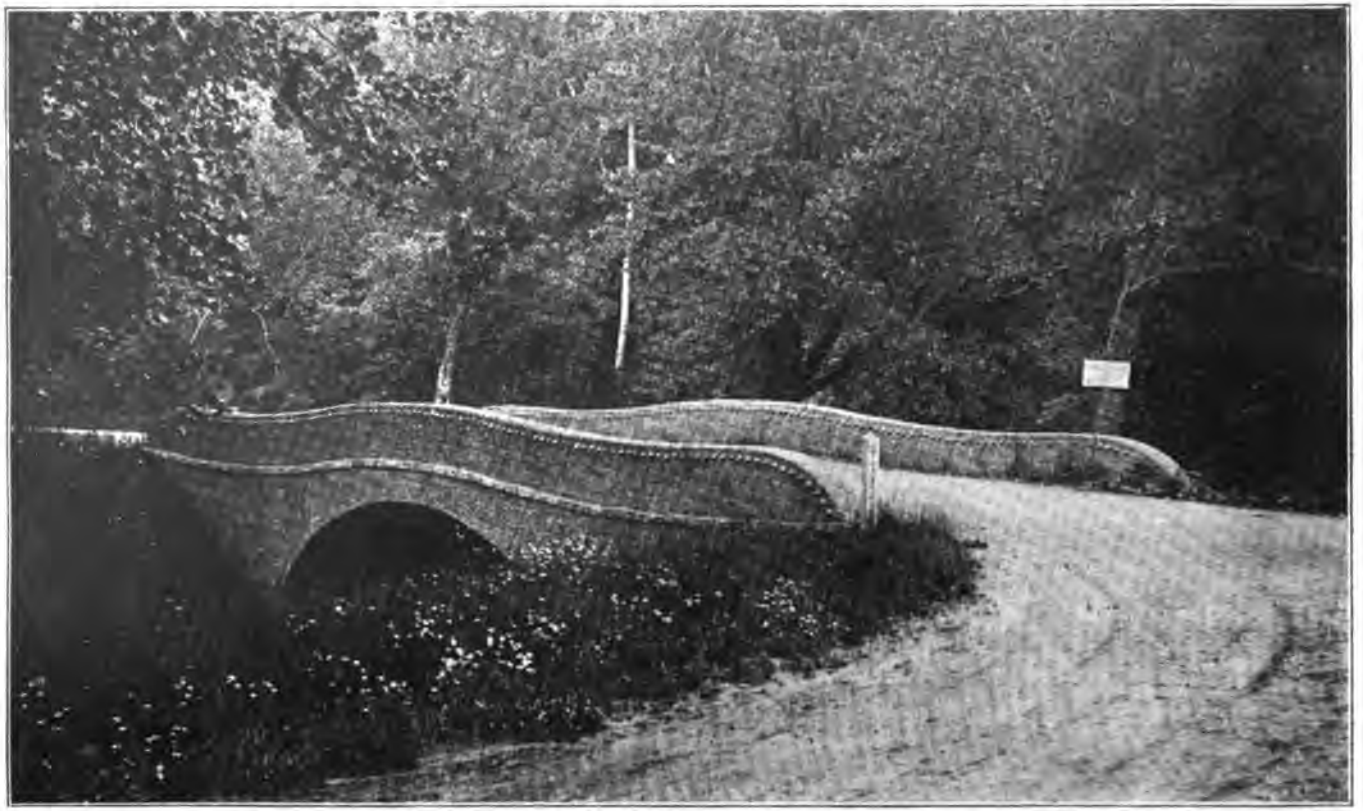


CEMENT AND PEBBLE BRIDGE IN ROCK CREEK PARK, WASHINGTON, D. C.

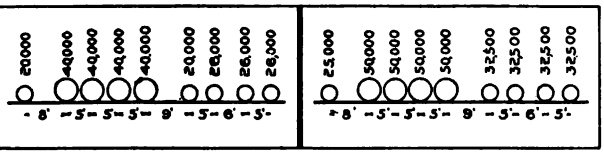
TABLE XIX

Safe Spans for I-Beams

(Based on unit-stress of 10 000 lbs. per square inch in extreme fibre)

Size of I-Beam	Weight per Foot (Lbs.)	Moment of Inertia I-Beam	Engine Class E 40 — Safe span C to C of Bearings			Engine Class E 50 — Safe span C to C of Bearings		
			1 Beam per rail	2 Beams per rail	3 Beams per rail	1 Beam per rail	2 Beams per rail	3 Beams per rail
9 in.	25	92	.....	5 ft. 3 in.	7 ft. 9 in.	.....	4 ft. 9 in.	7 ft. 0 in.
9 "	30	102	.....	5 " 9 "	8 " 9 "	.....	5 " 0 "	7 " 0 "
9 "	35	112	.....	6 " 6 "	9 " 6 "	.....	5 " 3 "	7 " 9 "
10 "	30	135	3 ft. 6 in.	6 " 9 "	10 " 0 "	3 ft. 0 in.	5 " 9 "	8 " 6 "
10 "	35	163	4 " 3 "	8 " 6 "	11 " 0 "	3 " 6 "	7 " 0 "	10 " 0 "
10 "	40	175	4 " 6 "	9 " 0 "	12 " 0 "	3 " 9 "	7 " 6 "	11 " 0 "
12 "	31½	218	4 " 9 "	9 " 3 "	12 " 9 "	4 " 0 "	7 " 9 "	11 " 3 "
12 "	40	274	5 " 9 "	11 " 6 "	13 " 9 "	4 " 9 "	9 " 6 "	12 " 3 "
12 "	50	332	7 " 3 "	12 " 6 "	15 " 3 "	6 " 0 "	11 " 3 "	13 " 6 "
12 "	65	403	8 " 6 "	13 " 9 "	16 " 0 "	7 " 0 "	12 " 3 "	15 " 0 "
15 "	42	443	7 " 6 "	12 " 9 "	15 " 6 "	6 " 3 "	11 " 6 "	14 " 0 "
15 "	50	515	8 " 6 "	13 " 9 "	16 " 3 "	7 " 3 "	12 " 6 "	15 " 3 "
15 "	60	619	10 " 6 "	15 " 3 "	19 " 0 "	8 " 9 "	13 " 6 "	16 " 9 "
15 "	70	718	11 " 6 "	16 " 6 "	20 " 9 "	10 " 3 "	14 " 9 "	18 " 6 "
15 "	80	774	12 " 0 "	17 " 3 "	21 " 6 "	10 " 9 "	15 " 3 "	19 " 3 "
18 "	55	809	11 " 0 "	16 " 0 "	20 " 0 "	9 " 6 "	14 " 0 "	17 " 9 "
18 "	65	890	11 " 9 "	16 " 9 "	20 " 9 "	10 " 6 "	15 " 0 "	18 " 9 "
18 "	75	1 023	12 " 6 "	18 " 0 "	23 " 6 "	11 " 6 "	16 " 0 "	20 " 0 "
18 "	80	1 063	12 " 9 "	18 " 9 "	23 " 0 "	11 " 9 "	16 " 6 "	20 " 6 "
18 "	90	1 188	13 " 6 "	19 " 6 "	24 " 6 "	12 " 3 "	17 " 6 "	21 " 9 "
20 "	65	1 180	12 " 9 "	18 " 9 "	23 " 0 "	11 " 6 "	16 " 6 "	20 " 9 "
20 "	75	1 277	13 " 3 "	19 " 6 "	24 " 0 "	12 " 0 "	17 " 3 "	21 " 3 "
20 "	85	1 453	14 " 3 "	21 " 0 "	26 " 0 "	12 " 9 "	18 " 6 "	23 " 0 "
20 "	90	1 502	14 " 6 "	21 " 3 "	26 " 6 "	13 " 0 "	18 " 9 "	23 " 3 "
20 "	100	1 650	15 " 3 "	22 " 3 "	28 " 0 "	13 " 6 "	19 " 9 "	24 " 6 "
24 "	80	2 112	15 " 9 "	23 " 0 "	29 " 3 "	14 " 0 "	20 " 6 "	25 " 9 "
24 "	85	2 182	16 " 0 "	23 " 9 "	29 " 9 "	14 " 6 "	21 " 0 "	26 " 3 "
24 "	90	2 356	17 " 0 "	24 " 6 "	30 " 9 "	14 " 9 "	21 " 6 "	27 " 3 "
24 "	95	2 427	17 " 3 "	25 " 0 "	31 " 6 "	15 " 0 "	22 " 0 "	27 " 9 "
24 "	100	2 497	17 " 6 "	25 " 3 "	32 " 0 "	15 " 3 "	22 " 6 "	28 " 0 "

Diagram of Engines



first cost, but are disadvantageous on account of the eccentric connections which necessitate heavy brackets to resist part of their reaction. They are also somewhat undesirable on account of the fact that, the ties deflecting, most of the load is carried by the inner I-beam. However, I-beams for stringers and for short-span bridges (see Fig. 121) are much used in present practice, and give good results. Figs. 121 to 127 show the standard open floor sections of the Lehigh Valley Railroad. Table XIX gives the required number of I-beams, together with their weight, which are to be used for short-span bridges or as stringers in panels of given length.

Solid floors consist of angles and plates, channels and plates, or other shapes. They extend transversely across the bridge from truss

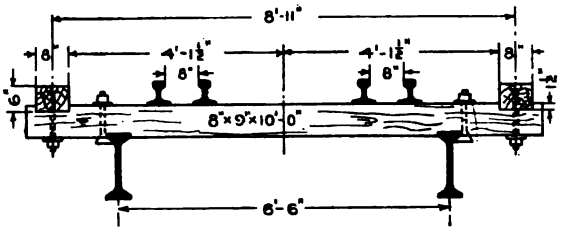


Fig. 121. Sectional View Showing Open-Floor Construction of Railroad Bridge of Short Span, Single Track. I-Beams used for Stringers. Lehigh Valley Standard.

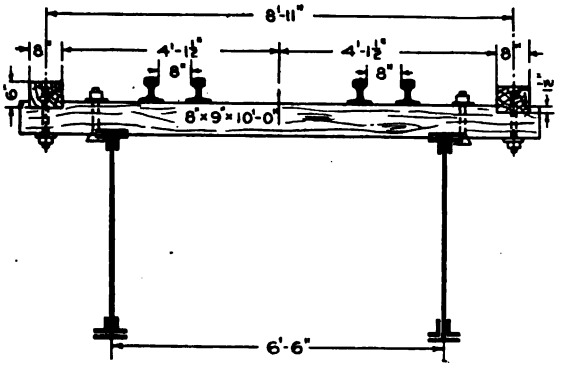


Fig. 122. Section of Open-Floor Construction of Deck-Girder and Truss Bridge, Single Track. Lehigh Valley Standard. Plate-Girders used for Stringers.

to truss, the lower chords, in case of truss bridges, being made heavy enough to act as girders as well as tension members. Figs. 128 to 130 show sections of solid floors. The ballast is laid directly upon these solid floors, which are first covered with a good damp-proof paint. The floors should also be supplied with good drainage facilities.

Concrete is sometimes laid directly upon the steel floor, and the ballast put upon this concrete,

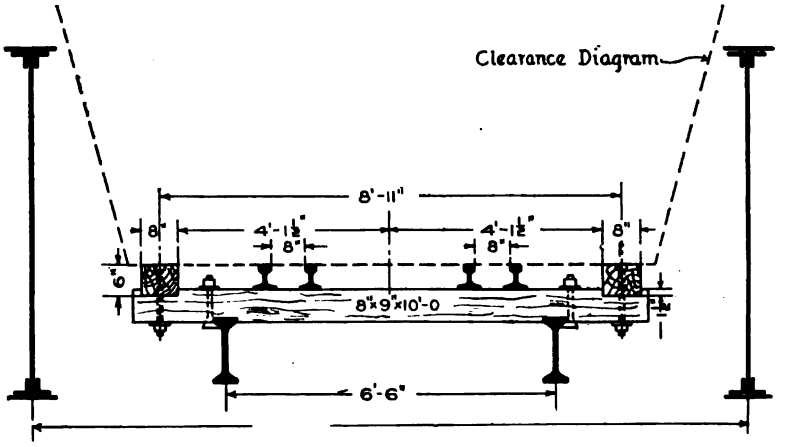


Fig. 123. Floor Construction of a Through-Girder Bridge, Single Track. Lehigh Valley Standard.

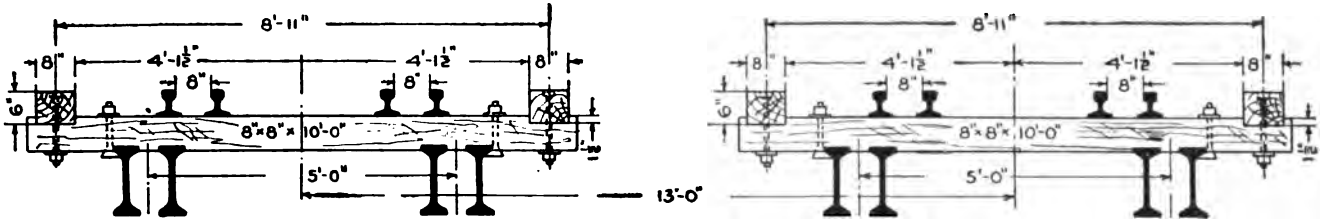


Fig. 124. Floor Construction of Double-Track I-Beam Bridge, Lehigh Valley Standard.

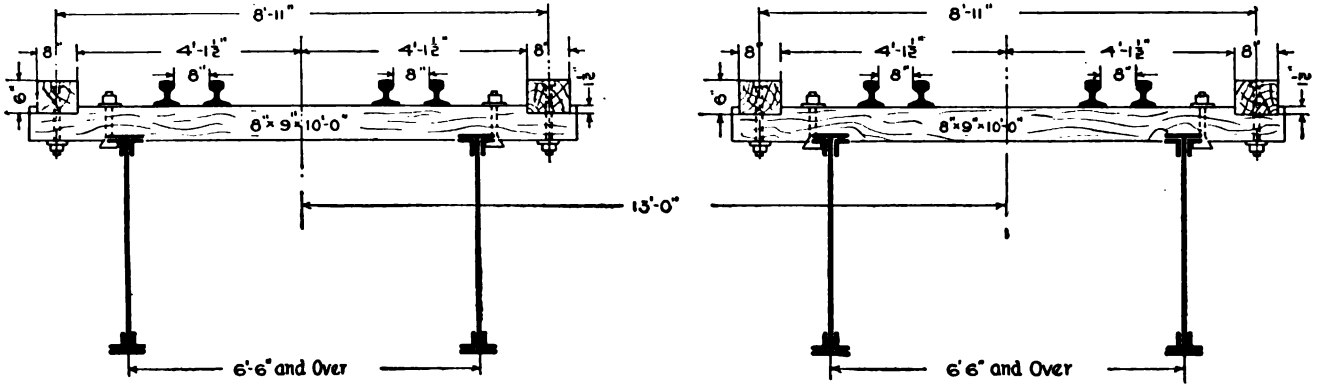


Fig. 125. Floor Construction of Double-Track Deck-Girder and Truss Bridge, Lehigh Valley Standard.

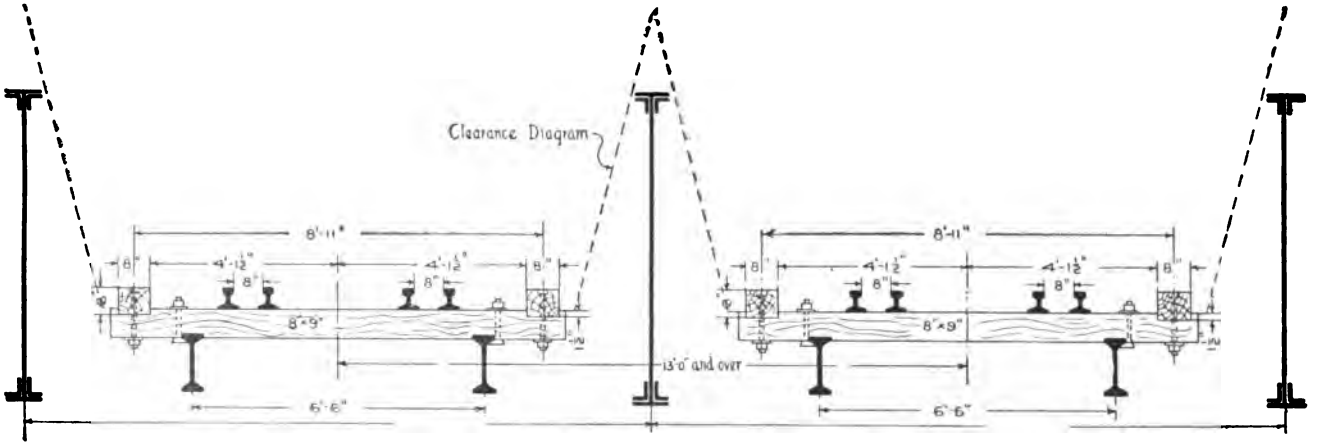


Fig. 126. Floor Construction of Double-Track Through-Girder Bridge, Lehigh Valley Standard.

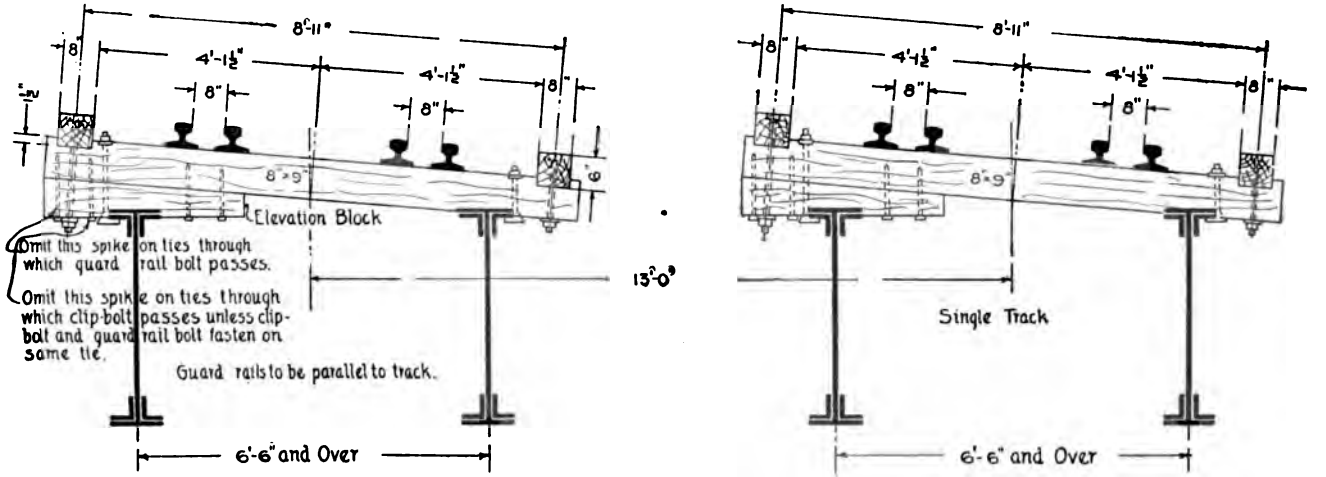


Fig. 127. Floor Construction of Double-Track Deck-Girder and Truss Bridge on Curve, Lehigh Valley Standard.

which has previously had a layer of some good waterproofing applied on its upper surface.

67. Practical Considerations. The possibilities of the rolling mill and the various shops of a bridge company, such as the drafting room, forge, foundry, templet shop, assembling shop, and riveting and finishing shop, and also the shipping and erecting facilities, should be well known in order to make the most economical use of them. This requisite knowledge comes only from experience. The best way to obtain this experience without being actually employed in the shops, is to go into the shops every chance that presents itself, keep your eyes and ears open, and ask all the questions you can. The use to be made of handbooks of the various steel manufacturers is given in Part I of "Steel Construction," and should be thoroughly studied before going further. Some one of these handbooks is indispensable to persons designing steel structures. That of the Carnegie Steel Company (edition 1903) is one of the best, and will be frequently referred to in the present text.

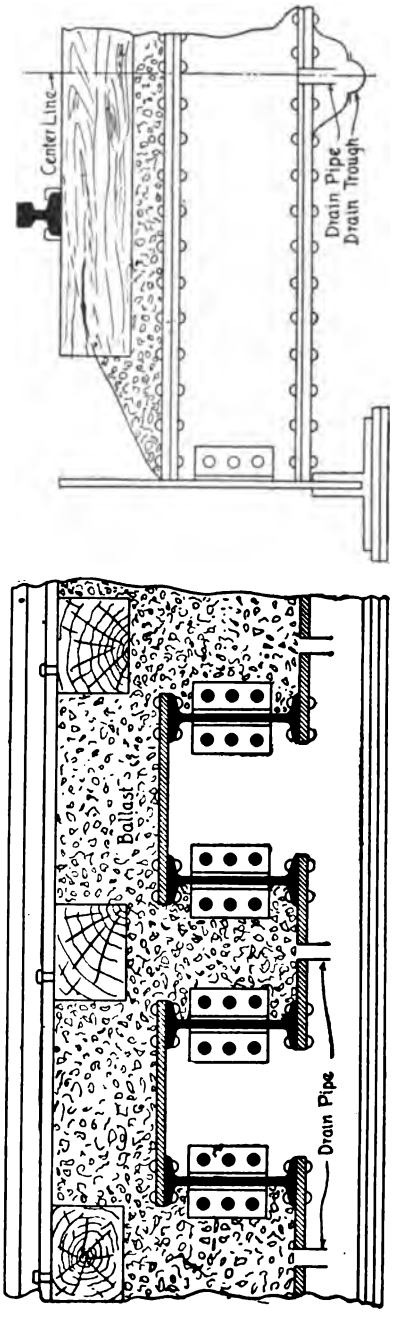


Fig. 128. Longitudinal and Transverse Section of Solid-Floor Construction.

Copies may be procured from the Carnegie Steel Company, Frick Building, Pittsburg, Pa. The usual price to students is 50 cents, to others $2.00.

DESIGN OF A PLATE-GIRDER RAILWAY-SPAN

68. **The Masonry Plan.** In some cases the general dimensions of the masonry are limited; such a case, for example, would occur in the crossing of a street or narrow waterway. Here the length of the span and the distance above the street or the surface of the water,

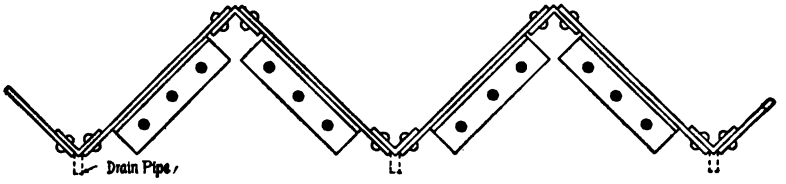


Fig. 129. Solid-Floor Construction of Plates and Angles.

are the limited dimensions. The span and under-clearance may be unlimited, as in the case of a country stream crossed by a roadway which is a considerable distance above the surface of the water. The term *unlimited* is not here used in its exact meaning, as the span in this case is really limited by the cost, which rapidly increases with the length of the span.

In some cases, as when the engineer is in a bridge office, the masonry plans are sent in by the railroad. In such cases many of the limited dimensions are fixed. The most usual dimensions to be fixed are the elevation of base of rail, the elevation and size of the bridge seat, and the length of the span under coping. These limit the depth of the girder, or the depth of the floor if it be a through girder, and also limit the length of the bearing plates at the end. Fig. 131, the masonry plan of a road crossing, shows in general what can be expected. All the dimensions usually fixed are given, and those marked x and y may or may not be, but x should never be less than 3 feet.

69. **Determination of the Class.** As before mentioned, the deck plate-girder should be used if possible, since its cost is less. There are some cases, however—such as track elevation in cities — where the additional cost required to elevate the track so as to use a

deck plate-girder will more than balance the saving in its favor. In such cases the through plate-girder is used.

The case whose design is under consideration will be taken similar to that of Fig. 131, and the span will therefore be a deck one.

70. **Determination of the Span, Center to Center.** Fig. 132 shows the various spans—namely, *under coping, center to center of end bearings,* and *over all.* The span under coping is that span from under coping to under coping lines of the abutments, and is so chosen as to give the required distance between the abutments at their base. The span center to center is equal to the span under coping *plus* the length of one bearing plate. The span over all is the extreme length of the girder. The length of the bearing plate is influenced by the width of the bridge seat, and also by the maximum reaction of the girder. The length should seldom be greater than 18 inches and never greater than 2 feet, as the deflection of the girder will cause a great amount of the weight to come on the inner edge of the bearing plate and also on the masonry, in which case the masonry is liable to fail at that point and the bearing plates are over-stressed.

Cast-steel bearings are now almost universally used. They decrease the height of the masonry, and distribute the pressure more evenly and for a greater distance over the bridge seat. When these castings are used, the bearing area between them and the girder may be made quite small, thus doing away to a great extent with the

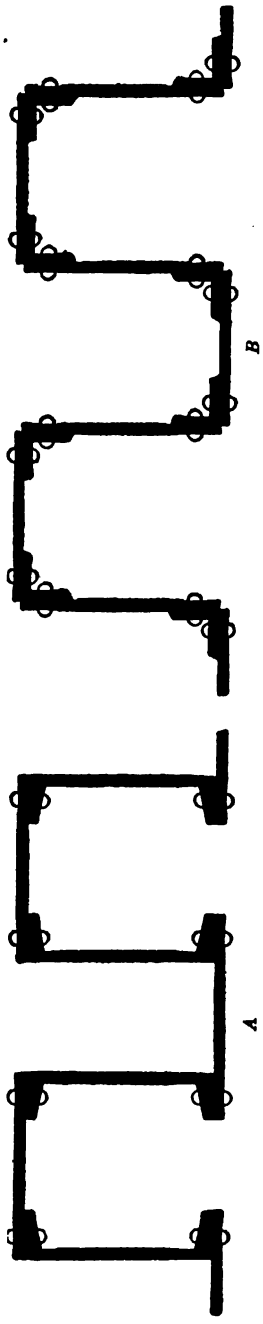


Fig. 130. Methods of Solid-Floor Construction. *A*—Plates and Channels; *B*—Plates and Angles.

deteriorating effect due to the deflection of the girder as mentioned above. Fig. 133 shows the end of a girder equipped with a cast-steel pedestal. Table XX gives the length of the bearing on the

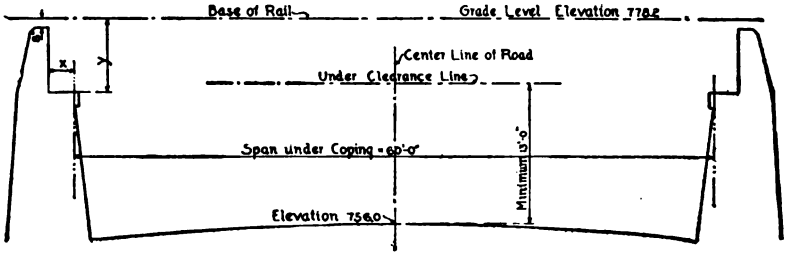


Fig. 131. Masonry Plan of a Road Crossing.

masonry for various spans, Cooper's E 40 loading being used and cast-steel pedestals being employed.

TABLE XX

Length of Masonry Bearings

Span	Length of Bearing
15 to 24 feet	12 inches
25 to 44 "	16 "
45 to 69 "	21 "
70 to 79 "	23 "
80 to 89 "	29 "
90 to 115 "	36 "

As an example, let it be required to determine the span center to center of a deck plate-girder of 60-foot span under coping, the

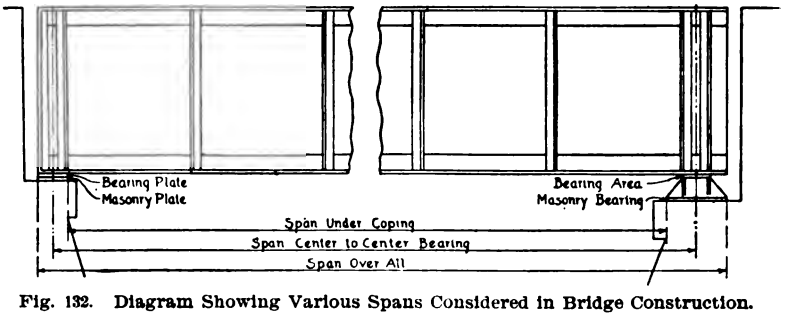


Fig. 132. Diagram Showing Various Spans Considered in Bridge Construction.

loading being E 40. From Table XX it is seen that the length of the masonry bearing will be 21 inches, and therefore the span center

to center of bearings will be $60 + 2 \times (\frac{1}{2} \times 1$ ft. 9 in.) = 61 feet 9 inches.

In Articles 71 to 77 the above girder will be designed; and also such information as is of importance regarding the subject-matter of each article will be treated. The dead- and live-load shears are computed by the methods of Part I, and are given in Fig. 134.

71. **Ties and Guard-Rails.** The length of ties for single-track bridges is 10 feet. For double-track bridges the length is in most cases the same. In some double-track bridges, however, either each tie or every third tie extends entirely across the bridge. In other cases every third tie on one track extends to the opposite track, thus acting as a support for the foot-walk which is laid upon them. It is the best practice to limit the length of the ties on double-track bridges to 10 feet, since, if they extend into the opposite track in any way whatsoever, unnecessary expense is incurred whenever repairs or renewals are made, because both tracks must necessarily be disturbed to some extent.

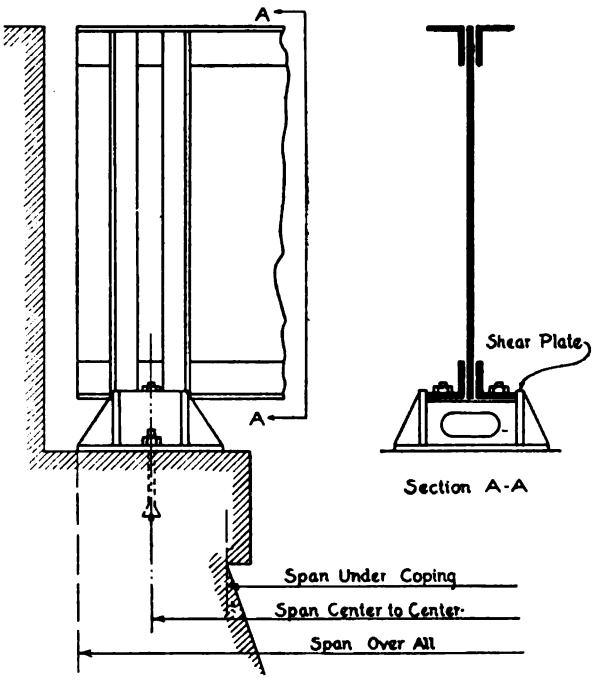


Fig. 133. End of a Girder Equipped with a Cast-Steel Pedestal.

The size of the ties varies with the weight of the engines and the spacing of the stringers or girders on which they rest. They are usually sawed to size instead of hewn, and the following sizes may be purchased on the open market—namely, 6 by 8, 7 by 9, 8 by 9, 9 by 10,

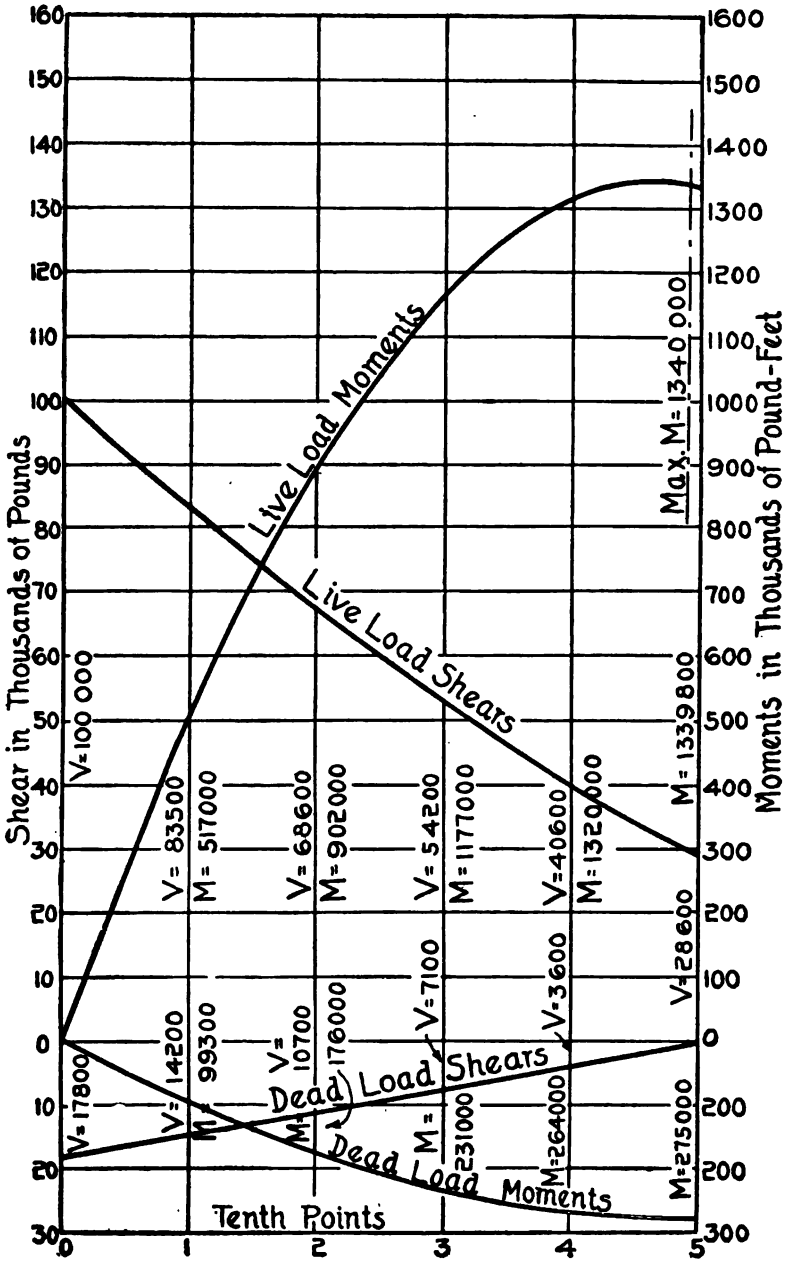


Fig. 134. Shear and Moment Diagram.

and 10 by 12 inches. Larger sizes may be obtained on special order.

The elevation blocks (see Fig. 127) should be of length to suit the width of the cover-plates and the spacing of the supports. They are usually made of the best quality of white oak, since the cost of renewal is great enough to demand that they be made of material as permanent as possible.

The guard-rails should be placed in accordance with the specifications (see Articles 13 and 14). Some railroads specify that the guard-rails shall be in 24-foot lengths unless the bridge is shorter than 24 feet, in which case one length of timber should be used. For method of connection and other details, consult Figs. 121 to 127. The guard-rails and the ties are usually made of Georgia long-leaf yellow pine, prime inspection. Other wood, such as chestnut, cedar, and oak, may be used.

In addition to the wooden guard-rail, a steel guard-rail usually consisting of railroad rails is placed within about 8 inches of the track rail.

In designing ties, the problem is that of a simple beam symmetrically loaded with two equal concentrated loads, the weight of the rail and tie itself usually being neglected. For the case in hand, which is that of a deck plate-girder, loading E 40, the concentrated load for which the tie must be designed is, according to Specifications (Article 23, 3d part), 8 333 pounds. According to Article 23, 100 000 pounds is on four wheels. This gives 25 000 pounds on one wheel, and according to Article 15, one-third of this, or 8 333 pounds, will come on one tie. Fig. 135 shows the condition of the loading, the space center to center of rail being taken as 4 feet 10 inches. Some designers take this distance as 5 feet; but as the standard rail head is about 2 inches, and the standard gauge 4 feet 8½ inches, the distance here taken seems to be the more logical one.

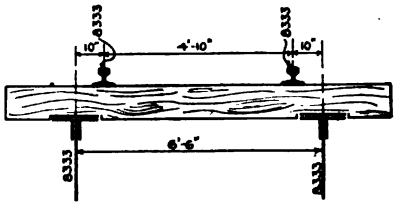


Fig. 135. Distribution of Loading on Ties of Deck Plate-Girder Bridge.

The formula to be used in the design of this beam is that given in "Strength of Materials," and is $M = \frac{SI}{c}$. In this case $M = 10 \times$

8 333 = 83 330 pound-inches. In the above formula, $I = bd^3 \div 12$, and $c = d \div 2$, and therefore $\frac{I}{c} = \frac{bd^2}{6}$. Substituting the value of the moment in the above formula, and solving for S, there results:

$$S = \frac{499\ 980}{bd^2}.$$

For a 6 by 8-inch tie, the unit-stress would then be:

$$S = \frac{499\ 980}{6 \times 8^2} = 1\ 310 \text{ pounds.}$$

If a 7 by 9-inch tie is used, the unit-stress is found to be 880 pounds. Since according to Article 15 of the Specifications, the unit-stress cannot be greater than 1 000 per square inch, it is necessary to use a 7 by 9-inch tie. If the engine loading had been E 50, the moment would have been 100 000 pound-inches, and then the stress in a 7 by 9-inch tie would be 1 060 pounds per square inch, and the stress in an 8 by 9-inch tie would be 930 pounds, which would necessitate the use of the latter.

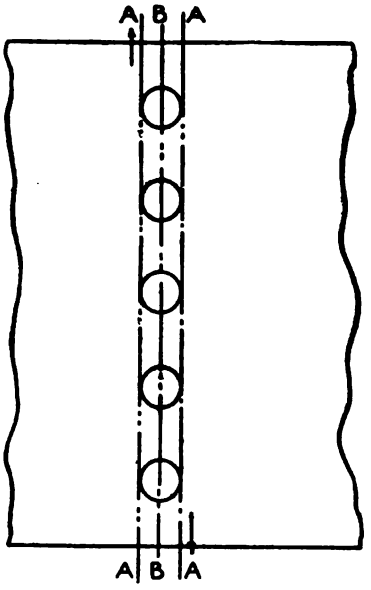


Fig. 136. End Rivets Transferring Shear to Web.

The guard-rails on this bridge will be placed according to the Lehigh Valley standard, and hence their inner face will be 4 feet 1½ inches from the center of the track.

Elevation blocks will not be required, as the bridge is on a tangent.

72. The Web. The economic depth of the web, according to Article 59, Part I, will be:

$$\text{Depth} = \frac{61 \text{ ft. } 9 \text{ in.}}{0.005 \times 61 \text{ ft. } 9 \text{ in.} + 0.543} = 72.5.$$

The depth might be taken as 72 inches, but 74 inches will be decided upon, as this will decrease the area of the flange and also will not affect the total weight to any great extent. The unit-stress for shear

is 9 000 pounds per square inch (see Specifications, Articles 40 and 41).

The maximum shear in the girder occurs at the end, where it is 117 800 pounds. The area required for the web is then 117 800 ÷ 9 000 = 13.09 square inches, and the required thickness is 13.09 ÷ 74 = 0.177 inch. This latter value cannot be used, since, on account of Article 82 of the Specifications, no material less than $\frac{3}{8}$ inch can be used. The web plate will therefore be taken as 74 in. by $\frac{3}{8}$ in. in size.

Some engineers insist that the net section of the web should be considered. Consider Fig. 136, the shear being transferred to the web by the end rivets. The web will not tend to shear along the section *B–B*, in which case the rivet-holes should be subtracted; but it will shear along section *A–A*, a section which is unaffected by the rivet-holes. The web splice should come at one of the stiffeners, and will therefore be considered in Article 76.

73. **The Flanges.** This portion of the girder is usually built either of two angles or of two angles and one or more plates. In heavy girders where the flange areas are large, additional area is obtained by using side plates or side plates and four angles. Sometimes two channels are used in the place of side plates and angles. Fig. 137 shows the different methods of constructing the top flanges of girders. The lower flanges are usually of the same construction. Fig. 137 *b* has the web extending beyond the upper sur-

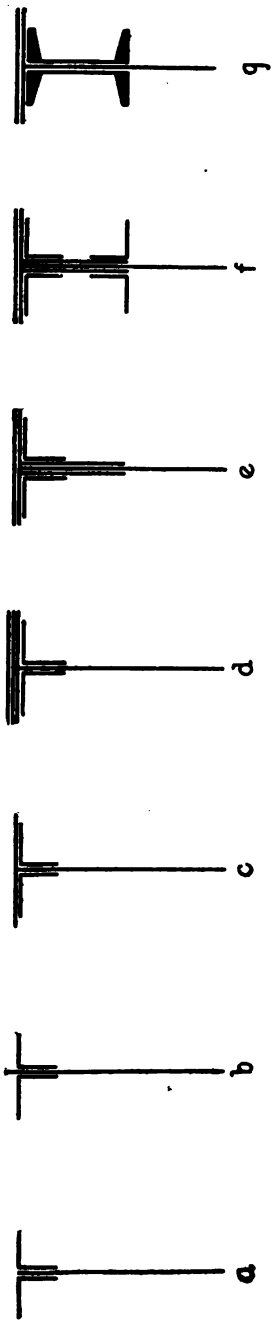


Fig. 137. Methods of Constructing Top Flanges of Girders.

faces of the upper flange angles. This is done in order that the ties may be dapped over it, and thus prevent the labor usually required for cutting holes in the lower face of the tie in order to allow for the projecting rivet-heads. Fig. 137 *g* is usually uneconomical, since the thinness of the channel web requires a great many rivets to sufficiently transmit the shear from the web to the flange, and also since the cover-plates must be very narrow.

Specifications usually state that the flanges shall have at least one-half of the total flange area in the angles, or that the angles shall be the largest that are manufactured. The largest angles are not usually employed, since their thickness is greater than three-quarters of an inch and therefore the rivet-holes must be bored, not punched. The reason for this is that the depth of the rivet-hole is too great in proportion to its diameter, and on this account the dies used for punching frequently break. Also, the punching of such thick material injures the adjacent metal, which makes it undesirable. In reality the flange area of only the short-span girders is small enough to allow the flange area to be taken up by the angles.

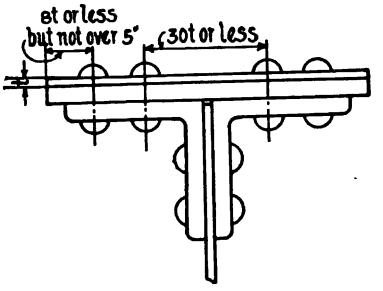


Fig. 138. Diagram Showing Relation between Thickness of Cover-Plates and Position of Rivets.

In choosing the thickness of the cover-plates, care should be taken so that the outer row of rivets will not come closer to the outer edge of the plate than eight times the thickness of the thinnest plate. In case eight times the thickness of the plate is greater than 5 inches, then 5 inches should be the limit. Also, the distance between the inner rows of rivets should not exceed thirty times the thickness of the thinnest plate. These limitations are placed by Article 77 of the Specifications, and Fig. 138 indicates their significance.

The determination of the required flange area depends upon the distance between the centers of gravity of the flanges; and in order to determine this exactly, the area and composition of the flanges should be known. The above condition requires an approximate design to be made, the supposition being that the flanges consist of two angles and one or more plates as shown in Fig. 138.

The distance back to back of angles will be taken as $74 + 2 \times \frac{1}{8} = 74\frac{1}{4}$ inches. Article 74 of the Specifications requires $\frac{1}{16}$ inch; but $\frac{1}{8}$ inch is better practice, since the edges of the web plate are very liable to overrun more than $\frac{1}{16}$ inch. Some specifications require $\frac{1}{4}$ inch.

In the computation of the approximate flange area, the center of gravity of each flange will be assumed as one inch from the back of the angles. The approximate effective depth is then $74\frac{1}{4}$ less 2×1 inch, which equals $72\frac{1}{4}$ inches. The approximate stresses in the flange areas are:

$$\text{For dead load, } \frac{275\,000 \times 12}{72.25} = 45\,600 \text{ pounds.}$$

$$\text{For live load, } \frac{1\,340\,000 \times 12}{72.25} = 222\,000 \text{ pounds.}$$

The approximate flange areas are now obtained by dividing these amounts by the allowable unit-stresses for dead and live load, which are (see Specifications, p. 8, Article 31): 20 000 and 10 000 pounds per square inch respectively; and the resulting areas are:

$$\text{For dead load, } \frac{45\,600}{20\,000} = 2.28 \text{ square inches.}$$

$$\text{For live load, } \frac{222\,000}{10\,000} = 22.20 \text{ square inches.}$$

These amounts give a total of 24.48 square inches as the approximate net flange area required.

It will be assumed that one-half the total area, or $24.48 \div 2 = 12.24$ square inches, is to be taken up by angles. If 12.24 sq. in. is distributed over two angles, then $12.24 \div 2 = 6.12$ square inches is the net area for one angle. Of course it is not to be assumed that the area of the angle chosen must be exactly 6.12, but that this 6.12 square inches is the approximate area of the angle to be chosen, and the net area of the angle (see Specifications, Article 149) must not be $2\frac{1}{2}$ per cent less than this, although it may be greater.

From Steel Construction, Part I, Table VII, or from the Carnegie Handbook, p. 117, a 6 by 6 by $\frac{3}{4}$-inch angle gives a gross area of 8.44 square inches and a net area of $8.44 - 2 \times (\frac{7}{8} + \frac{1}{8}) \times \frac{3}{4} = 6.94$ square inches, $\frac{7}{8}$-inch rivets being used and so spaced that two rivets are taken out of each angle (see Specifications, Article 63, and Fig. 139). A 6 by 4 by $\frac{13}{16}$-inch angle, giving a gross area of 7.47 and a net area of 6.66 square inches, one rivet-hole being out,

could have been used, but $\frac{13}{16}$ inch is too thick to punch, and therefore the above angle is chosen.

The required net area of the cover-plate is now found to be $24.48 - 2 \times 6.94 = 10.60$ square inches. Since the legs of the angles are 6 inches and the thickness of the web is $\frac{3}{8}$ inch, the outer edges of the angles are $12\frac{3}{8}$ inches apart; and since the cover-plate must extend somewhat over the edges of the angle, and the width of the cover-plate should be in the even inch, the width of the cover-plates must be at least 14 inches, as shown in Fig. 139.

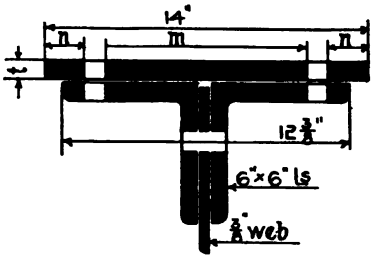


Fig. 139. Calculation of Size of Angles and Cover-Plate.

On account of the 1-inch rivet-holes to be deducted, the real or net width of the cover-plate is: $2 \times n + m = 14 - 2 \times 1 = 12$ inches. The thickness of all the cover-plates at the center is now:

$$t = \frac{10.60}{12} = 0.885 \text{ inch—say, } \tfrac{7}{8} \text{ inch.}$$

A thickness of $\frac{7}{8}$ of an inch is decided upon, for the reason that plates are rolled only to the nearest sixteenth of an inch.

The approximate section at the center has now been determined, and is:

2 Angles 6 by 6 by $\frac{3}{4}$-inch	=	13.88 sq. in. net.
Cover-plates $\frac{7}{8}$ inch thick	=	10.50 sq. in. net.
Total	=	24.38 sq. in. net.

This approximate section must now be examined, and, if it shows too great an excess or a deficiency, must be revised.

In order to determine the effective depth the distance between the centers of gravity of the flanges must first be computed, the gross areas being used. Theoretically, perhaps, the net areas should be used; but this is an unnecessary refinement, since the effect on the final result is of no practical importance.

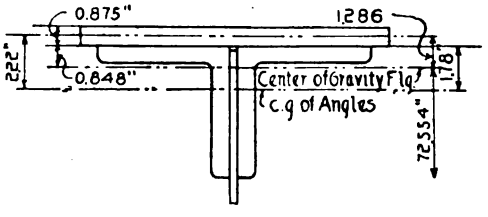


Fig. 140. Determination of Center of Gravity.

In computing the center of gravity (see Fig. 140), the axis is taken at the center of the cover-plates, as this reduces the moment of the cover-plates to zero. The distance of the center of gravity of the angles from their back (Carnegie Handbook, p. 117, column 6) is 1.78 inches. The distance of this center of gravity from the center of the cover-plate, is $1.78 + 0.875 \div 2 = 2.22$ inches.

Gross area of the angles $= 2 \times 8.44 = 16.88$ sq. in.
" " " " cover-plates $= \frac{7}{8} \times 14 = 12.25$ sq. in.
Total $= 29.13$ sq. in.

The center of gravity is now found to be $16.88 \times 2.22 \div 29.13 = 1.286$ inches from the center of the cover-plate, and $1.286 - 0.875 \div 2 = 0.848$ inch from the back of the angle. The effective depth h_e is $74.25 - 2 \times 0.848 = 72.554$ inches, and the required flange areas are:

$$\frac{275\,000 \times 12}{72.554 \times 20\,000} = 2.272 \text{ sq. in. for dead load.}$$

$$\frac{1\,340\,000 \times 12}{72.554 \times 10\,000} = 22.200 \text{ sq. in. for live load.}$$

$$\text{Total} = 24.472 \text{ sq. in.}$$

The values of the moments, as taken from the curves, must be multiplied by 12 in order to reduce them to pound-inches.

A total of 24.86 square inches is given by the section approximately designed, and the difference between that and the section as above determined is: $(24.472 - 24.38) \div 24.472 = 0.38$ per cent, and as this is less than $2\frac{1}{2}$ per cent (see Specifications, Article 149), it may be used without any further change. If there should have been a deficiency or an excess greater than $2\frac{1}{2}$ per cent, then it would have been necessary to revise. In case a revision of section is necessary, the size and thickness of the angles generally remain the same as those taken in the approximate design, the thickness of the cover-plates being decreased or increased as the case may be.

The total thickness of the cover-plates, $\frac{7}{8}$ inch, is too thick to be punched. In such cases as this, the section is made up of two or more plates whose total thickness is equal to that required. If plates of more than one thickness are decided upon, then their thickness should decrease from the flange angles outward. For the case in hand, one plate $\frac{3}{8}$ inch thick and one plate $\frac{1}{2}$ inch thick will be decided upon. The flange section at the center as finally designed is:

Shape	Net Section	Gross Section
2 Angles 6 by 6 by $\frac{3}{4}$ in.	13.88 sq. in.	16.88 sq. in.
1 Cover-plate 14 by $\frac{3}{8}$ in.	4.50 "	5.25 "
1 Cover-plate 14 by $\frac{1}{2}$ in.	6.00 "	7.00 "
Total	24.38 "	29.13 "

The above is the section required at the center of the girder; for any other point it will be less, decreasing toward the end, where it will be zero. Evidently, then, the cover-plates will not be required to extend the entire length. The following analysis will determine where they should be stopped. If the load were uniform, the moment

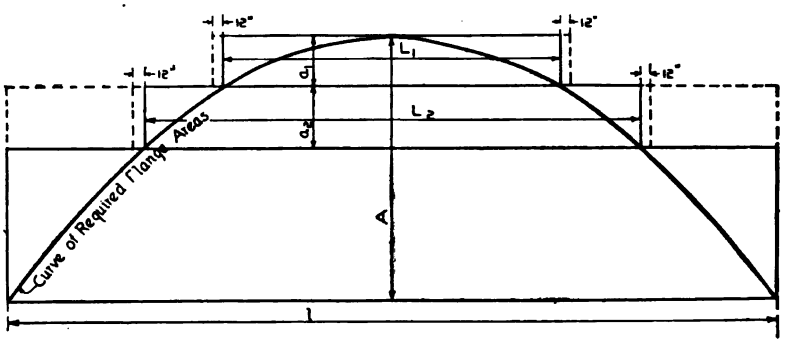


Fig. 141. Diagram Showing Curve of Required Flange Areas.

curve would be a parabola. Although under wheel loading the curve of moments is not a parabola, yet it is sufficient for practical purposes to consider it as such. The curve of flange areas, like that of moments, is to be considered a parabola (see Fig. 141).

Let a_1 = Net area, in square inches, of the outer cover-plate;
a_2 = Net area, in square inches, of the next cover-plate;
a_3, etc. = Net areas of the other cover-plates;
A = Net area of all the cover-plates and the flange angle.

Then, from the properties of the parabola,

$$L = l\sqrt{\frac{\Sigma a}{A}},$$

where L = Length of cover-plate in question;
l = Length of span, center to center;
a = Net area of that cover-plate and all above it; and
A = Total net area of the flange, $\frac{1}{8}$ of the gross area of the web not being considered in this quantity.

The lengths of the cover-plates for the section above designed (see Fig. 141) are:

$$L_1 = 61.75\sqrt{\frac{4.50}{24.38}} = 26.45 \text{ feet.}$$

$$L_2 = 61.75\sqrt{\frac{4.50 + 6.00}{24.38}} = 40.00 \text{ feet.}$$

One foot is usually added on each end of the cover-plate as theoretically determined above. The results are also usually rounded off to the nearest half-foot. This is done in order to allow a safe margin because of the fact that the curve of flange areas is not a true parabola. The final measurements of the cover-plates are:

14 in. by $\frac{3}{8}$ in. by 28 ft. 6 in. long.
14 in. by $\frac{1}{2}$ in. by 42 ft. 6 in. long.

In most cases the cover-plate next to the angle on the top flange only is made to extend the entire length of the girder. Although this is not required for flange area, it is done in order to provide additional stiffness to the flange angles toward the ends of the span, and to prevent the action of the elements from deteriorating the angles and the web by attacking the joint at the top (see broken lines, Fig. 141, for length of first cover-plate extended).

EXAMPLES FOR PRACTICE

1. The dead-load moment equals 469 000 pound-inches; and the live-load moment, 4 522 000 pound-inches. Design a flange section entirely of angles, if the distance back to back of angles is $45\frac{1}{4}$ inches.

2. The dead-load moment is 3 340 000 pound-inches, and the live-load moment, 21 235 000 pound-inches. Design a flange section using 6 by 6-inch angles and three 14-inch cover-plates, the distance back to back of flange angles being $78\frac{1}{4}$ inches.

3. In each of the above cases, design the flange section considering that $\frac{1}{8}$ of the web area is taken as effective flange area. (For demonstration of the methods to be employed in the solution of this problem, see the succeeding text.)

While the section of a plate-girder is composite—that is, it consists of certain shapes joined together, and is not one solid piece—nevertheless these shapes are joined so securely that the section may be considered as a solid one and its moment of resistance computed accordingly. Let Fig. 142 be considered.

The moment of resistance of the section is:

$$M = \frac{S\left(A \times \frac{h_e^2}{2^2} + A \times \frac{h_e^2}{2^2} + \frac{th^3}{12}\right)}{\frac{h}{2}},$$

in the derivation of which the moment of inertia of the flange about its own neutral axis is considered as zero, and A equals the net area of one flange. Now, as the values of h_e and h seldom differ by more

than one inch, for all practical purposes they may be considered as equal. The above expression then reduces to:

$$M = S \times h\left(A + \frac{th}{6}\right)$$

$$= S \times h \text{ (net area of flange + one-sixth gross area of web)}$$

Since the rivet-holes decrease the moment of resistance of the web, one-sixth of the gross area cannot be considered, as is theoretically indicated in the above formula. It is common practice to take one-eighth, instead of one-sixth, of the gross web area. Substituting this value in the above equation, and transposing, there results:

$$\text{Area of flange} + \tfrac{1}{8} \text{ gross web area} = \frac{M}{Sh}.$$

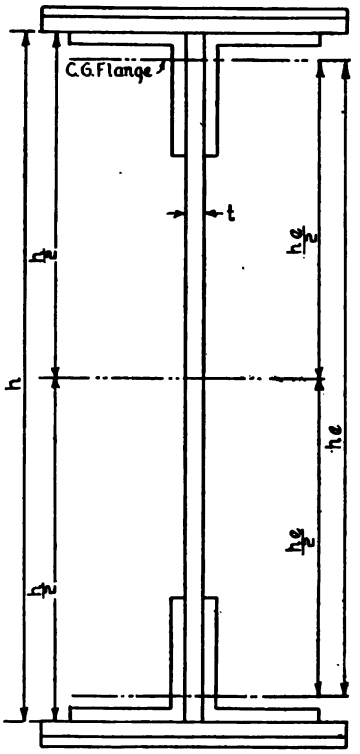


Fig. 142. Section of Plate-Girder.

The flange section will now be designed for the moments previously given, considering $\frac{1}{8}$ of the gross web area as efficient in withstanding the moment.

The gross area of the web is $74 \times \frac{3}{8} = 27.75$ square inches; and $\frac{1}{8}$ of this is 3.47 square-inches. The total approximate amount of flange area required is, as in the first case, 24.48 square inches.

According to the above formula, $\frac{1}{8}$ of the web area, or 3.47 square inches, may be considered as flange area, and therefore $24.48 - 3.47 = 21.01$ square inches, is the approximate area of the angles and cover-plates of the flange. The approximate area of one angle is then $21.01 \div (2 \times 2) = 5.25$ square inches. A 6 by 6 by $\frac{9}{16}$-inch angle gives the gross area of 6.43 square inches and, two rivet-holes being deducted, a net area of 5.305 square inches (see "Steel Construction," Part I, Table VIII, or Carnegie Handbook, p. 117). As this is quite close to the approximate area determined above, this angle will be taken. The ap-

proximate area of the cover-plates is $21.01 - 2 \times 5.305 = 10.40$ square inches. As before, the gross width of the cover-plate will be taken as 14 inches. The thickness is then $\frac{10.40}{14 - 2(\frac{7}{8} + \frac{1}{8})}$ $= 0.867$ inch—say $\frac{7}{8}$ inch.

The gross area of the angles being 12.86 square inches, and that of the cover-plates 12.25 square inches, the center of gravity of the section is found, by a method similar to that previously employed, to be 1.10 inches from the center of the cover-plate, or $1.10 - 0.438 = 0.662$ inch from the back of the flange angles. This makes the effective depth 72.93 inches.

For this section, the true live-load flange stress is $(1\,340\,000 \times 12) \div 72.93 = 221\,000$ pounds, and the actual dead-load flange stress is $(275\,000 \times 12) \div 72.93 = 45\,400$ pounds. The actual areas required for the live and dead load are 22.10 and 2.27 square inches, which are obtained by dividing the above flange stresses by 10 000 and 20 000 pounds, respectively. The total required area is the sum of the two areas above, and is equal to 24.37 square inches. The total area required in the flange angles and cover-plates is therefore 24.37, less $\frac{1}{8}$ the gross area of the web, 3.47, which leaves 20.90 square inches. The same angles as decided upon before will be used. This gives a required area for the cover-plates of $20.90 - 10.61 = 10.29$ square inches. The required thickness is then $10.29 \div (14 - 2) = 0.857$—say $\frac{7}{8}$ inch. The following section of the flange will therefore be decided upon:

Shape	Net Section	Gross Section
2 Angles 6 in. by 6 in. by $\frac{9}{16}$ in.	10.61 sq. in.	12.86 sq. in.
1 Cover-plate 14 in. by $\frac{3}{8}$ in.	4.50 "	5.25 "
1 Cover-plate 14 in. by $\frac{1}{2}$ in.	6.00 "	7.00 "
Total =	21.11 "	25.11 "

As the total net area above is within $2\frac{1}{2}$ per cent of the required net area, that section will be taken (see Specifications, Article 149). Note that in this case, the thickness of the cover-plates in the final design is the same as that determined in the preliminary design. Also note that the total net area is about 4 square inches, or 20 per

cent, less than in the flange as first designed, in which case none of the area of the web was considered as withstanding the bending moment.

The $\frac{1}{2}$-inch cover-plate on the top flange will extend the entire length of the grider, and is therefore 62 feet 9 inches long. The lengths of the other cover-plates are:

$$\text{For } \tfrac{1}{2}\text{-inch plate at the bottom, } L = 61.75\sqrt{\frac{10.50}{21.11}} = 43.5 \text{ feet.}$$

$$\text{For each } \tfrac{3}{8}\text{-inch plate, } L = 61.75\sqrt{\frac{4.50}{21.11}} = 28.5 \text{ feet.}$$

One foot should be added to each of the above lengths at each end, thus making the total lengths 45 feet 6 inches and 30 feet 6 inches, respectively.

EXAMPLES FOR PRACTICE

1. If the span is 63 feet center to center, compute the length of the cover-plate. The section consists of two angles 6 by 6 by $\frac{5}{8}$ in.; one cover-plate 14 by $\frac{1}{2}$ in.; and one cover-plate 14 by $\frac{7}{16}$ in.; two rivet-holes being taken out of each angle and each cover-plate.

2. If the span is 87 ft. 9 in. center to center, compute the length of the cover-plate if the flange consists of two angles 6 by 6 by $\frac{3}{4}$ in., and four cover-plates 16 by $\frac{9}{16}$ in., two rivet-holes being taken out of each angle and each cover-plate.

In determining the area of plates, the tables in the Carnegie Handbook, pp. 245 to 250, are convenient. In order to obtain plates

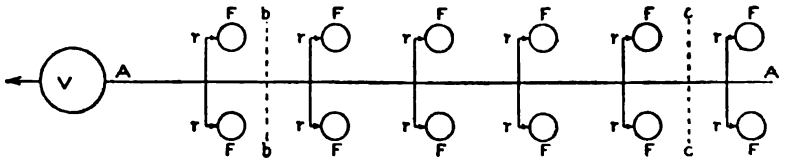


Fig. 143. Diagram Illustrating Transference of Shear from Web to Flanges by Rivets.

whose widths are greater than $12\frac{3}{4}$ inches, see the note in the right-hand column on page 250, Carnegie Handbook. For another presentation of the above subject-matter, see "Steel Construction," Part IV. pp. 252, 254, and 261.

The spacing of the rivets in the flanges is a matter of considerable importance; the shear is transferred from the web to the flanges, where it becomes flange stress. This is done by the rivets, each rivet taking as much flange stress as is allowed by the Specifications. The conditions are similar to those shown in Fig. 143, where V represents

an object exerting a pull on a long, thin plate $A-A$ which has, at various points along this length, small objects F attached to it by means of pegs $r-r$. These small objects F hold the plate $A-A$ in equilibrium. Here V represents the shear which tends to cause the movement; $A-A$, the web; $r-r$, the rivets; and $2F$ the amount of flange stress taken by each rivet.

At section $c-c$ the total amount in the web to be transformed is $2F$; at section $b-b$ it is $10F$. From this it is seen that enough rivets $r-r$ must be put in between the sections $b-b$ and $c-c$ to take up $10F-2F=8F$; hence it is proved that the rivets between any two sections of the flange take up the difference in flange stress between those two sections.

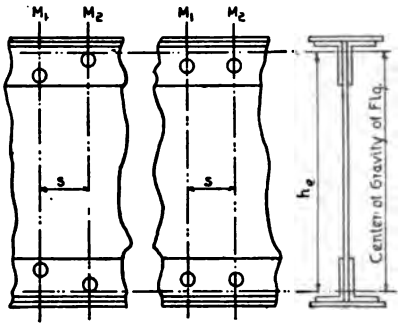


Fig. 144. Determination of Rivet Spacing.

The discussion just given will be the means of giving us the number of rivets required between any two sections; but it does not give us the rivet spacing between these two sections. In order to determine the rivet spacing at any particular point, the following analysis is presented (see Fig. 144).

Let M_1 = Moment at one section;
M_2 = Moment at another section nearer center of girder than the section where M_1 occurs;
V = Shear at section where M_1 occurs;
v = Amount of flange stress one rivet can transfer; or it is the stress on one rivet;
s = Distance between the two sections;
n = Number of rivets between the two sections.

Then,

$\frac{M_1}{h_e}$ = Flange stress due to moment M_1;

$\frac{M_2}{h_e}$ = Flange stress due to moment M_2;

$\frac{M_2}{h_e} - \frac{M_1}{h_e}$ = Difference of flange stress between the two sections;

$$\left(\frac{M_2}{h_e} - \frac{M_1}{h_e}\right) \div v = n, \text{ Number of rivets required in space } s \ldots (1)$$

If the above sections be taken close enough together so that the number of rivets required will be 1 (that is, $n = 1$), then V can be considered as constant between the two sections, and then the moment $M_2 = M_1 + Vs$ (see Article 44, Part I). Substituting in Equation 1, above, there results:

$$\left(\frac{M_1 + Vs}{h_e} - \frac{M_1}{h_e}\right) \div v = 1,$$

from which,

$$s = \frac{vh_e}{V},$$

which is the formula for the rivet spacing in the vertical parts of the flanges of any girder, providing the flange is not subjected to localized loading. It is to be used for the rivet spacing in both the top and bottom flanges of through girders, but not in the *top* flanges of deck plate-girders for railroad service. It is to be used, however, in the *bottom* flanges of deck plate-girders for railroad service. The distance h_e is not ordinarily used, the distance between rivet lines being used instead (see Fig. 145). The rivet spacing in the cover-plates and horizontal legs of the angles is made to stagger with that in the vertical legs, and usually the staggering is with every other rivet in the vertical flange. The term *stagger* signifies that the rivets in the top flange are not placed opposite the rivets in the vertical legs of the flange angles—or, that in case there are two lines of rivets in the vertical legs of the angle, a rivet near the outer edge of the cover-plate is placed in the same section where a rivet occurs near the lower edge of the vertical legs of the angle, and *vice versâ*.

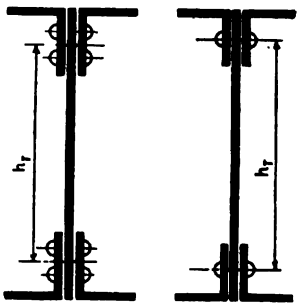


Fig. 145. Determination of Rivet Spacing.

EXAMPLES FOR PRACTICE

1. Determine the rivet spacing at a section where the shear is 147 200 pounds, the value of one rivet 4 920 pounds, and the effective depth of the section 84¼ inches.

Ans. 2.82 inches.

2. Determine the stress on a rivet at a section where the shear is 299 400 pounds, the spacing 2½ inches, and the effective depth of the girder 84¼ inches.

Ans. 8 870 pounds.

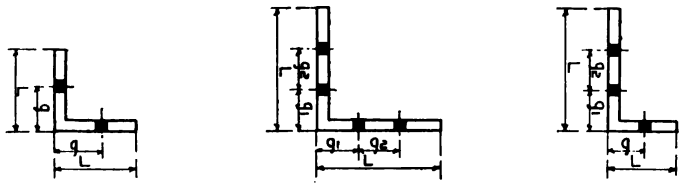
The rivet spacing is usually determined at the tenth-points, and a curve is plotted with the spacing as ordinates and the tenth-points as abscissæ. The rivet spacing at any intermediate point can be determined from this curve. When one-eighth the gross section of the web is considered as flange area, then only that proportion of the shear which is transferred to flanges is to be considered in computing the rivet spacing, on account of the fact that some of the shear is transferred directly to the bending moment in the web.

In order to determine the distance between rivet lines, the *gauge*, or distance out from the back of the angles to the place where the rivets must be placed, must be known for different lengths of legs. Table XXI gives the standard gauges, and also the diameter of the largest rivet or bolt which is allowed to be used in any sized leg. No gauges should be punched otherwise unless your large experience or instructions from one higher in authority demand it, and this should be so seldom that indeed it might be said never to be necessary.

TABLE XXI

Standard Gauges for Angles

(All dimensions given in inches)



L	g	Maximum Rivet or Bolt	L	g	Maximum Rivet or Bolt	L	g	Maximum Rivet or Bolt
8	4½	⅞	3½	2	⅞	2	1⅛	½
7	4	⅞	3	1¾	⅞	1¾	1	½
6	3½	⅞	2¾	1⅝	¾	1½	⅞	⅜
5	3	⅞	2½	1⅜	⅝	1¼	¾	⅜
4	2¼	⅞	2¼	1¼	⅝	1	9/16	¼

L	g_1	g_2	L	g_1	g_2
8	3	3	6*	2½	2¼
7	2½	3	5	2	1¾
6	2¼	2½			

*When thickness is ¾ inch or over.

The distance between rivet lines for the girder being designed (see Fig. 146), is, in the first case:

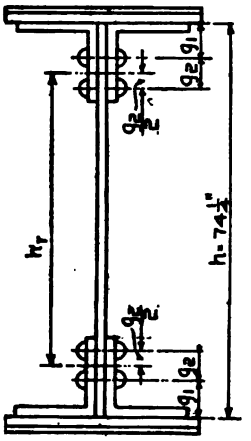


Fig. 146. Determination of Distance between Rivet Lines.

$$h_r = h - \left(2g_1 + \frac{2g_2}{2}\right)$$
$$= 74.25 - (2 \times 2\tfrac{1}{2} + 2\tfrac{1}{4})$$
$$= 67.00 \text{ inches.}$$

In the second case, where $\frac{1}{8}$ of the web is considered, the above distance is $74.25 - (2 \times 2\frac{1}{4} + 2\frac{1}{2}) = 67.25$ inches. The computations and the rivet spaces at the tenth-point, and at the ends of the cover-plates in the bottom chord of the plate-girder, are shown for each case in Table XXII. The value of v is the value of a $\frac{7}{8}$-inch rivet in bearing in a $\frac{3}{8}$-inch web (see Specifications, Article 40, and Carnegie Handbook, p. 195, second table). This value is 4 920 pounds.

In the first column, 7.98− indicates that the end of the cover-plate next to the flange is 7.98 feet from the end of the girder, and that this section is taken just to one side of that point, the side being that nearest the end of the girder. In a similar manner, 7.98+ indicates that the section is taken to that side of the point which is nearest the center of the girder. A like interpretation should be placed on 15.55− and 15.55+, the point under consideration in this case being the end of the outer or top cover-plate.

In the fifth column, values are given which indicate that portion of the shear which is transferred to the flanges. For example, $\frac{97\,700}{(14.08 \div 10.61)} = 74\,700$, and the difference between 97 700 and 74 700 represents that portion of the shear which is taken up directly by the web in the form of bending moment. An inspection of the headings of the third and fourth columns will tend to make this matter clearer.

Where there is local loading, as in the top flange, the rivets, in addition to the stress caused by the transferring of web stresses, are stressed by the vertical action of the angles being pressed downward by the ties and the consequent upward pressure of the web. Accord-

TABLE XXII

Rivet Spacing in Bottom Flange

Flange Taking All the Moment

Section	Total Shear (Pounds)	h_r (Inches)	v (Pounds)	Rivet Spacing (Inches)	Remarks
0	117 800	67	4 290	2.80	
1	97 700	67	4 290	3.38	
2	79 300	67	4 290	4.16	
3	61 300	67	4 290	5.38	
4	44 200	67	4 290	7.46	See Specifications, Art.54
5	28 600	67	4 290	11.52	

One-Eighth of Web Area Considered

h_r = 67.25 inches; v = 4 920 pounds

Section	Total Shear (Pounds)	Net Flange Area plus $\frac{1}{8}$ Web Area (Sq. Inches)	Net Flange Area (Sq. Inches)	Reduced Shear (Pounds)	Rivet Spacing (Inches)
0	117 800	14.08	10.61	88 800	3.75
1	97 700	14.08	10.61	74 700	4.45
7.98−	90 000	14.08	10.61	67 900	4.88
7.98+	90 000	20.08	16.61	74 800	4.42
2	79 300	20.08	16.61	65 800	5.02
15.55−	67 500	20.08	16.61	56 000	5.81
15.55+	67 500	24.58	21.11	58 100	5.69
3	61 300	24.58	21.11	52 700	6.28
4	44 200	24.58	21.11	38 000	8.71
5	28 600	24.58	21.11	24 600	13.42

ing to Article 15 of the Specifications, the weight of one driver is distributed over three ties (see Fig. 147).

Let $\frac{W}{l}$, $= w$, the load per linear inch caused by one wheel W, which load is assumed to be uniformly distributed over the distance l;

$ws = v_1$, the vertical load or stress that comes on one rivet in the space s;

$v = \frac{Vs}{h_r}$, the stress on a rivet due to the distribution of flange stresses when s is a space, and V the shear at that point.

When these two stresses act on the rivet, the maximum stress will be v_u, the ultimate amount that it is allowed to carry, and this will act as shown in Fig. 147: Then,

$$v_u = \sqrt{v^2 + v_1^2}$$

$$= \sqrt{\left(\frac{V_s}{h_r}\right)^2 + (ws)^2}$$

$$= s\sqrt{\left(\frac{V}{h_r}\right)^2 + w^2},$$

from which,

$$s = \frac{v_u}{\sqrt{\left(\frac{V}{h_r}\right)^2 + w^2}},$$

which gives the spacing at any point in the girder flange under localized loading. Note that if w equals zero—that is, if there is no localized loading—there results:

$$s = \frac{v_u}{\sqrt{\left(\frac{V}{h_r}\right)^2 + 0}}$$

$$= \frac{v_u h_r}{V},$$

which is the same as previously deduced for flanges without localized loadings.

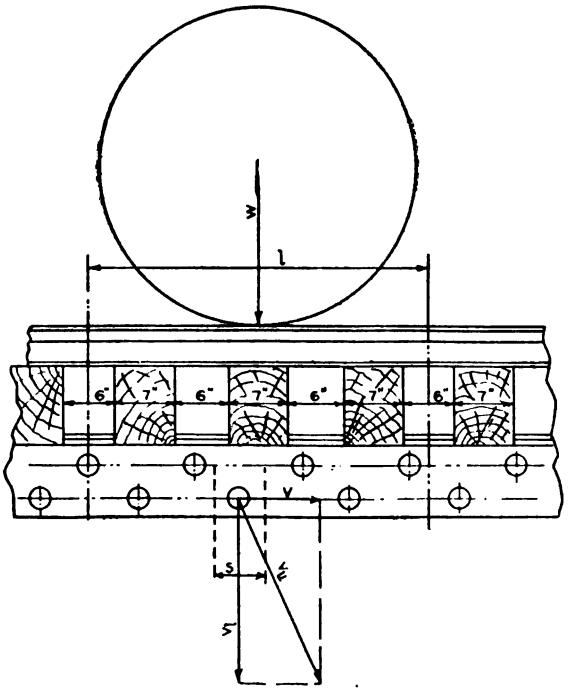


Fig 147. Rivet Spacing Determined by Stresses Distributed under Localized Loading.

The rivet spacing for the top flange of the girder which is being designed is given in Table XXIII. Here $W = 20\,000$; $l = (3 \times 7 + 3 \times 6) = 39$ inches; $w = \frac{20\,000}{39} = 513$; $h_r = 67$ inches; and $v_w = 4\,920$ pounds, which is the bearing of a $\frac{7}{8}$-inch rivet in the $\frac{3}{8}$-inch web. The top cover-plate is run the entire length of the span.

TABLE XXIII

Rivet Spacing in Top Flange

Flange Taking All the Moment

Section	w^2	Total Shear (Pounds)	$\left(\frac{V}{h_r}\right)^2$	$\sqrt{\left(\frac{V}{h_r}\right)^2 + w^2}$	Rivet Space (Inches)
0	262 600	117 800	3 080 000	1 825	2.70
1	"	97 700	2 140 000	1 550	3.17
2	"	79 300	1 390 000	1 285	3.83
3	"	61 300	840 000	1 050	4.68
4	"	44 200	435 000	835	5.88
5	"	28 600	181 500	660	7.45

One-Eighth of Web Area Considered

$w = 513$; $h_r = 67\frac{1}{4}$ inches; $v_u = 4\ 920$ pounds.

Section	w^2	Reduced Shear (Pounds)	$\left(\frac{V}{h_r}\right)^2$	$\sqrt{\left(\frac{V}{h_r}\right)^2 + w^2}$	Rivet Space (Inches)
0	262 600	97 600	2 100 000	1 538	3.20
1	262 600	81 000	1 450 000	1 315	3.74
2	262 600	65 800	985 000	1 100	4.50
15.55 −	262 600	56 000	695 000	980	5.02
15.55 +	262 600	58 100	765 000	1 014	4.85
3	262 600	52 700	616 000	938	5.24
4	262 600	38 000	320 000	763	6.44
5	262 600	24 600	134 000	629	7.82

The points other than the tenth-points referred to in the first column are for sections taken just to the left and right of the top cover-plate. The values of the reduced shears given in the third column are obtained as has been previously explained. Although the rivet spacing in the lower flange is considerably greater than that in the upper flange, and accordingly a smaller number of rivets would be required, yet the spacing in the lower flange is made the same as that in the upper. Convenience in the preparing of plans and facility in manufacture make this action economical. Theoretical spacing greater than 6 inches should be dealt with according to Article 54 of the Specifications.

The values of the rivet spacing given in Tables XXII and XXIII are plotted in Fig. 148. Note that the effect of the localized loading is to decrease the rivet spacing, and also note that the effect increases from the ends toward the center.

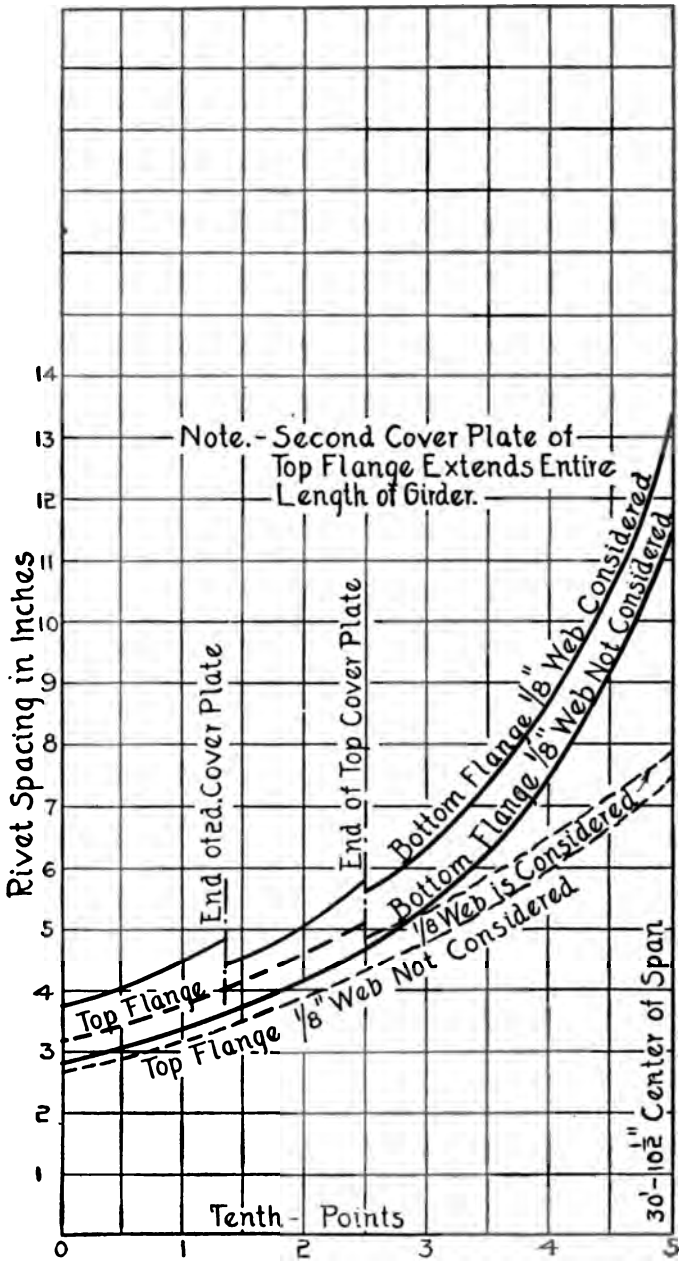


Fig. 148. Plotted Values of Rivet Spacing Given in Tables XXII and XXIII.

The size of the flange angles and the width of the cover-plates for different spans, are a matter of choice. Once the size is determined, the thickness can be computed. The sizes very generally adopted in practice are as follows:

Spans	Angles	Width of Cover-Plate
15 to 20 feet	5 x 3½ inches	none
20 " 25 "	6 x 4 "	"
25 " 40 "	6 x 6 "	"
40 " 75 "	6 x 6 "	14 inches
75 " 100 "	6 x 6 "	16 "
100 " 120 "	8 x 8 "	20 "

For another method of the presentation of this subject, see "Steel Construction," Part IV, pp. 264 to 268.

EXAMPLE FOR PRACTICE

1. Determine the rivet spacing for the top chord of a plate-girder, loading E 40, and 7 by 9-inch ties being used. The web is $\frac{3}{8}$ inch thick; distance from back to back of angles, 6 feet 6½ inches; flange angles, 6 by 6 by ½-inch; and cover-plate, 14 by $\frac{5}{8}$-inch, two $\frac{7}{8}$-inch rivets being out of each. First, consider the flange as taking all the bending moment; and second, consider one-eighth the gross area of the web. The total unreduced shear is 80 000 pounds in both cases.

Ans. 3.21 inches; 3.76 inches.

74. **Lateral Systems and Cross-Frames.** There are two methods in use in common practice in determining where the panels of the lateral bracing shall fall—namely, (1) To choose the number of panels so that the panel points come opposite the stiffness, and (2) to choose the number of panels so that the placing of the panel points is independent of the stiffener spacing. The lateral systems should be of the Warren type; and in both of the above cases the angles that the diagonals make with the girder should not be greater than 45 degrees. Also, it is best to have all panels the same length and to have an equal number of panels. This latter condition will simplify the drafting very much, since one-half of one girder can be drawn and the other half will be symmetrical, the opposite girder being similar to the one drawn, but being left-handed.

The members of the lateral systems will take tension or compression according to the direction the wind blows. Cross-frames are placed at intermediate points to stiffen the girders. These are

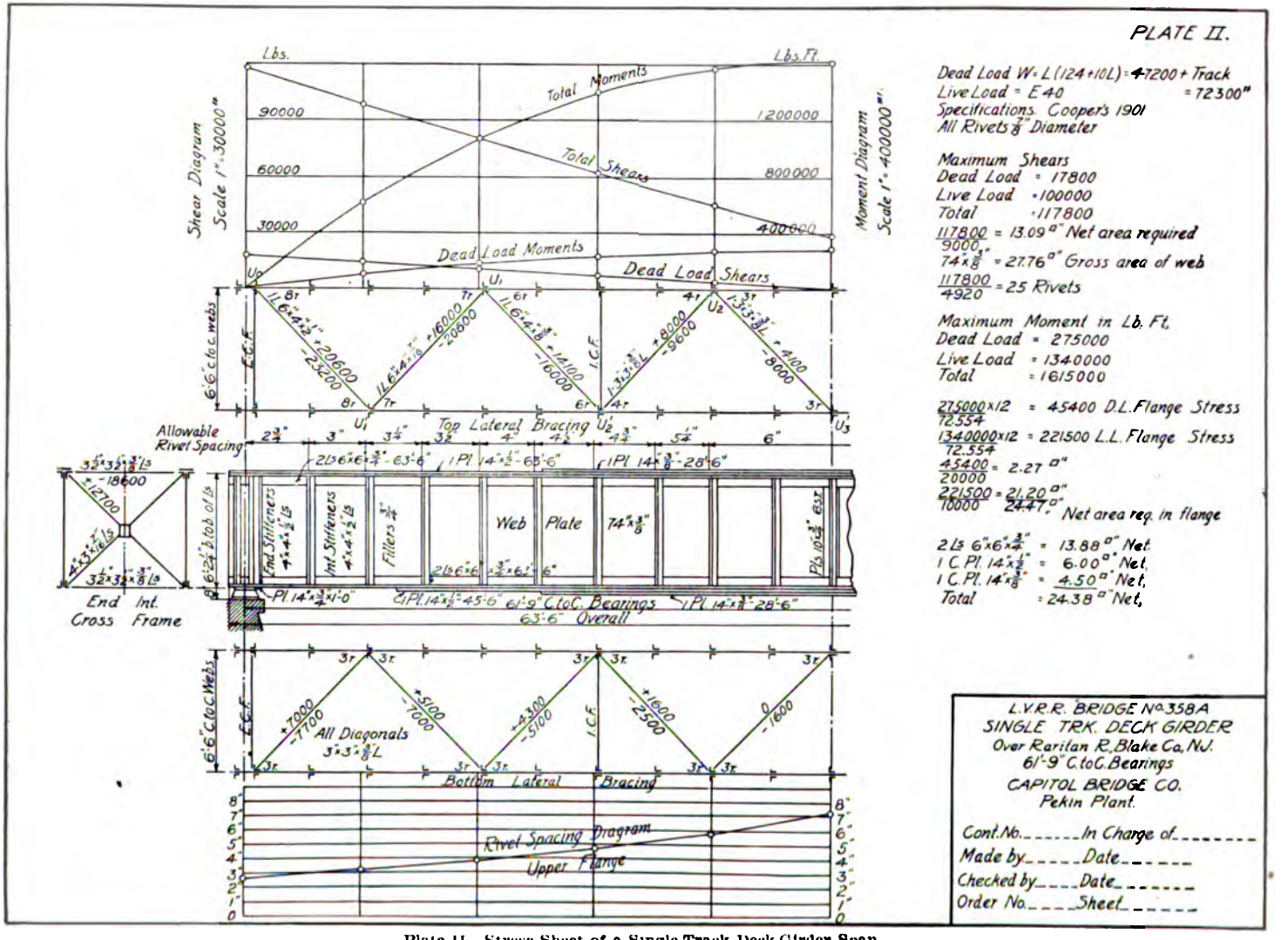


Plate II. Stress Sheet of a Single-Track Deck-Girder Span.

diagonal bracings (see Plate II), and are placed at certain intervals according to the judgment of the engineer. Good practice demands that their number should be about as indicated below:

Span of Girder	Number of Cross-Frames
0 to 20 feet	2
20 to 35 "	3
35 to 70 "	4
70 to 85 "	5
85 to 110 "	6

The above is not intended to serve as a hard and fixed rule. Variations from the limits given are to be made as the case demands. In all cases they are put at the panel points of the bracing, the top and bottom parts acting as sub-verticals in the lateral system. Also, the cross-frames should divide the span into equal parts if possible. In cases where that is not possible, the shortest divisions should come near the ends of the spans.

If the panel points are to be located at the stiffeners, the number of panels is a function of the depth of the girder (see Specifications, Articles 47 and 48). In this case the number of panels is given by:

$$N = \frac{\text{Span in inches.}}{\text{Depth of girder in inches}},$$

no fraction being considered. As an example, let it be required to determine the number of panels in a girder 85 feet center to center of bearings, the depth being $90\frac{1}{4}$ inches back to back of angles. Then,

$$N = \frac{85 \times 12}{90.25} = 11.3, \text{ or, say, } 11 \text{ panels.}$$

Each panel will then be 92.8 inches long. This, according to Article 47 of the Specifications, being greater than 5 feet, would not be allowed as a space between two stiffeners; but one stiffener can be placed in between, and then the panel points will come at every other stiffener. The cross-frames should be five in number.

The arrangement of panels and cross-frames is shown in Fig. 149. Here the cross-frames are marked *C. F.*, and the broken lines represent the lower lateral system.

In case an even number of panels were desired, then ten would be the number chosen and the general arrangement would be as shown in Fig. 150. The length of a panel would be $85 \times 12 \div$

10 = 102 inches, or 8 feet 6 inches, which would allow of one stiffener in between and still keep the stiffener spacing within the limit of 5 feet.

The cross-frames at the ends of the span are designated as *end cross-frames*, and those in between are designated as *intermediate cross-frames*.

In case the spacing of the stiffeners is not required to be such as to coincide with the panel points of the lateral bracing, the panel length will depend upon the spacing of the girders, being equal to or

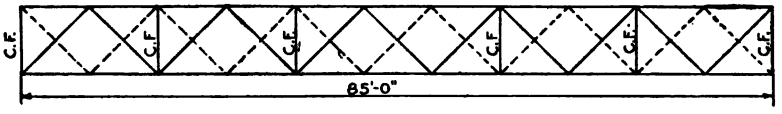


Fig. 149.

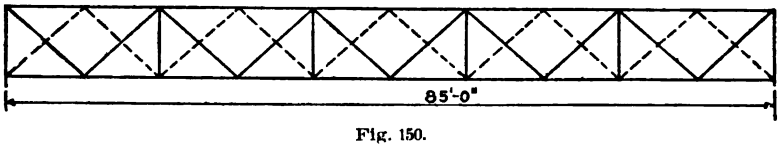


Fig. 150.

Arrangements of Panels and Cross-Frames.

greater than the spacing in order to keep the angle which the diagonals make with the girder less than 45 degrees. In this case,

$$N = \frac{\text{Span in feet}}{\text{Distance center to center of girders in feet}}.$$

For the girder considered on page 174, the number of panels would be $\frac{85}{7.5} = 11.3$—or, say, 11 panels—if odd numbers were to be used, and 12 if even numbers were to be desired.

For the case in hand, the panel points of the bracing will be taken at the stiffeners, and an even number of panels will be used. Then,

$$N = \frac{61.75 \times 12}{74.25} = 9.98 \text{ (say 10)}.$$

The arrangement of the panels and cross-frames, and also the maximum stresses in the diagonals, are shown in Plate II, the stresses being determined according to Article 50, Part I, "Bridge Analysis," and Article 24 of the Specifications. All the wind is taken as acting on one side of the bridge; and no overturning effect, either on the girder

or on the train, is considered. Also, note that the wind stresses in the flanges are not considered. Should the student determine these, he will find them too small to be considered according to Specifications, Article 39.

Before designing the lateral diagonals which consist of one or two angles, Articles 31, 33, 34, 35 (last portion), 38, 40, 63, and 83 of the Specifications should be carefully studied. The upper lateral bracing is to be designed first. Carnegie Handbook, pp. 109 to 119, is to be used.

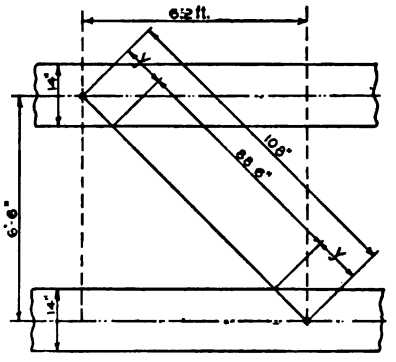


Fig. 151. Determination of Length of Diagonal in Lateral Bracing.

The member U_0U_1 must be designed for a compressive stress equal to $23.20 + 0.8 \times 20.6 = -39.68$, and a tensile stress of $20.6 + 0.8 \times 20.6 = +37.08$. The length of the diagonal measured from center to center of girders is $\sqrt{6.5^2 + 6.2^2} = 9$ feet, or 108 inches. In reality the length is not 108 inches, as the cover-plate takes off a certain amount, as shown in Fig. 151. The true length, which is to be taken as a column length in designing, is $108 - 2y$, and y is readily computed to be 9.70 inches, thus making the true length 88.6 inches. The least allowable rectangular radius of gyration is obtained from the relation that the greatest value of $\frac{l}{r} = 120$, and therefore the least value of $r = \frac{l}{120} = 0.74$. It will be assumed that a 6 by 4 by $\frac{9}{16}$-inch angle with an area of 5.31 square inches will be sufficient. Here the length equals 88.60 inches, and the least rectangular radius of gyration is 1.14; hence,

$$P = 13\,000 - 60 \times \frac{88.60}{1.14}$$
$$= 8\,330 \text{ pounds per square inch.}$$

The required area is $\frac{39\,360}{8\,330} = 4.73$ square inches. As the angle assumed has an area of 5.31 square inches, which is considerably greater than the 4.73 square inches required, this angle cannot be used, and other assumptions must be made until the area of the angle

assumed and the required area as computed are equal or very nearly so.

A 6 by 4 by $\frac{1}{2}$-inch angle with an area of 4.75 square inches will now be assumed. The length is 88.60 inches as before, and the least rectangular radius of gyration is 1.15. The unit-load $P = 8\,340$ pounds per square inch, and the required area is $\frac{39\,360}{8\,340} = 4.72$ square inches. As the area of the angle assumed and the required area as computed are very close, this sized angle will be used.

The section must now be examined for tension, and in order that both legs of the angle may be considered as effective section, both legs must be connected at the end. The area required will be $\frac{36\,760}{18\,000} =$ 2.04 square inches., Considering one rivet-hole is taken out of the angle, the net area is $4.75 - 1 \times (\frac{7}{8} + \frac{1}{8}) \times \frac{1}{2} = 4.25$ square inches, which is amply sufficient.

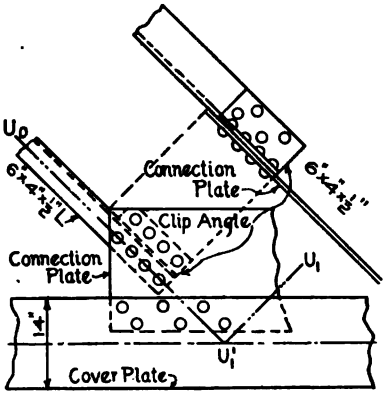


Fig. 152. Method of Connecting Angle Legs in Lateral Bracing and Cross-Frames.

If the 4-inch leg only were assumed to be connected, the gross area would be $4 \times \frac{1}{2} = 2.00$ square inches, and the net area would then be $2.00 - 1 \times (\frac{7}{8} + \frac{1}{8}) \times \frac{1}{2}$-inch $= 1.50$ square inches, which is not sufficient. If the 6-inch leg were connected, the area would be sufficient. See Fig. 152 for method of connection and rivets.

The number of rivets required (see Specifications, Article 38 and 40) is computed as follows: If the member were not subjected to both tension and compression, the number of rivets required in single shear would be:

$$n = \frac{39\,360}{(9\,000 + 50 \text{ per cent of } 9\,000) \times 0.6013}$$
$$= \frac{39\,360}{8\,100}$$
$$= 4.86 \text{ (say 5).}$$

But according to Article 38 of the Specifications, this number must be increased 50 per cent, and accordingly $4.86 \times 1\frac{1}{2} = 7.29$ (say 8)

shop rivets are to be used. In the above formula, 0.6013 is the area of the cross-section in square inches of a $\frac{7}{8}$-inch rivet. In order that both legs should be connected, a clip angle as shown in Fig. 152 is used; and the same number of rivets must go through both legs of the clip angle, since the stress in the vertical leg of the main angle is transferred to the clip angle and from there into the connecting plate.

The above number of rivets makes the joint safe so that it will not shear off in the plane between the connection plate and the horizontal leg of the angle. The joint must also be designed so that there will be sufficient rivets in bearing to prevent them from tearing out of the connecting plate. The number required is:

$$n = \frac{39\,360}{(15\,000 + 50 \text{ per cent of } 15\,000) \times \frac{7}{8} \times \frac{3}{8}}$$
$$= \frac{39\,360}{7\,380}$$
$$= 5.34.$$

This 5.34 must be increased 50 per cent, making a total of $5.34 \times 1.5 = 8.01$ = say, 8 shop rivets as before.

The above rivets are shop rivets, since it is assumed that the span, being a small one, will be riveted complete in the shop and shipped to the bridge site ready to place in position without any further riveting. In case the girders are shipped separately, then the lateral bracing must be riveted up in the field; and according to last part of Article 40 of the Specifications, the rivets, being field rivets, must have the allowed unit-stresses reduced one-third, which is equivalent to having the number of shop rivets increased 50 per cent. This will make the required number of field rivets $8 \times 1.5 = 12$.

As a rule, the connection plates are $\frac{3}{8}$ inch thick, seldom more. Also, the members of the upper lateral system are connected on the lower side of the connection plate in order not to interfere with the ties. Note that the use of the clip angles requires a smaller connection plate than would be necessary if these angles were not used, since in the latter case all the rivets must then be placed in one row in the horizontal leg of the angle.

The number of rivets required in the connection plate and the flange of the girder must be sufficient to take up the component of that member parallel to the girder. For the case in hand, the number (see Fig. 153) is:

$$\frac{x}{8} = \frac{6.2}{9.0},$$

from which,

$$x = 5.5 \text{ (say 6) rivets.}$$

Additional rivets should also be put in, in order to take up the component of the other lateral diagonal which meets at this point.

The member $U_1' U_1$ is to be designed for a maximum compressive stress of $20.6 + 0.8 \times 16.0 = -33.4$. A 6 by 4 by $\frac{7}{16}$-inch angle with an area of 4.18 square inches will be assumed. The least rectangular radius of gyration is 1.16. The unit allowable load is:

$$P = 13\,000 - 60 \times \frac{88.6}{1.16} = 8\,420$$

pounds per square inch.

The required area is $\frac{33\,400}{8\,420} =$ 3.97 square inches. As this is very near the area assumed, and as trials with other angles do not give required areas which come any closer, this angle will be used.

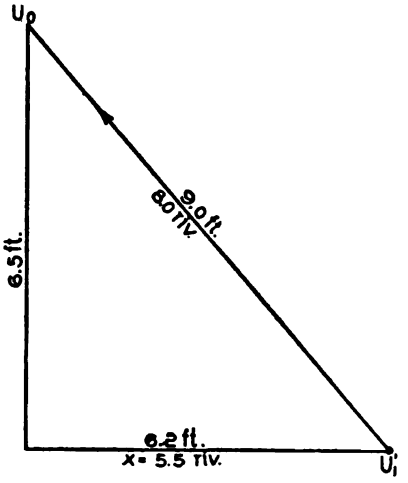


Fig. 153. Calculation of Rivets in Connection Plate and Flange of Girder.

The rivets required in single shear are:

$$\frac{33\,400}{8\,100} \times 1.5 = 6.21 \text{ (say 7) shop rivets, and}$$

$$6.21 \times 1.5 = 9.3 \text{ (say 10) field rivets.}$$

The rivets required in bearing in the $\frac{3}{8}$-inch connection plate are:

$$\frac{33\,400}{7\,380} \times 1.5 = 6.78 \text{ (say 7) shop rivets, and}$$

$$6.78 \times 1.5 = 10.17 \text{ (say 11) field rivets.}$$

The above computations show the joint to be weakest in bearing, and therefore 7 shop or 11 field rivets must be used. It is not necessary to investigate this member for tension, as the computations for the first diagonal indicate that the area will be sufficient, both legs being connected.

The member U_1U_2' must be designed for a maximum compressive stress of $16.0 + 0.8 \times 14.1 = -27.28$. A 6 by 4 by $\frac{3}{8}$-inch

angle with an area of 3.61 square inches and a least rectangular radius of gyration of 1.17 will be assumed. The unit-stress P, as computed from the formula in the Specifications, is 8 460 pounds per square inch; and the required area is $\frac{27\,280}{8\,460} = 3.23$ square inches. This angle will be used, as the given and required areas are close together, and as the next smaller angle—a 6 by $3\frac{1}{2}$ by $\frac{3}{8}$-inch angle with an area of 3.42 square inches—gives a required area of 3.58 square inches, thus being too small.

The rivets required in single shear are:

$$\frac{27\,280}{8\,100} \times 1.5 = 5.04 \text{ (say 5) shop rivets, and}$$

$$5.04 \times 1.5 = 7.6 \text{ (say 8) field rivets.}$$

The rivets required in bearing in a $\frac{3}{8}$-inch web are:

$$\frac{27\,280}{7\,380} \times 1.5 = 5.54 \text{ (say 6) shop rivets, and}$$

$$5.54 \times 1.5 = 8.3 \text{ (say 9) field rivets.}$$

In order to make the joints safe, 6 shop or 9 field rivets should be used.

The member $U_2'U_2$ must be designed for a maximum compressive stress of $9.6 + 0.8 \times 8.0 = -16.00$. A $3\frac{1}{2}$ by 3 by $\frac{3}{8}$-inch angle with an area of 2.30 square inches and a least rectangular radius of gyration of 0.90 will be assumed. The unit-stress P is 7 090 pounds per square inch, and the required area is $\frac{16\,000}{7\,090} = 2.26$ square inches. As the required and the actual areas are very close together, this angle will be used.

The rivets required in single shear are:

$$\frac{16\,000}{8\,100} \times 1.5 = 2.96 \text{ (say 3) shop rivets, and}$$

$$2.96 \times 1\tfrac{1}{2} = 4.44 \text{ (say 5) field rivets.}$$

By computation similar to the above, it is found that 4 shop or 5 field rivets are required in bearing. Since the bearing requires the most rivets to make the joint safe, 4 shop or 5 field rivets must be used.

If the Specifications would have allowed a $3\frac{1}{2}$ by $3\frac{1}{2}$ by $\frac{5}{16}$-inch angle with an area of 2.09 square inches, this angle would have exactly fulfilled the requirements, the required area being 2.09 square inches.

The member U_2U_3' must be designed for a maximum compres-

sive stress of $8.0 + 0.8 \times 4.1 = -11.28$. A 3 by 3 by $\frac{3}{8}$-inch angle with an area of 2.11 square inches and a least radius of gyration of 0.91 will be assumed. In this case the unit-stress is 7 160, and the area required is 1.58 square inches. The required area is considerably less than the area of the angle assumed; but it must be used, since it is the smallest allowed by the Specifications, which require that the material shall not be less than $\frac{3}{8}$-inch, and from Table XXI it is seen that 3 inches is the smallest size leg in which a $\frac{7}{8}$-inch rivet can be used.

The stresses in all the members of the lower lateral system are less than the stresses in the member just designed, and therefore all members of the lower lateral system will be made of one 3 by 3 by $\frac{3}{8}$-inch angle.

For the last member designed in the upper lateral system, and for all members in the lower lateral system, 3 shop or 5 field rivets will be required at the ends. These are more than sufficient to take up the stress, but it has been found that less than three rivets do not make a good joint.

The stress sheet, Plate II, shows the general arrangement of the lateral system, the number of rivets in the connections and also in the connection plates where they join the flanges.

The *intermediate cross-frames* do not lend themselves to a theoretical design, since the stresses which come upon them are not easily ascertained. It is good practice to require that all members be of the sizes as given below:

Span of Girder (in Feet)	Angles (in Inches)	Rivets	
		Shop	Field
30 to 65	$3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$	3	4
65 to 110	$4 \times 4 \times \frac{3}{8}$	4	5

The angles in the intermediate cross-frames will therefore be $3\frac{1}{2}$ by $3\frac{1}{2}$ by $\frac{3}{8}$-inch.

The *end cross-frames* (see Fig. 154) act in a manner somewhat similar to the portal bracing in a bridge, since they transfer all the wind which comes on the top chord and on the train to the abutment. This load, which acts at the level of the ties, is in this case (see Article 24 of the Specifications):

$$P = \frac{600 \times 61 \text{ ft. } 9 \text{ in.}}{2} = 18\,525 \text{ pounds.}$$

It is usually assumed that half of this is transferred to the point a by means of a–b, and from there down a–b' to the masonry. The other half goes directly down b–a' to the masonry. This causes stresses as shown in Fig. 154. Note that the stress in a–b will always be compression; but the stresses in the diagonal will be either tension or compression according to the direction the wind blows. The member a–b will be a $3\frac{1}{2}$ by $3\frac{1}{2}$ by $\frac{3}{8}$-inch angle. To form the connections at its end, 3 shop or 5 field rivets will be used.

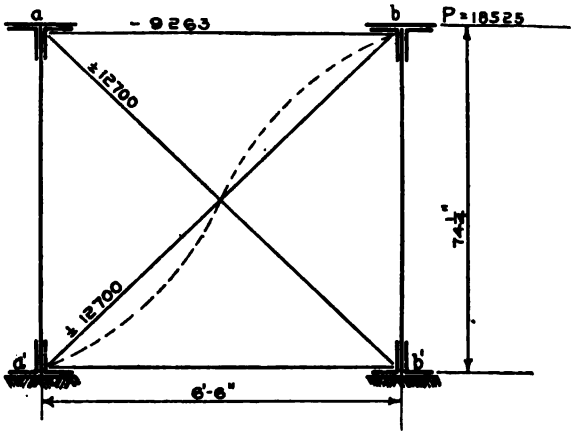


Fig. 154. Action of End Cross-Frames.

The maximum compressive stress for which the diagonals are to be designed is $12.70 + 0.8 \times 12.70 = -22.86$. Here the length is 108 inches if the angle tends to bend one way; but if it bends as shown by the broken lines in Fig. 154, the length will be one-half of this. For this reason, angles with unequal legs should be used, the longer leg extending outward. This allows the greatest rectangular radius of gyration to be used.

A 4 by 3 by $\frac{7}{16}$-inch angle with an area of 2.87 square inches and a radius of gyration of 1.25 will be assumed. The unit-load P is computed to be 8 750 pounds, and the required area is therefore $22\,860 \div 8\,750 = 2.61$ square inches. This does not coincide very closely with the given area, but will be used since this angle comes nearer to fulfilling the condition than any of the other sizes rolled. The joint will require more rivets in bearing than in single shear. It is not necessary to perform the complete computations in order to determine this, since a comparison of the values of a rivet in single shear and in bearing shows that the value in bearing is less than that

in single shear, and therefore the number of rivets required in bearing will be greater than that number required in single shear. The number of rivets required in bearing is:

$$\frac{22\,860}{4\,920 \times 1.5} = \text{say, 4 shop rivets, and}$$

$$4.00 \times 1.5 = 6 \text{ field rivets.}$$

75. **The Stiffeners.** According to Article 47 of the Specifications, these should be placed at certain intervals whenever the unit-shear is greater than

$$S = 10\,000 - 75 \times \frac{74}{3/8} = 10\,000 - 14\,800 = -4\,800 \text{ pounds.}$$

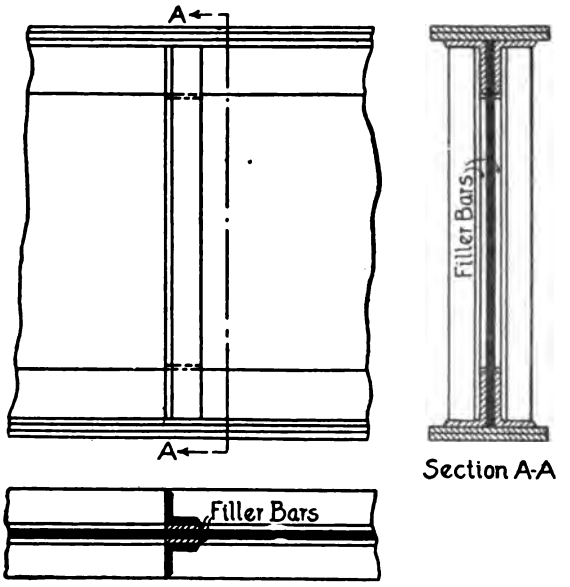


Fig. 155. Use of Straight Stiffeners, with Filler Bars.

This negative sign signifies that whenever the unit shearing stress is greater than zero, the stiffeners must be placed throughout the entire length of the span at distances not to exceed 5 feet.

The intermediate stiffeners should have the outstanding leg long enough to give good support to the flange angle (see Fig. 155). The filler bars or fillers are put in so as to allow the stiffener angles to remain straight throughout their entire length; otherwise they will have to be bent as shown in Fig. 156. This bending is called *crimping*. Stiffeners must also conform to Article 48 of the Specifications. This would require a different sized stiffener at each point, and also a different number of rivets in each stiffener. This is not usually done in practice. In practice the stiffener for the first intermediate

point is designed, and the remainder are made the same in size and have the same number of rivets. An exception to this is where a stiffener comes at a web splice. In this case the size is usually kept the same, but the number of rivets is changed somewhat to conform to the requirements of the splice design.

The second intermediate stiffener comes at the first tenth-point, and is 6.175 (say 6.2) feet from the end, since it is at the first panel point, or opposite the first panel point, of the lateral system. The first stiffener will be 3.1 feet from the end; and scaling off the value of the shear at this point (see Fig. 134), it is found to be 108 000

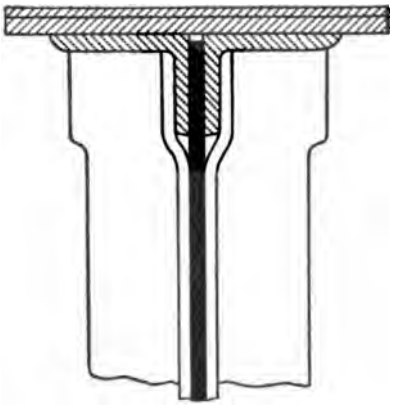


Fig. 156. Crimping of Stiffener Angles where No Filler Bars are Used.

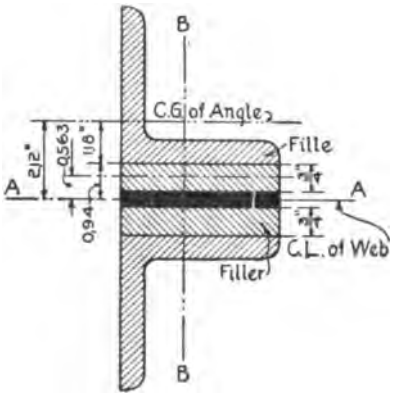


Fig. 157. Section of Intermediate Stiffener Construction.

pounds. Here the length l to be used in the formula for the unit allowable compressive stress is $74\frac{1}{4} - 2 \times \frac{3}{4} = 72.75$ inches, the $\frac{3}{4}$ being the thickness of the flange angle. The section of the material which according to Article 48 of the Specifications is to be considered as a column, is shown in Fig. 157. The assumed column cannot bend about the axis $B-B$, but about the axis $A-A$, and therefore the radius of gyration about the axis $A-A$ must be computed. The moment of inertia of the fillers and the web plates about their own axes is considered as zero.

A 4 by 4 by $\frac{1}{2}$-inch angle with an area of 3.75 square inches will be assumed to be sufficient to withstand the stress. The moment of inertia of this and the filler bars and the web portion is

$$I_{A-A} = 2\,(5.55 + 3.75 \times 2.12^2 + 3.00 \times 0.563^2) = 46.70.$$

The radius of gyration, then, is, $\sqrt{\frac{46.70}{15.00}} = 1.764$, and the unit-stress computed with this value and a length of 72.75 inches is 8 140 pounds. The required area is now determined to be 108 000 ÷ 8 140 = 13.27 square inches. The value 15.00 used in the above computation for the radius of gyration is the value of the area of the angles, the filler bars, and the web portion. A 5 by $3\frac{1}{2}$-inch angle with the 5-inch leg out would have given better support to the flange, but would not make so good a job, as it would have extended about $\frac{1}{8}$ inch beyond the curved part of the horizontal leg of the flange angle.

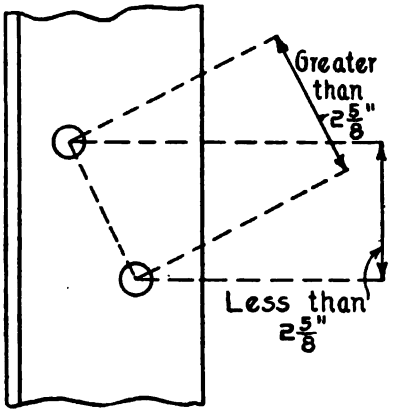


Fig. 158. Rivets Placed in Two Rows to Give Necessary Number and Spacing.

The bearing determines the number of rivets in this case. The number is 108 000 ÷ 4 920 = 22 shop rivets in the web.

The angle must now be investigated in order to see if these 22 rivets can go in one row without being closer together than $2\frac{5}{8}$ inches, which is three diameters of the $\frac{7}{8}$-inch rivet. The total length in which these rivets must be placed is 72.25 inches, and therefore we have 72.25 ÷ 22 = 3.3 inches as a spacing. Since this is greater than $2\frac{5}{8}$ inches, 22 rivets can be placed in one row. If the spacing as determined above had been less than $2\frac{5}{8}$ inches, it would have been necessary to use two rows of rivets spaced as shown in Fig. 158; and then the distance center to center would be more than $2\frac{5}{8}$ inches, although the spacing in a vertical line would be less than that.

The four angles at the ends of the girders are called the *end stiffeners*. These are placed in pairs on opposite sides of the web (see Plate II, Article 74).

The total end shear is 117 800 pounds, and this is assumed to be carried by the two pairs of end-stiffener angles, each carrying one-half. This amount would require lighter angles than the angles used for intermediate stiffeners. It is the customary practice to make them the same size and thickness as the intermediate stiffeners,

additional strength being allowed in order to withstand the effects of the end cross-frame when in action.

The bearing determines the number of rivets required in each pair of stiffeners. The number required is $\frac{117\,800}{2 \times 4\,920} = 12$ shop rivets.

Some engineers arbitrarily choose the stiffeners regardless of the shear, enough rivets, however, being put in the end stiffeners to take up all the shear; and the spacing in the intermediate stiffeners is made the same. One noted engineering firm determines its stiffeners according to the following:

Flange Angle		Stiffeners	
Horizontal Leg	Thickness	End	Intermediate
4 in.	Any	3 x 3 x ½ in.	3 x 3 x ⅜ in.
5 in.	Any	4 x 4 x ½ in.	3½ x 3½ x ⅜ in.
6 in.	Over ¾ in.	4 x 4 x ½ in.	3½ x 3½ x ⅜ in.
6 in.	Less than ¾ in.	5 x 3½ x ½ in.	5 x 3½ x ⅜ in.
8 in.	Any	6 x 6 x ¾ in.	6 x 4 x ⅜ in.

No rational method has as yet been determined for ascertaining the stresses in the stiffeners of plate-girders. Results obtained by placing extensometers on the stiffeners of actual plate-girders appear to indicate that the stresses are very small, in fact in most cases not being greater than 1 500 or 2 000 pounds per square inch.

PROBLEMS FOR PRACTICE

1. Design, according to Cooper's Specifications, the end stiffeners if the shear is 150 000 pounds, the distance back to back of angles is 6 feet 6½ inches, the web ⅜ inch thick, and the flange angle 6 by 6 by ½-inch. Use fillers.

2. Design the intermediate stiffeners for the girder of Problem 1, above, where the shear is equal to 75 000 pounds. Use crimped stiffener angles. Note that in this case the angles lie close against the web, no filler bars being used in between.

76. **The Web Splice.** Web splices are required because of the fact that wide plates cannot be rolled sufficiently long. Web splices should be as few as possible, and good practice demands that they be placed at the same points as the stiffener angle.

The tables on page 30 of the Carnegie Handbook give the extreme length of plates which can be procured for any given width. The

length of plates for widths which are not given in these tables, should be taken equal to the length of the next plate given whose width is less than that of the desired plate. From the first table it is seen that a 74 by $\frac{3}{8}$-inch plate can be rolled up to 400 inches, or 33 feet 4 inches, in length. Therefore, if the girder under consideration is spliced at the center, the web plates will be required to be $\frac{61 \text{ ft. } 9 \text{ in.}}{2} = 30$ feet $10\frac{1}{2}$ inches, which value does not exceed the 33 feet 4 inches as given above.

According to Articles 46 and 71 of the Specifications, a plate must be placed on each side of the web as shown in Fig. 159, and enough rivets placed in each side to take the total shear. The total thickness of both plates, and also their length, must be sufficient to stand the total shear, but must not be less than $\frac{3}{8}$ inch.

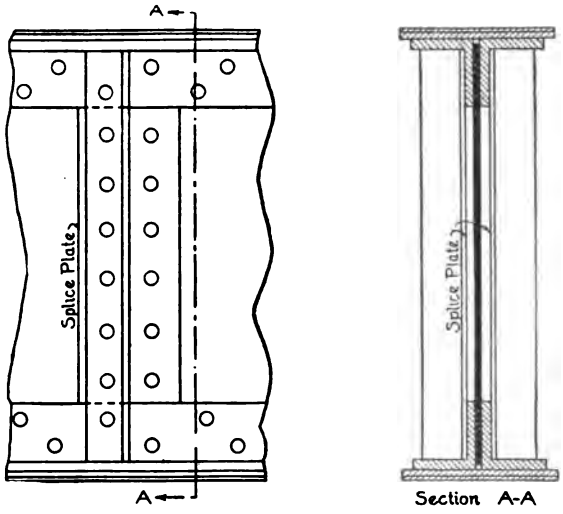


Fig. 159. Splice Plates Placed on Each Side of Web.

The total shear at the center of the girder under consideration (see Fig. 134, p. 150) is 28 600 pounds. The area required in each of the two splice plates is $\frac{28\,600}{2 \times 9\,000}$ $= 1.59$ square inches; and as their length is 62.25 inches, the thickness must be $1.59 \div 62.25 = 0.0255$ inch, but they must be made $\frac{3}{8}$ inch thick according to the Specifications. The width should be somewhat greater than twice the width of the stiffener angle leg. This would make the width in this case about 10 inches.

The bearing governs the number of rivets required in this case, and they are $28\,600 \div 4\,920 = 5.81$, say 6, shop rivets. More rivets than this will be required by practical considerations, as indicated by

Article 54 of the Specifications or in order to make the spacing in the stiffener angle the same as that in the other stiffeners. This detail is to be left to the draftsman, the required number only being put on the stress sheet

In case $\frac{1}{8}$ of the gross area of the web is considered as efficient flange area, then provision must be made in the splice for the bending moment which the web takes. A very economical and efficient splice is shown in Fig. 160. The horizontal plates take the stress due to the moment, and the vertical plates take the stress due to the shear.

The web equivalent is 3.47 square inches and the total moment is 1 615 000 pound-feet, which is composed of 275 000 pound-

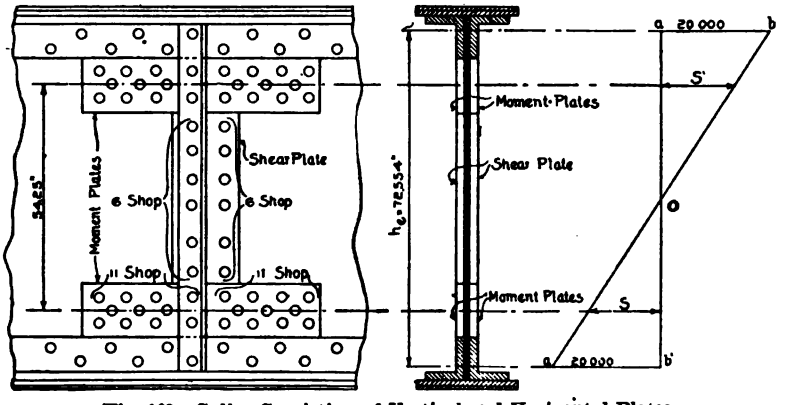


Fig. 160. Splice Consisting of Vertical and Horizontal Plates.

feet due to dead load and 1 340 000 pound-feet due to live load. Therefore that proportion of the 3.47 which is taken up by the dead load is:

$$\frac{275\,000}{1\,615\,000} \times 3.47 = 0.59 \text{ square inch;}$$

and that proportion taken up by the live load is:

$$\frac{1\,340\,000}{1\,615\,000} \times 3.47 = 2.88 \text{ square inches.}$$

The equivalent flange area is assumed to act at the center of gravity of the flange; and the bending moments equivalent to the above areas are, for dead load:

$$0.59 \times 20\,000 \times 72.554 = 856\,000 \text{ pound-inches;}$$

and for live load:

$$2.88 \times 10\,000 \times 72.554 = 2\,090\,000 \text{ pound-inches.}$$

These bending moments must be taken up by the horizontal splice plates The stresses in these plates (see Fig. 160) are, for dead load:

$$\frac{856\ 000}{54.25} = 15\ 780 \text{ pounds;}$$

and for live load,

$$\frac{2\ 090\ 000}{54.25} = 38\ 500 \text{ pounds.}$$

While the allowable unit-stresses are a maximum at the center of gravity of the flange and are those given by the Specifications, they decrease rapidly towards the center of the girder, being zero at the neutral axis of the entire section. The unit allowable stress at the center of the horizontal plates will not be so great as the maximum allowable, but will be proportional to the distance from center (see Fig. 160). The horizontal plates will be taken 8 inches in width. The unit-stresses are easily determined by means of the similar triangles *oab* and *oab'*. The dead-load stress is determined from the proportion:

$$\frac{S'}{20\ 000} = \frac{\frac{54.25}{2}}{\frac{72.25}{2}},$$

and is 14 950 pounds. For live load, the unit-stress will be one-half of this amount, or 7 475 pounds.

The area required for this plate is, for dead load, $\frac{15\ 780}{14\ 950} = 1.05$ square inches, and for live load $\frac{38\ 500}{7\ 475} = 5.16$ square inches, making a total of 6.21 square inches for both plates. Assuming two rivet-holes out of the section, the net width is $8 - 2\,(\frac{7}{8} + \frac{1}{8}) = 6$ inches; and the required thickness for one plate is $\frac{6.21}{2 \times 6} = 0.52$, say $\frac{9}{16}$ inch.

The joint will be weakest in bearing in the $\frac{3}{8}$-inch web. The number of rivets required is:

$$\frac{15\ 670 + 38\ 500}{4\ 920} = 11 \text{ shop rivets.}$$

The design of the shear plate is as follows: The shear is 28 600 pounds, and the required area is $\frac{28\ 600}{9\ 000} = 3.18$ square inches. As the length of the plate is $46\frac{1}{4}$ inches, the required thickness is $\frac{3.18}{2 \times 46.25}$

= 0.034 inch, but on account of the Specifications it cannot be less than $\frac{3}{8}$ inch thick. It will, however, be made $\frac{9}{16}$ inch thick, since it will then fill out even with the horizontal tension plates and no filler will be required. Bearing in the web plate decides the number of rivets, which is:

$$\frac{28\,600}{4\,920} = 6 \text{ shop rivets.}$$

The width of this shear plate should be, as before, 10 inches. The same conditions limiting the spacing of the rivets apply here as in the case where the splice was designed for shear only. The length of the horizontal plates should be sufficient to get in all the rivets, and this is a detail which is left to the judgment of the draftsman.

PROBLEMS FOR PRACTICE

1. A plate-girder is 87 ft. 9 in. center to center of end bearings. The dead-load moment is 9 125 000 pound-inches, and the live-load moment is 38 265 000 pound-inches, the total shear at the section being 202 700 pounds. The web is 90 by $\frac{7}{16}$-inch, and the flange angles are 6 by 6 by $\frac{3}{4}$-inch. Design the web splice when no part of the web is considered as taking bending moment.

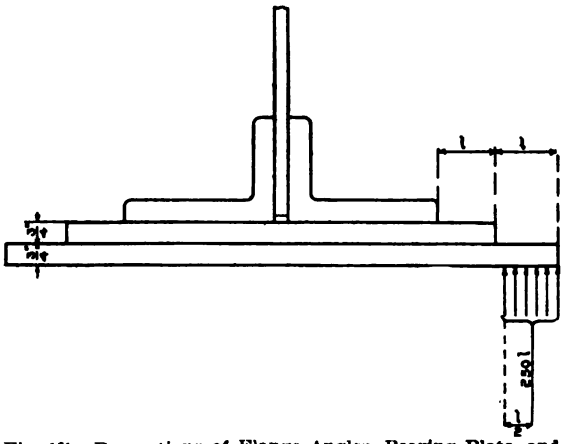


Fig. 161. Proportions of Flange Angles, Bearing Plate, and Masonry Plate.

2. For the girder of Problem 1, above, design the splice when $\frac{1}{3}$ of the gross area of the web is considered as effective flange area.

77. **The Bearings.** Articles 113 to 119 of the Specifications should be carefully studied before proceeding; also Article 87. Article 70 should be referred to, and the remarks there made about bearings should be read. In case the length of bearing is such as to allow a simple $\frac{3}{4}$-inch plate, care must be taken that the bearing plate does not extend past the flange angles more than 2 inches, or that the masonry plate does not extend past the bearing plate over 2 inches. Reference to "Steel Construction," Part II, p. 96, to Fig. 161,

and to the discussion which follows, will explain the reason of this.

$$M = \frac{SI}{c} = 250 \times l \times \frac{l}{2};$$

$$I = \frac{bd^3}{12} = \frac{1 \times (\frac{3}{4})^3}{12};$$

$$c = \frac{d}{2} = \frac{\frac{3}{4}}{2} \qquad \therefore \frac{I}{c} = \frac{1 \times 9}{16 \times 12};$$

and as $s = 10\,000$,

$$250\,\frac{l^2}{2} = \frac{10\,000 \times 9}{16 \times 12},$$

from which,

$$l = 1.94, \text{ say 2 inches.}$$

In case it is desirable to have a simple masonry plate instead of a cast-steel pedestal, and to have the plate extend over the sides of the angles a distance greater than 2 inches, then some arrangement must be made for supporting the projecting portion. Fig. 162 shows one of the methods most commonly used. Notwithstanding the bracing of the gusset plates, the masonry plate is not adequately supported, the greater proportion of the stress coming upon the ends.

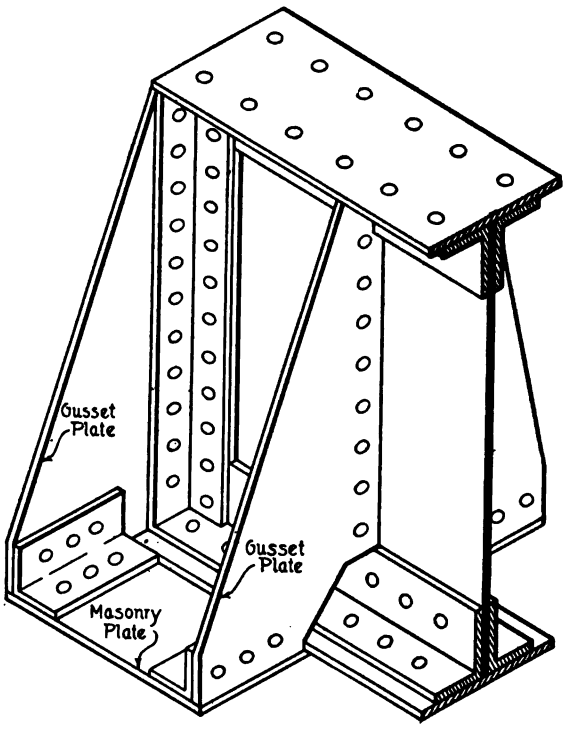


Fig. 162. Arrangement where Masonry Plate is Used instead of Cast-Steel Pedestal.

The disadvantage of having the masonry plate too long is plainly shown by Fig. 163. Here the girder is shown deflected under a live load, the rear end of the plate being tilted up and the greater part

of the pressure coming upon the forward end. The use of this style of plate is not to be recommended for spans over 40 feet.

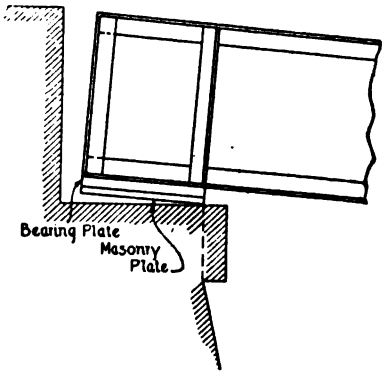


Fig. 163. Effect of Having Masonry Plate Too Long.

The design for the bearing of the girder under consideration will now be made. The total reaction of one girder must now be computed. This will be due to the weight of the steel in the girder, to the weight of the track, and to the reaction produced by the E 40 loading when wheel 2 is directly over the end support. This reaction is:

Weight of Steel, $\frac{(123.5 + 10 \times 61.75)\,61.75}{4}$	=	11 430 pounds
Weight of Track, $\frac{400}{2}$ $(61.75 + 1.75)\,\frac{1}{2}$	=	6 350 "
Reaction Due to Engine Loading	=	99 700 "
Total	=	117 480 pounds.

The square inches of bearing surface required is $\frac{117\,480}{250} = 470$; and, as the length is 1 foot 9 inches, or 21 inches, the total width of the cast-steel pedestal will be $\frac{470}{21} = 22.4$, say 23 inches, or 1 foot 11 inches.

A bearing plate must be riveted to the lower flange where it rests upon the pedestal. The pedestal must be so constructed as to allow this bearing plate to set in it. Hand-holes should be provided in the casting in order to allow the bolts which connect the casting to the girder to be inserted. These bolts should be at least $\frac{3}{4}$ inch in diameter. Anchor bolts $\frac{7}{8}$ inch thick and at least 8 inches long should be provided and fox-bolted to the masonry. The thickness of the metal in all parts of the casting should be at least $1\frac{1}{4}$ inches. The details of the pedestal are given in Fig. 164, the length of the bearing being made 12 inches so as to allow one rivet to be driven in the flange angle in the space between the end stiffeners.

Allowance should be made for a variation of 150 degrees in tem-

perature. The coefficient of expansion for steel per unit of length is 0.0000065, and the amount of expansion for 150 degrees of temperature will be:

$$0.0000065 \times (61 \text{ ft. } 9 \text{ in.}) \times 150 = 0.06 \text{ foot.}$$

This is about $\frac{3}{4}$ inch, and therefore the holes in the flanges at one end of the girder should be made oblong and long enough to allow the

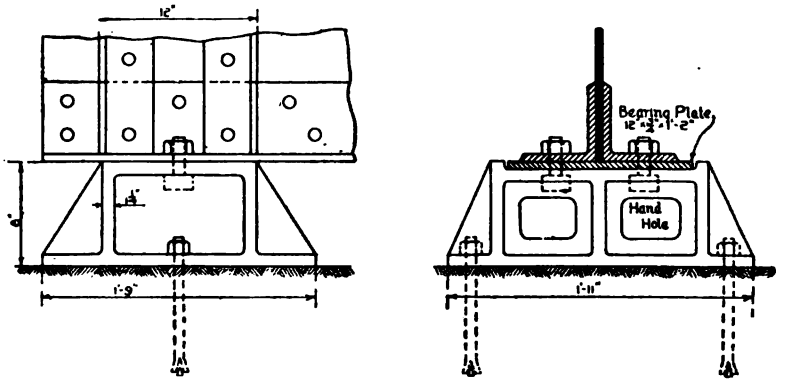


Fig. 164. Side and End Elevations Showing Construction of Pedestal and Connection to Bearing Plate.

girder to move $\frac{3}{4}$ inch, or $\frac{3}{8}$ inch either backward or forward from a central position. In determining the length of this slotted hole (see Fig. 165), it must be noted that the $\frac{7}{8}$-inch bolt takes up part of this hole, and therefore its length should be $\frac{7}{8} + \frac{3}{4}$ = say $1\frac{3}{4}$ inches. The width of the hole should be sufficient to allow for any over-run in the diameter of the bolt. It should be at least $1\frac{1}{4}$ inches wide.

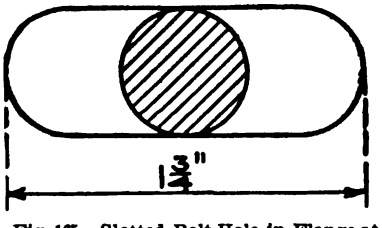


Fig. 165. Slotted Bolt-Hole in Flange at End of Girder to Allow for Contraction and Expansion Due to Temperature Changes.

PROBLEMS FOR PRACTICE

1. Determine the distance center to center of bearings, and the size of the masonry plate, for a plate-girder of 40-foot span under coping, the loading being E 40.

Ans. 41 ft. 4 in.; 350 square inches.

(Note—Interpolate values in Table I, Cooper, p. 30.)

2. If the girder span is 78 feet under coping, and the loading E 40, determine the maximum end reaction and the width of the masonry plate. Ans. 147 130 pounds; $24\frac{1}{2}$ inches.

78. **The Stress Sheet.** Plate II (p. 172) shows the stress sheet for

the girder which has just been designed. It represents the best modern practice in that it gives, in addition to the sizes of all the sections, the curves for the maximum shears and moments, the rivet-spacing curve, and the number of rivets required in the different parts of the structure. This general form has been adopted by one of the largest bridge corporations in this country, and is to be recommended since it gives the draftsman all necessary data and thus prevents the loss of time by an inexperienced man in recomputing certain results. The results just referred to are the shears, the moments, the rivet spacing, and the number of rivets required in the various parts. Formerly it was not customary to give this information on the stress sheet, and the draftsman was therefore required to do all this computation which had previously been worked out by the designer but had not been placed on the stress sheet in available form, and thus unnecessary loss of time resulted.

DESIGN OF A THROUGH PRATT RAILWAY-SPAN

79. **The Masonry Plan.** The same remarks which are made in Article 67 apply here. In this case the length of the masonry plate is usually determined by considerations relative to the number and length of the rollers in the bearing, and not by the bearing per square inch upon the masonry, the size of the plate as determined by the above considerations being usually much larger than if it had been determined by the unit bearing stress. A preliminary design of the masonry plate is usually made in a manner similar to that done in the case of the plate-girder; or the length of the masonry plate may be approximately determined from the following:

Span	Length of Masonry Plate	
	Fixed End	Roller End
100 feet	23 inches	23 inches
125 "	26 "	26 "
150 "	28 "	28 "
175 "	31 "	31 "
200 "	35 "	35 "

The above masonry plates are for single-track bridges, with or without end floor-beams, the length being the same in either case.

80. **Determination of the Span.** The determination of the span is made in exactly the same manner as described in Article 68. Care should be taken, in case end floor-beams are not used, to allow for the pedestal stones, which are square stones resting directly upon the bridge seat, and upon the top of which rest the masonry plates of the stringers. Their height must, of course, be such as to keep the stringers level. In case these stones are used, their size must be determined; and if it is greater than that of the bearing or masonry plates, then their size determines the width of the bridge seat and the span center to center of bearings.

81. **The Ties.** In the design of the ties, as well as in all the design which follows, the Specifications of the American Railway

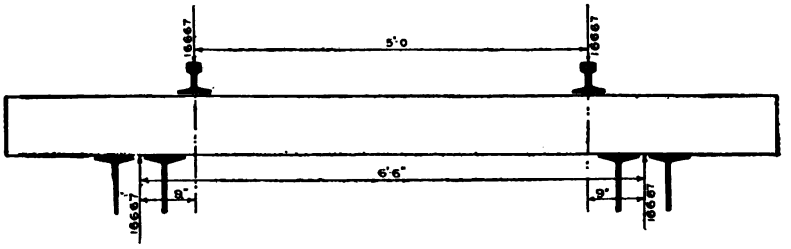


Fig. 166. Spacing of Stringers and Rails, and Position of Loads.

Engineering & Maintenance of Way Association will be followed. Whenever reference is made to these specifications, the number of the article will be enclosed in parentheses, as "(5)," which signifies that Article 5 of the Specifications is to be referred to.

The stringers in the bridge in question will be taken 6 ft. 6 in. center to center. The maximum loading (7) is such as to bring 8 333 pounds on one tie, and to this must be added 100 per cent for impact, making a total of 16 667 pounds. In order to illustrate the method of assuming the distance, center to center of rails, as 5 feet, that distance will be used in this case. The maximum moment will then be $9 \times 16\,667$ = say, 150 000 pound-inches. The size of the tie will be determined as in Article 71, the allowable unit-stress being 2 000 pounds per square inch (5). If a 7 by 9-inch tie is used, the unit-stress will be 1 590 pounds. If a 6 by 8-inch tie is used, the unit-stress will be 2 340 pounds. It is evident that a 7 by 9-inch tie must be used. See Fig. 166 for spacing of stringers and

rails, and for position of the loads. Note that, although impact is taken into account in this case, the size of the tie is the same as that designed for the plate-girder, although the unit allowable stress also differs.

82. **The Stringers.** The width, center to center of trusses, will be assumed as 17 feet, since this is sufficient to clear the clearance diagram in cases of single-track bridges of spans less than 250 feet.

The span which is to be designed in the following articles is a through-Pratt with 7 panels of 21 feet each, making a total span, center to center of bearings, of 147 feet 0 inches. See Plate III (p. 251). Rivets $\frac{7}{8}$ inch in diameter will be used throughout, except in channel flanges.

The length of the stringers end to end will be 21 feet, and according to Cooper's Specifications, p. 32, the maximum moment for the live load will be 226 000 pound-feet per rail. The coefficient of impact (9) will be $\left(\frac{300}{21 + 300}\right) = 0.935$, and therefore the moment due to impact will be $0.935 \times 226\,000 \times 12 = 2\,535\,000$ pound-inches, making a total of 5 247 000 pound-inches due to live load.

The section modulus for any particular beam is equal to the bending moment divided by the unit-stress, and this is equal to the moment of inertia divided by one-half the depth of the beam. This latter quantity is constant for any given beam, and for I-beams may be found in column 11, Carnegie Handbook, p. 98.

On account of the cheapness of I-beams, they will be used for stringers in this bridge; and sufficiently heavy shelf angles will be used to take up any distorting influences due to the eccentric connections which are unavoidable in this case. In case an I-beam had not been decided upon, the stringers would have been small plate-girders with a span of 21 feet and depth according to formulæ given. They would have been computed in exactly the same manner as a plate-girder span of 21 feet center to center of bearings.

Since the dead-load moment cannot be determined until the size of the stringer is known, an approximate design must first be made by using the live-load bending moment alone; and then, with the size determined in this manner, the extra section modulus required for the dead-load moment due to the weight of the beam and the track must be computed. If this extra section modulus, added to the one previously determined, is greater than that given by the beam in

question, a larger size beam must be used and a recomputation made.

The section modulus (17) required for live load only is $\frac{5\,247\,000}{16\,000}$ $= 327.9$. As this is too large for one beam, two beams will be used, thus giving a required section modulus of 164 for one beam. Two 24-inch 80-pound I-beams will be used, giving a total section modulus of $2 \times 174 = 348$.

Assuming the rails and ties to weigh 400 pounds per linear foot, the dead load per linear foot per stringer is $80 + \frac{400}{4} = 180$ pounds. The dead-load moment is therefore $\frac{180 \times 21 \times 21 \times 12}{8} = 119\,000$ pound-inches. This requires an additional section modulus of $\frac{119\,000}{16\,000} = 7.45$. This, added to the 164 as determined above, makes a total required section modulus of 171.25, which, being less than 174 (which is that for one I-beam), indicates that the above chosen beam is sufficient in strength, and it will therefore be used.

The number of rivets in the end connections will now be determined. The total end reaction for one I-beam is equal to the weight of one-half the beam, one-eighth the track in the panel, and one-half the maximum live-load reaction for one rail. These quantities are:

$\frac{1}{2}$ Live-Load Reaction $= \frac{51\,400}{2}$	$= 25\,700$ pounds.
Impact $= 25\,700 \times 0.935$	$= 24\,030$ "
Weight of Track $= \frac{400}{8} \times \frac{21}{2}$	$= 530$ "
Weight of Stringer $= \frac{21}{2} \times 80$	$= 840$ "
	Total $= 51\,100$ pounds.

The coefficient of impact is that for a loaded length of 21 feet.

From p. 177, Carnegie Handbook, sixth column, it is seen that the longest connection angle which can be used with a 24-inch I-beam is $20\frac{3}{4}$, say 20 inches. In this case the thickness of the connection angles must be $\frac{51\,100}{10\,000 \times 22} = 0.23$ inch; but according to (36), $\frac{3}{8}$ inch will be used. The angles chosen will be 6 by $3\frac{1}{2}$ by $\frac{3}{8}$-inch. The 6-inch leg will be placed against the web of the floor-beam in order to allow for sufficient room for rivets to be driven.

The rivets will tend to shear off at places between the webs of the stringer and floor-beam and the connection angles. They will also tend to tear out of the web of the stringer and out of the web of the floor-beam. As the thickness of this latter is not known, the determination of the rivets for this condition will be made under the next article. The bearing value of a $\frac{7}{8}$-inch rivet in a $\frac{1}{2}$-inch plate (19) is $\frac{7}{8} \times \frac{1}{2} \times 24\,000 = 10\,500$ pounds, and therefore $\frac{51\,100}{10\,500} = 5$ shop rivets are required in bearing in the web of the stringer. The value of a $\frac{7}{8}$-inch shop rivet in single shear (18) is $0.6013 \times 12\,000 = 7\,220$ pounds, and the number of rivets required to prevent shearing between the connection angles and the webs is $\frac{51\,100}{7\,220} = 7$ shop rivets. The value of a $\frac{7}{8}$-inch field rivet in single shear (18) is $0.6013 \times 10\,000 = 6\,013$ pounds, and therefore $\frac{51\,100}{6\,013} = 9$ field rivets are required to connect the connection angle to the web of the floor-beam. As mentioned above, the number of rivets in bearing in the web of the floor-beam will be determined in the next article; and if the number required for bearing is greater than 9, then that number must be used instead of 9. Fig. 167 shows the connection of the stringer to the floor-beam web, and also the number of rivets as determined above, in their proper positions. The distance between the webs of the stringers must be such as to prevent their flanges from touching at the top.

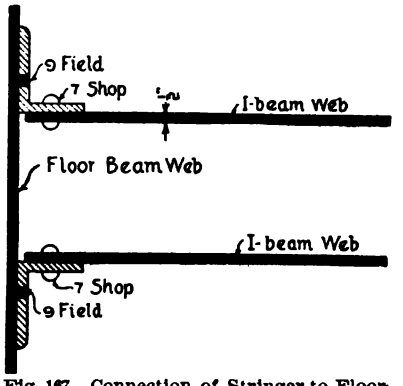


Fig. 167. Connection of Stringer to Floor-Beam Web; also Number of Rivets.

The stringers should be connected at the bottom by a system of lateral bracing of the Warren type. The size of these angles cannot be determined by theoretical considerations, but is usually chosen to be $3\frac{1}{2}$ by $3\frac{1}{2}$ by $\frac{3}{8}$-in. See Plate II (p. 172) for the general arrangement of this bracing.

83. **The Floor-Beams.** All floor-beams should be of sufficient depth to allow the use of small-legged connection angles at the ends

where they join the end-posts. The thickness of the web should also be greater than that which is theoretically computed, in order that sufficient bearing may be given so that the rivets for the stringer connections will not require the stringers to be of too great a depth. The depth of the floor-beam will, of course, vary somewhat with the length of the panel and with the loading, but should not be less than 36 inches in any case. A considerable variation in the depth will not affect the weight of the floor-beam or the bridge to an appreciable extent. A good plan is not to exceed a depth of 5 feet, with panel lengths of 25 feet and E 50 loading. In this bridge the depth of all intermediate floor-beams will be taken as 48 inches. It is good practice not to consider $\frac{1}{8}$ the web area when designing flanges of floor-beams and stringers, and the design here given does not consider the web as taking any bending moment.

The design of an intermediate floor-beam will now be made. The loads for which it is designed are the floor-beam reaction due to the live load (see Cooper, p. 32), the floor-beam reaction due to impact, the dead weight of the stringers and track, and the weight of the beam itself. The latter weight is distributed uniformly over the entire length of the beam, and the other loads act as concentrated loads spaced 6 feet 6 inches apart at equal distances from the center. The computation of the concentrated loads is as follows:

Live Load = ... 68 000 pounds

Impact $= 68\,000 \times \left(\frac{300}{37 + 300}\right)$ ..... 60 500 "

Dead Load of Stringer $= 2\left(\frac{21 \times 2 \times 80}{2}\right)$ .. 3 320 "

Dead Load of Track $= \frac{400}{2} \times 21$ ..... 4 200 "

Total = 136 020 pounds.

The moment at points under the loads (see Fig. 168) is 136 020 $\times$ 5.25 $\times$ 12 = 8 575 000 pound-inches. This is due to the concentrated loads only. The weight of one floor-beam may be approximately determined by the same formula as used to determine the weight of plate-girder spans; only, in place of the length of the span, the length of the panel must be substituted. The total weight of the above floor-beam, then, is:

$$W = 0.45 \times (123.5 + 10 \times 21) \times 21 = 3\,160 \text{ pounds.}$$

The dead-load moment at the center due to this weight will be:

$$\frac{Wl}{8} = \frac{3\,160 \times 17 \times 12}{8} = 80\,700 \text{ pound-inches,}$$

making a total moment at the center of the beam of 8 575 000 + 80 700

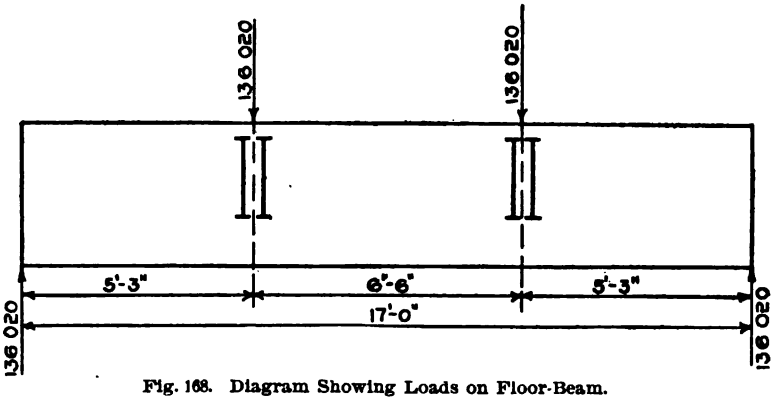


Fig. 168. Diagram Showing Loads on Floor-Beam.

= 8 655 700 pound-inches. Note that the dead-load moment at the center of the beam is added to the concentrated-load moment at the point where the concentrated load is applied. This will give the total moment at the center of the beam as shown by Fig. 169, since the concentrated-load moment is constant between the points of application. The end shear is readily computed to be 136 020 + 1 580 = 137 600 pounds. The curves of moments and shears are shown in Fig. 169.

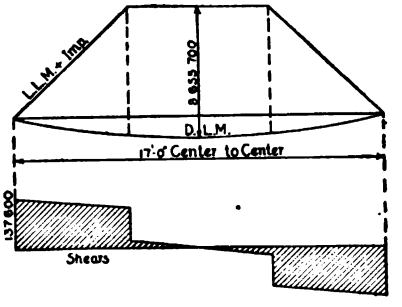


Fig. 169. Shear and Moment Diagram.

The total depth of the floor-beam, back to back of angles, is $48\frac{1}{4}$ inches; and the effective depth will, for approximate computation of the flange area, be taken as somewhat less, say $44\frac{1}{2}$ inches, since the flange angles will probably be 6 by 6-inch and the center of gravity of most of these angles lies about $1\frac{3}{4}$ inches from the back. The approximate flange stress is $\frac{8\,655\,700}{44.5} = 194\,500$ pounds, and the required net area (17) will be $\frac{194\,500}{16\,000}$

= 12.2 square inches. In assuming the size of the angle, it is to be remembered that when, as in this case, no cover-plates are used, no rivet-holes will be taken out of the top flange, and only one rivet-hole will be taken out of the vertical flange.

Two 6 by 6 by $\frac{5}{8}$-inch angles give a gross area of 7.11 square inches each, and a net section of $7.11 - 0.625 = 6.485$ square inches each, or 12.97 square inches net for both. As this is near the required area, these angles will be taken; and a recomputation will now be made with the actual effective depth, in order to see if sufficient variation in the areas occurs to require another angle to be taken. The actual effective depth is now $48\frac{1}{4} - 2 \times 1.84 = 44.57$ inches; and making computations with this, it will be found that a net area of 12.10 square inches is required. As this is practically the same as was determined at first, no change will be made in the size of the angle.

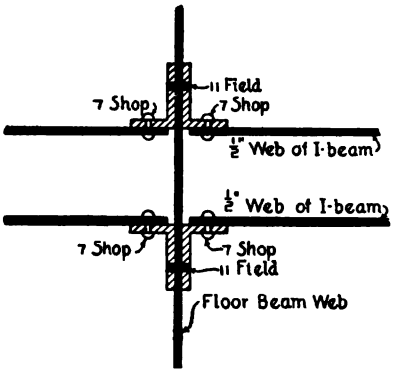


Fig. 170. Calculation of Number of Rivets through Connection Angles of Stringer and Floor-Beam Web.

The web is to be designed for a total shear of 137 600 pounds. The required area (18) is $\frac{137\,600}{10\,000} = 13.76$ square inches, and the required thickness is $\frac{13.76}{48} = 0.286$ inch; but on account of the Specifications (36), $\frac{3}{8}$ inch must be used. The web will accordingly be 48 by $\frac{3}{8}$-inch.

The determination of the number of rivets which go through the connection angle of the stringer and the web of the floor-beam can now be made. The value of a $\frac{7}{8}$-inch field rivet in bearing in a $\frac{3}{8}$-inch plate (19) is $\frac{7}{8} \times \frac{3}{8} \times 20\,000 = 6\,560$ pounds, and the total number required in one connection angle will be $\frac{136\,020}{2 \times 6\,560} = 11$ field rivets (see Fig. 170).

The pitch of the rivets in the flange can in this case be determined by the use of the formula:

$$s = \frac{vh_e}{V}.$$

Since the flange is of the same cross-section throughout, the value of the effective depth will not change, and it can therefore be used in the above equation instead of considering the value of the distance between rivet lines. The shear being practically constant from the connection of the stringers to the end of the floor-beams, the rivet spacing will be constant in this distance. It will be:

$$s = \frac{7\,880 \times 44.57}{137\,600} = 2.51 \text{ inches,}$$

the value of a $\frac{7}{8}$-inch shop rivet in bearing in the $\frac{3}{8}$-inch web being $\frac{7}{8} \times \frac{3}{8} \times 24\,000 = 7\,880$ pounds. This is seen to be less than $2\frac{5}{8}$ inches; but, as the angles have 6-inch legs, this spacing can be used in a horizontal direction; and the distance from center to center of rivets, which will be placed in rows on two gauge lines, will still be greater than $2\frac{5}{8}$ inches.

The shear between the stringer connections is practically zero, and therefore the spacing will be very large. Being over 6 inches, it will be subject to (37).

The connection angles at the ends of the floor-beams will be taken as 6 by $3\frac{1}{2}$ by $\frac{3}{8}$-inch, the 6-inch legs being against the web of the floor-beam. The other legs are chosen small in order that they may fit into the channels which will very likely be required for the posts; and according to the sixth column, p. 183, Carnegie Handbook, only $8\frac{1}{4}$ inches is available for this purpose. This $8\frac{1}{4}$ inches is taken from a 10-inch channel, since this is the smallest channel that can be used which will give room for connection and yet be in accordance with the Specifications. This is due to the fact that its web (36) is greater than $\frac{3}{8}$ inch. The rivets which connect the end angles to the floor-beam web are shop rivets, and those which connect the end angles to the posts are field rivets. Since the size of the post is not known, the thickness of its metal, of course, cannot be used, and therefore the number of rivets required in bearing in the post cannot be determined at this time.

The number of shop rivets required through the end angles and the floor-beam web is governed by the bearing of the rivets in the $\frac{3}{8}$-inch web of the floor-beam. The value of a $\frac{7}{8}$-inch shop rivet in bearing in a $\frac{3}{8}$-inch web (19), as has just been computed, is equal to 7 880 pounds, and the number of rivets required is $\frac{137\,600}{7\,880} = 18$.

The number of field rivets required in single shear to connect the end angles with the posts is $\frac{137\,600}{6\,013} = 23$. An even number of rivets will, of course, have to be used, one-half going into each of the $3\frac{1}{2}$-inch legs. See Fig 171 for the position and the number of rivets. It must be remembered that more than these numbers may be used by the draftsman on account of rivet spacing which may be required by conditions other than those of design.

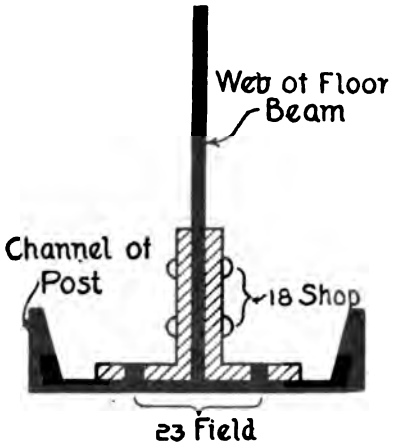


Fig. 171. Position and Number of Rivets to Connect End Angles with Posts.

The design of the end floor-beam is somewhat different from that of the intermediate floor-beams in that the load which comes upon it is considerably lighter, since this floor-beam takes the dead load of only one-half the panel and the live load due to the *maximum end reaction* for a stringer length instead of the floor-beam reaction for the stringer length (see Cooper, p. 32).

The maximum end shear is computed as follows:

End Shear for 21-foot Span	51 400 pounds.
Impact $= 51\,400 \times \left(\frac{300}{21 + 300}\right)$	48 000 "
Dead Load of Stringers $= \frac{80 \times 2 \times 21}{2}$	1 680 "
Dead Load of Track $= \frac{400 \times 21}{2 \times 2}$	2 100 "
Total	103 180 pounds.

The maximum moment due to the above load is $103\,180 \times 5.25 \times 12 = 6\,500\,000$ pound-inches. The weight of the beam may be assumed as 3 160 pounds. This is the same as was computed for the intermediate floor-beam, but will be used for this beam, since the size of the web will be the same as in the others; and, although the flange area will be less, the end connections will be somewhat heavier owing to the connection of the beam to the end-post and the roller bearing,

and this additional weight will cause the total weight of the end floor-beam to be about the same as that of the intermediate ones. The total moment at the center will then be 6 500 000 + 80 700 = 6 580 700 pound-inches.

The depth of the end floor-beam will be somewhat greater than the depth of the intermediate floor-beams. This is due to the fact that it extends downward a greater distance, resting upon the bearing plate, which comes directly upon the top of the rollers. The exact depth cannot, of course, be determined until after the roller bearings are designed; but it may be safely assumed as four or five inches deeper than the intermediate floor-beams, and in case this is not enough, the draftsman can easily fill in the remaining distance with filler plates, as this distance will not be very great. In case this depth is too great, the flange angles may be bent upward at the end, or a re-design may be made.

The depth will be assumed as 52 inches in this case. The effective depth will be assumed as 48 inches, and this gives an approximate flange stress of

$$\frac{6\,580\,700}{48} = 137\,000 \text{ pounds,}$$

and an approximate net flange area required of

$$\frac{137\,000}{16\,000} = 8.57 \text{ square inches.}$$

A 6 by 6 by $\frac{7}{16}$-inch angle gives a gross area of 5.06 square inches, and a net area of $5.06 - (\frac{7}{8} + \frac{1}{8})\ \frac{7}{16} = 4.62$ square inches. A recomputation with the true effective depth requires 8.42 square inches net. Two of these angles give 9.24 square inches; and as this coincides very closely with the required area, it will be used. The size of the web plate is 52 by $\frac{3}{8}$-inch.

The pitch or spacing of rivets in the flanges is:

$$\frac{7\,880 \times 48}{104\,760} = 3.62 \text{ inches.}$$

The maximum end shear as above computed is taken by two stringers; and therefore the number of rivets required in bearing to form the connection between the stringers, connection angles, and the floor-beam web is, for each angle:

$$\frac{104\,760}{2 \times 6\,560} = 8 \text{ field rivets.}$$

The value 6 560 in the above equation is the value of a $\frac{7}{8}$-inch field rivet in bearing in the $\frac{3}{8}$-inch web.

The number of rivets required in the end angles on the floor-beam is:

$$\frac{104\ 760}{7\ 880} = 14 \text{ shop rivets.}$$

These rivets go through the web of the floor-beam. The connection of the floor-beam to the end-post is made by means of field rivets and a large gusset plate. This gusset plate is usually $\frac{3}{8}$ inch in thickness. The number of rivets through the end connection angles and this gusset plate is governed by single shear, since the rivets will shear off between the angles and the gusset plate before they will tear out of the gusset plate, as the value of a rivet is greater in bearing than in shear. The number required is:

$$\frac{104\ 760}{6\ 013} = 18 \text{ field rivets.}$$

The general arrangement of the intermediate floor-beams is shown in Fig. 172. The ends of the lower flange are bent up as shown, in order to allow the I-bar heads or any other section of the lower chord to have clearance. This makes it necessary for the floor-beam web to be spliced at the ends, as shown. The distance which this plate should extend above the floor-beam proper depends upon the distance which the lower chord is bent up. In any case the length of the connection on the post should be at least equal to the depth of the floor-beam. Two splice plates, one on either side of the web, are placed here in a manner similar to that of a splice as designed in the plate-girder when shear only was considered. Here shear only is considered, and the number of rivets which must be on each side of the splice will be:

$$\frac{137\ 640}{7\ 880} = 18 \text{ shop rivets.}$$

The 7 880 which occurs in the above equation is the value of a $\frac{7}{8}$-inch rivet in bearing in a $\frac{3}{8}$-inch plate (19). Inspection of Plate II (p. 172) will make this design clearer. Plate II also shows the shape of the end floor-beams.

The small shelf angle shown in Fig. 172 should have sufficient rivets to prevent any twist of the stringers due to their being connected on one side of their web only. This number is a matter of

judgment. Experience seems to indicate that enough rivets to take up one-third of the total reaction of the stringers will be sufficient. This will require shop rivets, and the number will be:

$$\frac{103\,180}{3 \times 7\,220} = 5 \text{ shop rivets in single shear.}$$

84. **The Tension Members.** Tension members usually consist of long, thin, flat plates with circular heads forged upon their ends.

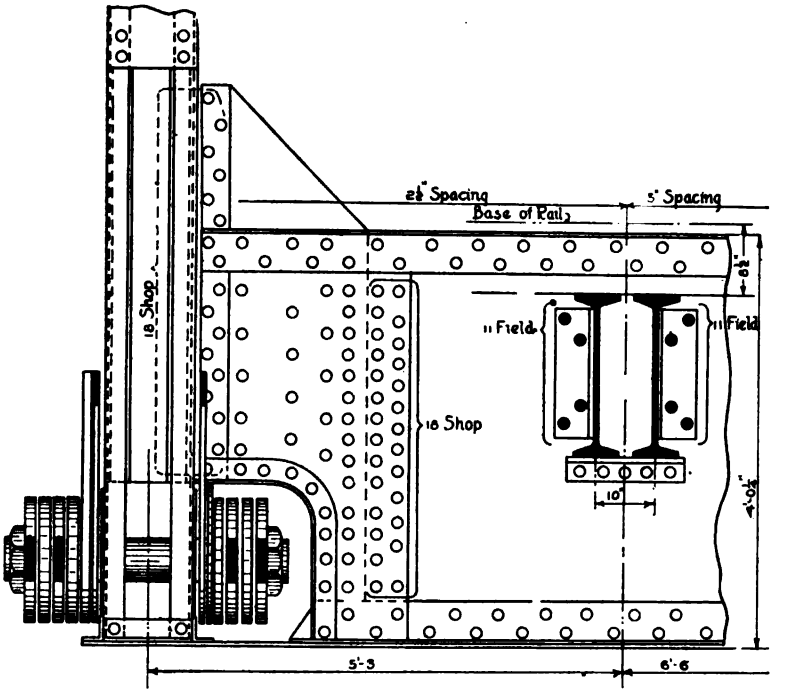


Fig. 172. General Arrangement of Rivets, Splices, Connections, etc., for Intermediate Floor-Beams.

These circular heads have holes punched through their centers and then very carefully bored. Through these holes are run cylindrical bars of steel called *pins*. These pins connect them with other members of the truss. See Carnegie Handbook, p. 212, for table of I-bars. The I-bars given are standard I-bars; and while departures from these widths and *minimum* thicknesses may be made, it may be done only at great cost to the purchaser. Note that there are no standard 9-inch I-bars. The thicknesses given are the *minimum* thicknesses

for that width of bar, and do not indicate that thicker bars of that width cannot be obtained; but on the contrary thicker bars of that width can be obtained, and this should be done, the minimum thickness as given in the table being avoided if possible.

It has been found that bars which have a ratio of thickness to width of about one-sixth give good service and are easy to forge. This relation gives us a rough guide which will enable us to determine the approximate width and thickness of any bar of a given area. Once the approximate dimensions are determined, the actual dimensions can be chosen from the market sizes of the material (see Carnegie Handbook, pp. 245 to 250).

An expression for the approximate depth of the bar will now be derived by using the above relation.

Let A = Area of bar, in square inches;
d = Width of bar, in inches;
t = Thickness of bar, in inches.

Then,

$$td = A;$$

also,

$$t = \frac{1}{6}d.$$

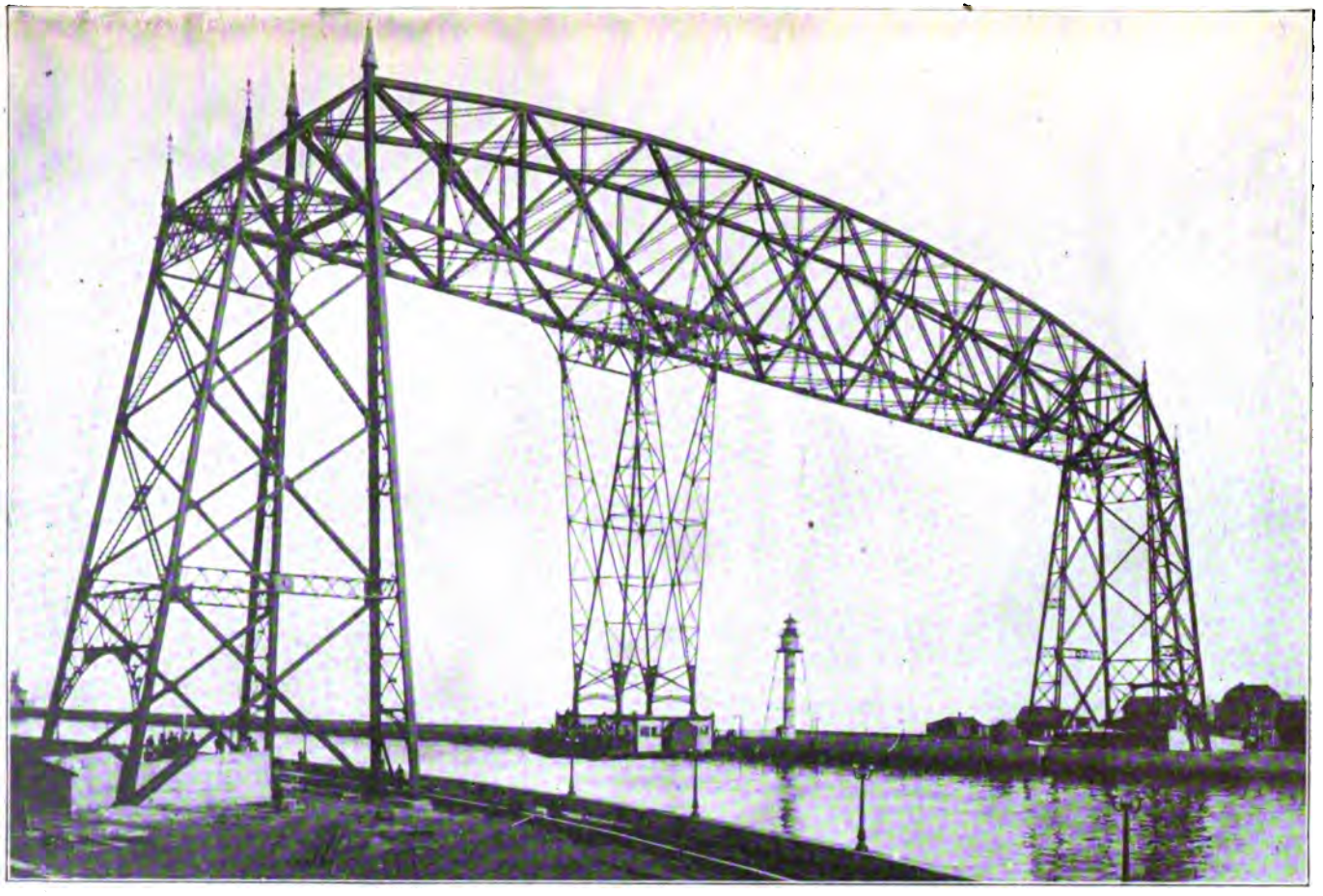
Substituting the value of t in the expression for A, there results:

$$d = \sqrt{6A}.$$

The stresses in all the members in the truss under consideration are computed by the method described in Part I, and are placed on the stress sheet, Plate III (p. 251). In the succeeding design, the student should obtain his stresses from Plate III without his attention being again called to the matter.

Table XXIV gives the tension members and their dead-load, live-load, impact, total, and unit stresses (15), together with the required area, the number of bars, the approximate depth of bars, and the final sizes used.

The first seven columns in Table XXIV are self-explanatory. The number of bars to be used in any particular case is a matter of judgment. One fast rule is that *an even number of bars should always be used, except in the case of counters, where one is permissible.* This is due to the fact that the placing of one of the main members in the center of the pin would create a large moment, and



AËRIAL BRIDGE OVER SHIP CANAL AT DULUTH, MINNESOTA

Built in 1904-05. View showing suspended car in transit. Structure rests on concrete piers, and consists of a riveted truss on riveted steel towers, the ferry car being supported by an inverted steel tower. Clear height above water, 135 ft.; clear span, 393.75 ft. Car has space for a street-car, two wagons, and two enclosed cabins; will carry 125,000 lbs.; passage made in a little over 1 minute. Motive power is electricity supplied from two separate sources, with hand-power available in case of failure of the electric current. Operates perfectly in high winds. Can handle over 33,000 passengers daily. Cost of structure, about $8,000.

VICTORIA FALLS BRIDGE OVER ZAMBESI RIVER, SOUTH AFRICA

Completed in 1907. Of cantilever type; total length, 650 ft.; central span, 500 ft.; depth of gorge, over 400 ft. The bridge is about on a level with the crest of the Falls, which are a short distance upstream. The river, which is a mile wide at the Falls, is here confined within a very narrow gorge. A development of the 35,000,000 wasted horse-power has been projected, the power to be transmitted long distances for various industrial enterprises.

therefore an excessively large pin would be required, and accordingly a very large head on the I-bar in proportion to its width —all of which are very undesirable and costly. In general the number of I-bars should be as small as possible, and they should be so chosen that the widths of the chord members *increase* from the ends toward the center of the truss, and the widths of the diagonals *decrease* from the ends toward the center of the truss.

The area of one bar is obtained by dividing the total stress by the number of bars and also by the allowable unit-stress. Thus, for the member L_2U_1, for example, the required area of one bar is:

$$\frac{295\,100}{2 \times 16\,000} = 9.22 \text{ square inches}$$

TABLE XXIV

Tension Members

Member	Stresses (in pounds)					Required Area*	No. of Bars	Area*: One Bar	$d = \sqrt{6A}$	Section Used*	Area* of Section Used
	Dead-Load	Live-Load	Impact	Total	Unit						
L_0L_2	41 000	114 700	78 500	234 200	16 000	16.64	..	....	....	.....	....
L_1U_1	13 000	68 000	60 500	141 600	16 000	8.85	..	....	....	.....	....
L_2U_1	47 900	143 900	103 300	295 100	16 000	18.44	2	9.22	7.44	7 x $1\frac{3}{8}$	9.64
L_2U_3	.....	28 000	23 100	25 900	16 000	1.62	1	1.62	....	$1\frac{3}{8}$ x $1\frac{3}{8}$	1.93
L_3U_2	24 000	95 200	72 600	191 800	16 000	11.98	2	5.99	6.00	6 x 1	6.00
L_2L_3	68 400	183 700	126 100	378 200	16 000	23.60	4	5.90	5.95	6 x 1	6.00
L_3L_4	82 000	218 400	149 100	449 500	16 000	28.20	4	7.05	6.50	6 x $1\frac{1}{4}$	7.50
L_3U_4	.....	54 200	44 100	98 300	16 000	6.14	2	3.07	4.30	4 x $\frac{7}{8}$	3.50

* Note—All areas in square inches, and all dimensions in inches.

Observe that the above table is not completely filled out with respect to the first two members given. This is on account of the requirements of the Specifications (80).

The approximate depth of this bar is determined by taking the square root of 6 times the area as above determined. It is:

$$d = \sqrt{6 \times 9.22} = 7.44 \text{ inches.}$$

As this is nearer 7 than 8 inches, a 7-inch bar will be chosen; and looking in the first column, Carnegie Handbook, p. 248, for an area which will be equal to or in excess of 9.22, it is found that a $1\frac{3}{8}$-inch bar satisfies this condition, and therefore the section of this member consists of two bars 7 by $1\frac{3}{8}$ inches.

According to (80), the first two sections for the lower chord are to be made of built-up members. This requires that instead of I-bars they are to be made of angles and plates, or, in case the stress is light, of channels. The depth of the section is limited by the size of the greatest I-bar head. As the diameter of the I-bar head depends upon the size of the pin, it cannot of course be determined accurately before the pin is designed. It is customary to assume the largest head, and to design the section so as to clear this. The size of the largest head for bars of given width is given in the Carnegie Handbook, p. 212.

The design of the member L_0L_2 will depend upon the size of the largest head of the 7-inch I-bar of the member U_1L_2. This is $17\frac{1}{2}$ inches; and in order that the head may have some clearance, it will be necessary to add $\frac{1}{2}$ inch to the top and the bottom, making a total of $18\frac{1}{2}$ inches. Since the flange angle, as in the case of plate-girders, will extend over the plate about $\frac{1}{4}$ inch, the plate itself may be 18 inches wide and still give sufficient clearance.

The total stress is 234 200 pounds, and the allowable unit-stress (15) is 16 000 pounds per square inch. The required net area, then, is:

$$\frac{234\,200}{16\,000} = 14.64 \text{ square inches.}$$

According to the Specifications, the thickness of the plate cannot be less than $\frac{3}{8}$ inch. The gross area of two 18 by $\frac{3}{8}$-inch plates is 13.5 square inches, and the gross area of four $3\frac{1}{2}$ by $3\frac{1}{2}$ by $\frac{3}{8}$-inch angles, which are assumed to be sufficient, is 9.92 square inches, thus making a total gross area of 23.42 square inches. If 5 rivet-holes are assumed to be taken out of each web, and one rivet-hole taken out of each angle, this will require a certain number of square inches to be

deducted from the section, and this is computed as follows:

Out of webs, $2 \times 5\,(\frac{7}{8} + \frac{1}{8}) \times \frac{3}{8} = 3.75$ sq. in.
Out of angles, $4\,(\frac{7}{8} + \frac{1}{8}) \times \frac{3}{8} \ldots = 1.50$ " "
Total $= 5.25$ sq. in.

The net area of the section is now determined to be $23.42 - 5.25 = 18.17$ square inches. This is somewhat greater than the required net area, but must be used, for according to (39), these are the smallest and thinnest angles that may be used.

Figs. 173 and 174 show the cross-section and the general detail at L_2. The width of the member cannot be determined until after the section of the end-post is computed, since it must fit inside of the end-post, the horizontal legs of the angles being cut off to allow this. The end-post, Article 87, is $14\frac{1}{2}$ inches inside. If it is assumed that all the pin-plates on the end-post are placed on the outside, and all those at L_0 on L_0L_2 are on the inside, then the width of L_0L_2, back to back of plates, must be $14 - (2 \times \frac{1}{2} + 2 \times \frac{3}{8}) = 12\frac{1}{4}$ inches or less, $\frac{1}{8}$-inch clearance being allowed between the sides of the angles and the web plates of the end-post (see Fig. 173).

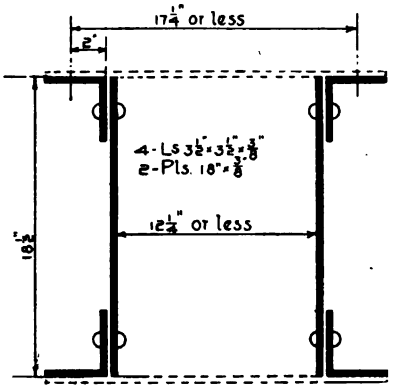


Fig. 173. Cross-Section Showing Construction of Lower Chord Member.

The total net section through the pin-hole at L_2 (26) must be $1\frac{1}{4} \times 18.17 = 22.7$ square inches, or 11.35 square inches for one side. The plate which is to increase the section must be on the outside, since the intermediate post U_2L_2 and the two I-bars of member U_1L_2 must go inside. The gross width of this plate is $11\frac{1}{4}$ inches (see Fig. 174), and the net width is $2w = 11\frac{1}{4} - 5 = 6\frac{1}{4}$ inches. The net area through the pin is:

Two $3\frac{1}{2}$ by $3\frac{1}{2}$ by $\frac{3}{8}$-in. angles $= 4.96$ square inches.
One 18 by $\frac{1}{2}$-in. plate $= 9 - 5 \times \frac{1}{2} = 6.50$ " "
Total $= 11.46$ square inches

Since this is greater than the 11.35 required, no plate will be necessary to fulfil (26) in this respect.

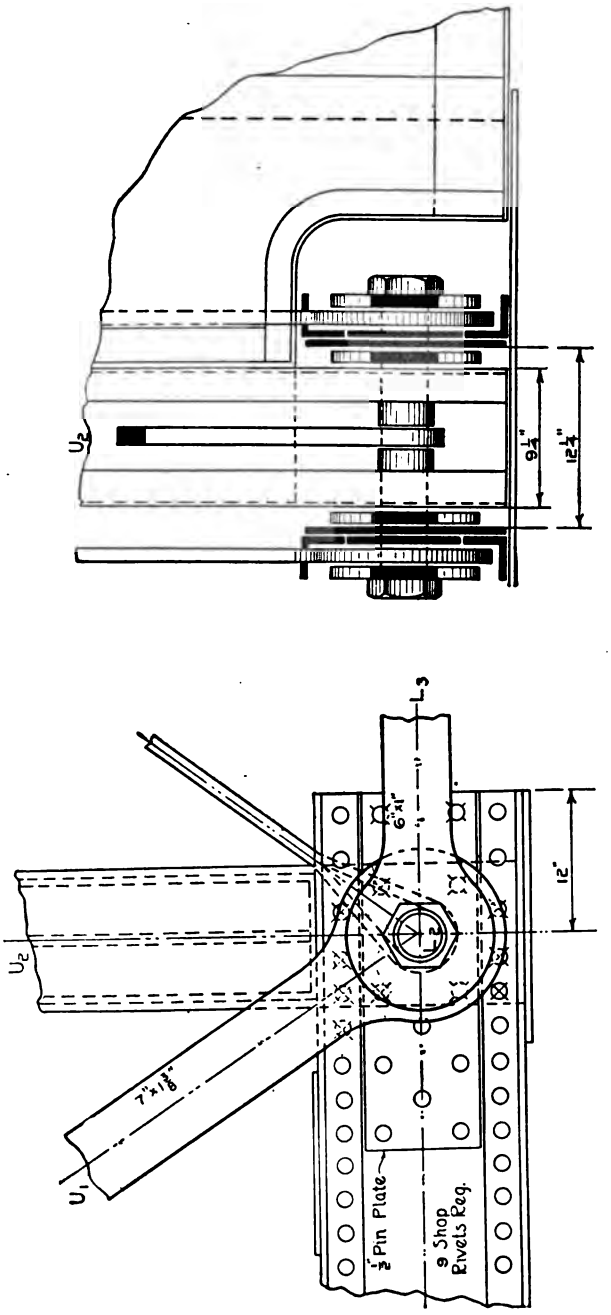


Fig. 174. Side Elevation and Transverse Section Showing General Details of Construction at Panel Point in Lower Chord.

Sufficient bearing area must be provided at this point. The total stress is 234 200 pounds, the total bearing area required is $\frac{234\,200}{24\,000} = 9.76$ square inches, and the total thickness for one side is $\frac{9.76}{2 \times 5} = 0.976$ inches. Since the thickness of the web is $\frac{1}{2}$ inch, the pin-plates must be $0.976 - 0.50 = 0.476$ inch (say $\frac{1}{2}$ inch) thick. A $\frac{1}{2}$-inch pin-plate must be used, and as the total thickness of the bearing area is now 1.00 inch, this pin-plate will take $\frac{0.50}{1.00} \times \frac{234\,200}{2}$ $= 58\,550$ pounds. The joint is weakest in shear, and will therefore require $\frac{58\,550}{7\,220} = 8 +$ (say 9) shop rivets.

In case it is necessary to put the member U_1L_2 on the outer side of L_0L_2, then the outer legs of the upper angles must be cut off to allow U_1L_2 to pass. This will decrease the section by an amount $(3\frac{1}{2} - \frac{3}{8}) \times \frac{3}{8} = 1.20$ square inches. Considering the pin-plate, which is $(18\frac{1}{2} - 2 \times 3\frac{1}{2}) - \frac{1}{4} = 11\frac{1}{4}$ inches, the $\frac{1}{4}$ inch being allowed for clearance between the edges of its flange angles, the total net section through the pin-hole on one side will be:

One Angle	$3\frac{1}{2}$ by $3\frac{1}{2}$ by $\frac{3}{8}$-in.	= 2.48	square inches
One Cut Angle	$3\frac{1}{2}$ by $3\frac{1}{2}$ by $\frac{3}{8}$-in.	= 1.31	" "
One Web	$(18 - 5)\ \frac{1}{2}$ sq. in.	= 6.50	" "
One Pin-Plate	$(11\frac{1}{4} - 5)\ \frac{3}{8}$ sq. in.	= 2.34	" "
	Total	= 12.63	square inches.

This is greater than 11.35 as required, and is therefore safe.

The distance from the center of the pin to the end must now be determined (26). The total net section of the body of the member is 18.17 square inches, or 9.09 square inches for one side, and the thickness of the web and the pin-plate is 1 inch. The distance from the pin to the end of the member is then $\frac{9.09}{1} = 9\frac{1}{8}$ inches, and the distance to the center of the pin is $9\frac{1}{8} + \frac{5}{2} = 11\frac{5}{8}$, say 12 inches (see Fig. 174). Rivets should be countersunk where necessary to prevent interference with I-bars. For signs, see "Steel Construction," Part III, p. 192, and Carnegie Handbook, p. 191.

At point L_0 of this member, the pin is $6\frac{1}{4}$ inches in diameter, and, as previously mentioned, the legs of the angles are cut (see Fig. 175). The total bearing area required for one side is $\frac{9.76}{2} = 4.88$, and the required thickness is $\frac{4.88}{6.25} = 0.781$ inch. Subtracting the thickness of the $\frac{1}{2}$-inch web from this gives 0.281 inch. A pin-plate $\frac{3}{8}$ inch thick must be used.

The net area through the pin (26) must be 11.35 square inches.

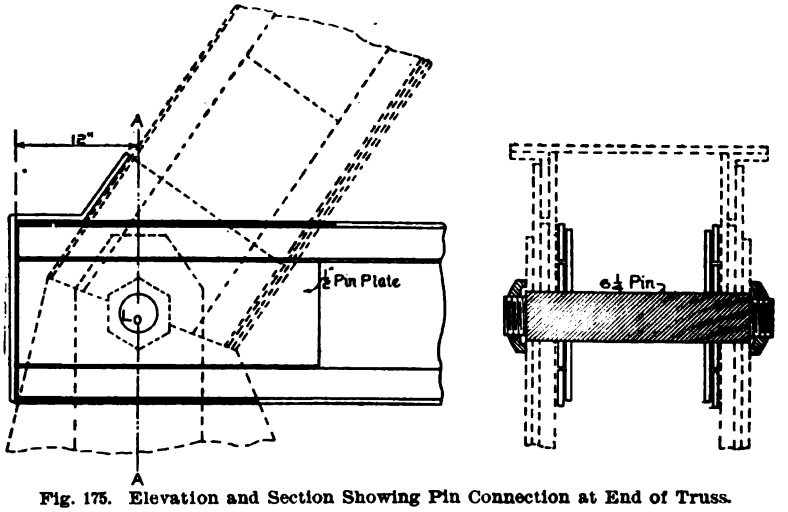


Fig. 175. Elevation and Section Showing Pin Connection at End of Truss.

This net area, remembering that the angle legs are cut and therefore their area is that of a bar $3\frac{1}{2}$ by $\frac{3}{8}$-inch, computed for one side, is as follows:

Two Angles, legs cut, $3\frac{1}{2}$ by $3\frac{1}{2}$ by $\frac{3}{8}$-in.	=	2.62 square inches
One Web $18 \times \frac{1}{2} - 6\frac{1}{4} \times \frac{1}{2}$ sq. in.	=	5.88 " "
One Pin-Plate $11\frac{1}{4} \times \frac{3}{8} - 6\frac{1}{4} \times \frac{3}{8}$ sq. in.	=	1.87 " "
Total		10.37 square inches.

This shows the section to be deficient, and the thickness of the pin-plate must be made $\frac{1}{2}$ inch. This gives a net area through the pin of 11.62 square inches.

The distance between rivet lines (see Fig. 173) is $17\frac{1}{4}$ inches, and (44) the tie-plates must be $17\frac{1}{4}$ (say 18) inches long, and their thickness $\frac{17.25}{50} = 0.346$ inch (say $\frac{3}{8}$ inch).

The lattice bars (45) must be $2\frac{1}{2}$ inches wide, and (47) must be double. From (45) and Table XXV, page 219, the thickness must be $\frac{7}{16}$ inch, the distance c being $17.25 \times$ secant $45° = 2$ ft. $0\frac{7}{16}$ in.

The design of the hip vertical U_1L_1 is also made in accordance with (80) of the Specifications. It will be assumed that the section consists of one 8 by $\frac{3}{8}$-inch plate, and four $3\frac{1}{2}$ by $3\frac{1}{2}$ by $\frac{3}{8}$-inch angles, since this is the lightest section that may be used according to the Specifications, the 8-inch plate being chosen as it gives some clearance between the inner edges of the legs of the angles.

The total stress in the member is 141 600 pounds, and the unit-stress is 16 000 pounds per square inch, thus requiring a net area of 8.85 square inches. The plate gives a net area of 2.20 square inches, and the four angles give a net area of 6.88 square inches, making a total of 9.08 square inches, one rivet-hole being taken out of each angle, and two out of the web, at any particular section. The net area is somewhat greater than that required, but must be used, as this is the minimum section allowed by the Specifications. Fig. 176 shows a cross-section of this member as above determined.

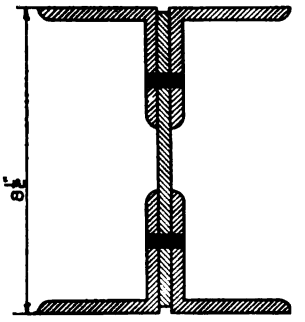


Fig. 176. Cross-Section of Hip Vertical.

This member will be connected to the upper chord and end-post by means of a pin which is $6\frac{1}{4}$ inches in diameter, the diameter of the pin being determined later. The total stress is 141 600 pounds, and this will be taken by two plates, one on either side of the member. The net section of the member is 9.08 square inches, and the section through the pin (26) must be 25 per cent in excess of this, making a total of 11.35 square inches, or 5.68 square inches for each plate. The total width of these plates will be taken as 12 inches, and this (see Fig. 177) will make the required thickness:

$$\frac{5.68}{2c} = \frac{5.68}{12.00 - 6.25} = 0.99 \text{ inch.}$$

Fig. 177 shows the details of these pin-plates. Since the above thickness is too great to be punched in one single piece, the above thickness will be made up of two plates, each $\frac{1}{2}$ inch thick. The area at section A–A must be equal to that of the body of the bar. It

is $12.00 \times \frac{1}{2} = 6.00$ square inches for one side, or 12.00 square inches for both sides. As this is greater than the 8.85 square inches as above computed, the area at A–A is sufficient, as is also the width of the plate, which was assumed as 12 inches.

One of the plates will be riveted directly to the member, and the other will be riveted to it as a pin-plate. The section back of the pin (26) must be equal to the net section in the body of the member. The net section is 4.54 square inches for one side, and the total thickness

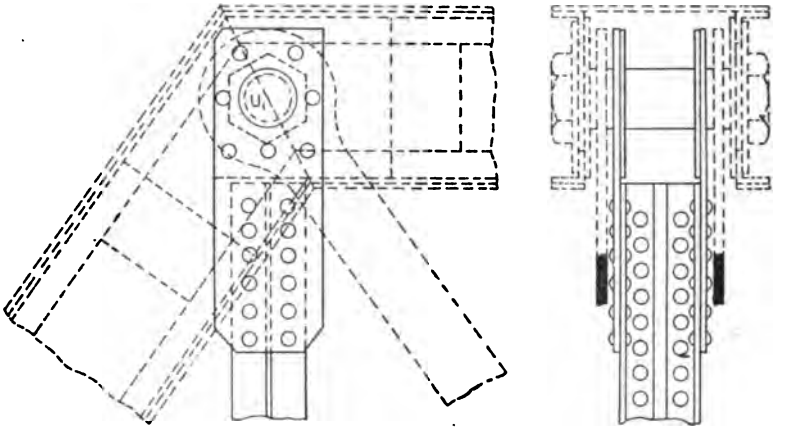


Fig. 177. Connection of Hip Vertical to Upper Chord and End-Post.

of the pin-plates is 1.125 inches, making the distance from the end of the member to the pin $\frac{4.54}{1.125} = 4\frac{1}{8}$ inches, and the distance to the center of the pin $4\frac{1}{8} + \frac{6\frac{1}{4}}{2} = 7\frac{1}{4}$ inches.

The joint between the plates and the main member will be weak in shear, the rivets tending to shear off between the $\frac{3}{8}$-inch angles and the plate, and also between the two plates themselves. As each side takes one-half of the above stress, the number of rivets required to connect the plates to the main member will be:

$$\frac{141\ 600 \div 2}{7\ 220} = 10 \text{ shop rivets,}$$

and the number of rivets required to connect the inner $\frac{1}{2}$-inch plate to the outer one which is connected to the member itself will be:

$$\frac{141\ 600 \div 4}{7\ 220} = 5 \text{ shop rivets.}$$

The distance from the center of the pin to the top of the main part must be greater than one-half the diameter of the largest I-bar head—that is, $17\frac{1}{2} \div 2 =$ say, 9 inches.

At the lower end, this member is connected to the bottom chord by means of a couple of clip angles and four or five rivets. Only sufficient rivets are required to prevent the sagging of the bottom chord, since the floor-beam is connected to the hip vertical above the lower chord, and hence no stress comes on the joint at the lower end (see Fig. 178).

The width of the plate has been assumed as 8 inches. This width is liable to be changed after the design of the intermediate

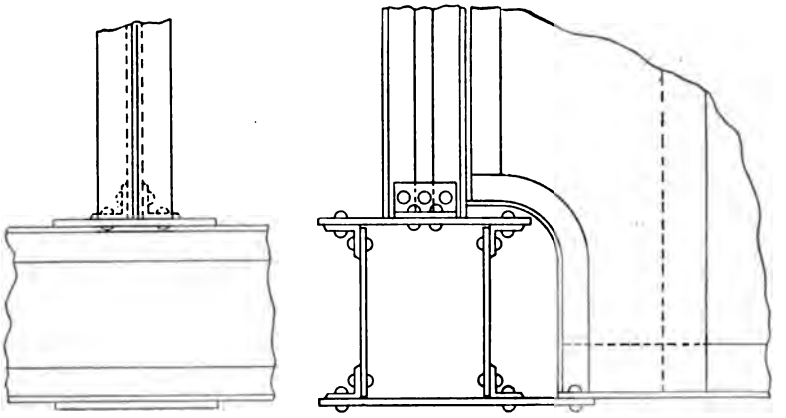


Fig. 178. Connection of Hip Vertical to Lower Chord.

posts has been made, since it will be economical to have all the intermediate floor-beams of the same length; and therefore the width of this plate will be changed so as to make the width of the hip vertical the same as the width of the intermediate posts.

85. **The Intermediate Posts.** The post U_2L_2 must be designed to stand a total stress of 163 600 pounds. Where possible, it is economical to make the intermediate posts out of channels, as this saves a large amount of riveting. As seen by the stress sheet, the length of these posts is 30.1 feet center to center of end pins. It is usually required that $\frac{l}{r}$ must not be more than 100, and this condition requires that the least radius of gyration cannot be less than $\frac{30.1 \times 12}{100} = 3.62$.

From Carnegie Handbook, p. 101, it is seen that a 12-inch 30-pound channel has a radius of gyration of 4.28, and will fulfil the conditions. The area of two of these channels is 17.64 square inches. The unit allowable stress (16) is:

$$P = 16\,000 - 70 \times \frac{30.1 \times 12}{4.28} = 10\,090 \text{ pounds per square inch.}$$

The required area is then determined to be $\frac{163\,600}{10\,090} = 16.2$ square inches; and as this coincides very closely with the area given, these channels are efficient and will be used.

Fig. 179 shows the cross-section of this post. The radius of gyration which was used above was the radius of gyration of the channels about an axis perpendicular to their web. The radius of gyration of the entire section about an axis perpendicular to the web will be the same as that of one channel. In order to have the sections safe, the radius of gyration about the axis *B–B* must be equal to or greater than the other. The radius of gyration about the axis *B–B* can be increased or decreased by spacing the channels. The exact distance which will make the two rectangular radii of gyration equal may be determined by the methods of "Strength of Materials," or it may be found in columns 14 and 15 of the Carnegie Handbook, p. 102. For any particular case it is equal to the value given in column 14, *plus* four times that given in column 15. For the channels under consideration, it is equal to $7.07 + 4 \times 0.704 = 9.89$ inches. Any increase in this distance will only tend to increase the radius of gyration about the axis *B–B*, and will make the post safer about that axis.

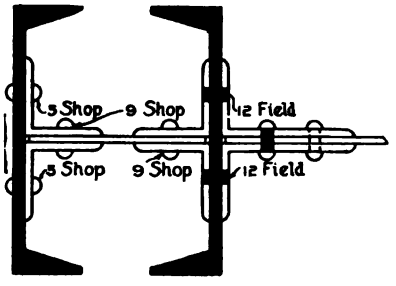


Fig. 179. Cross-Section of Intermediate Post, Showing Diaphragm.

Fig. 179 shows a diaphragm. The web of this diaphragm cannot be less than $\frac{3}{8}$ inch, and the size of the angles cannot be less than $3\frac{1}{2}$ by $3\frac{1}{2}$ by $\frac{3}{8}$-inch, as this is the least allowed by the Specifications. The function of this diaphragm is to transfer one-half of the floor-beam reaction to the outer side of the post. The rivets which connect the

angles to the diaphragm web are shop rivets, and (see design of floor-beam) must be $\frac{137\,600}{2 \times 7\,880} = 9$ in number. The rivets which connect the diaphragm angle with the outer channel of the post are also shop rivets, and are $\frac{137\,600}{2 \times 7\,220} = 10$ in number, 5 on each side. The same rivets which connect the floor-beam to the post go through the diaphragm angle on that side of the diaphragm next to the center of the bridge, and must therefore be field rivets and take the entire floor-beam reaction. These must be $\frac{137\,600}{6\,013} = 23$ in number, 12 on each side. The exact distance, back to back of the channels of the post, cannot be determined until after the top chord has been designed, since the post must slide up in the top chord and also leave room on each side for the diagonal members of the truss. The width is determined by the packing of the members at joint L_2 (see Fig. 174), and is found to be $9\frac{1}{4}$ inches. Since this is less than that required above, the post must be examined for bending about an axis parallel to the web of the channels.

According to the methods of "Mechanics" and "Strength of Materials," with the help of the Carnegie Handbook, p. 102, the moment of inertia about this axis is found to be 286.42, and the radius of gyration 3.96. The unit allowable compressive stress is then computed to be 9 580 pounds per square inch, and the required area $\frac{163\,600}{9\,580} = 17.10$ square inches, which, being less than 17.64, shows the section to be safe.

This member is connected to the top chord at its upper end by a 5-inch pin. The total stress is 163 600 pounds, and the total bearing area required is $\frac{163\,600}{24\,000} = 6.8$ square inches, or 3.4 square inches for each side (19). The total thickness of the bearing area for each side is $\frac{3.4}{5} = 0.68$ inch. The thickness of the web of a 12-inch 30-pound channel is 0.513 inch, which leaves $0.68 - 0.513 = 0.167$ inch as the thickness of the pin-plate, but it must be made $\frac{3}{8}$ inch according to the Specifications. Fig. 180 shows the arrangement of the plates and the rivets.

The sum total of the pin-plates and the channel web is 0.888 inch, and therefore on one side the stress transferred to the pin by means of the pin-plate, which is 0.375 inch, is $\frac{1}{2} \times \frac{0.375 \times 163\,600}{0.888}$ = 34 600 pounds. This plate will tend to shear off the rivets between it and the channel web, and therefore $\frac{34\,600}{7\,220}$ = 5 shop rivets are required.

The stress that is shown on the stress sheet is the stress in the post above the floor-beam. The stress in that part below the floor-

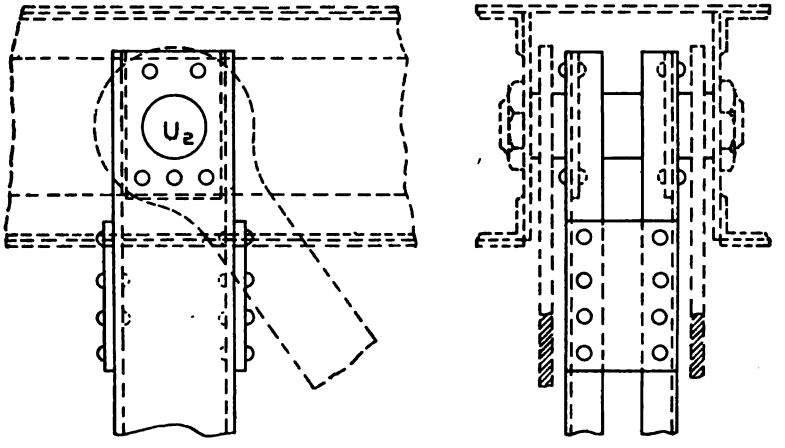
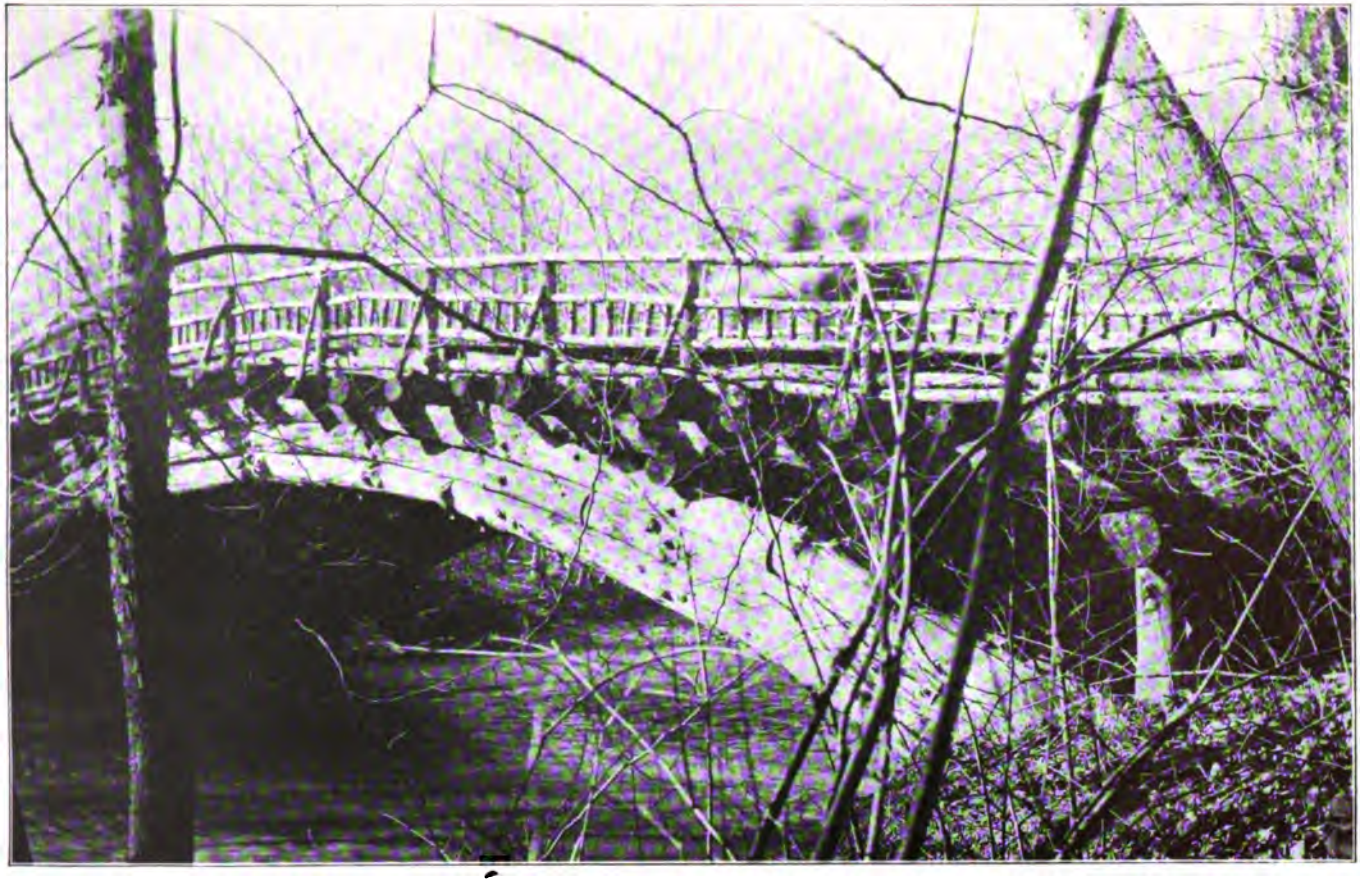


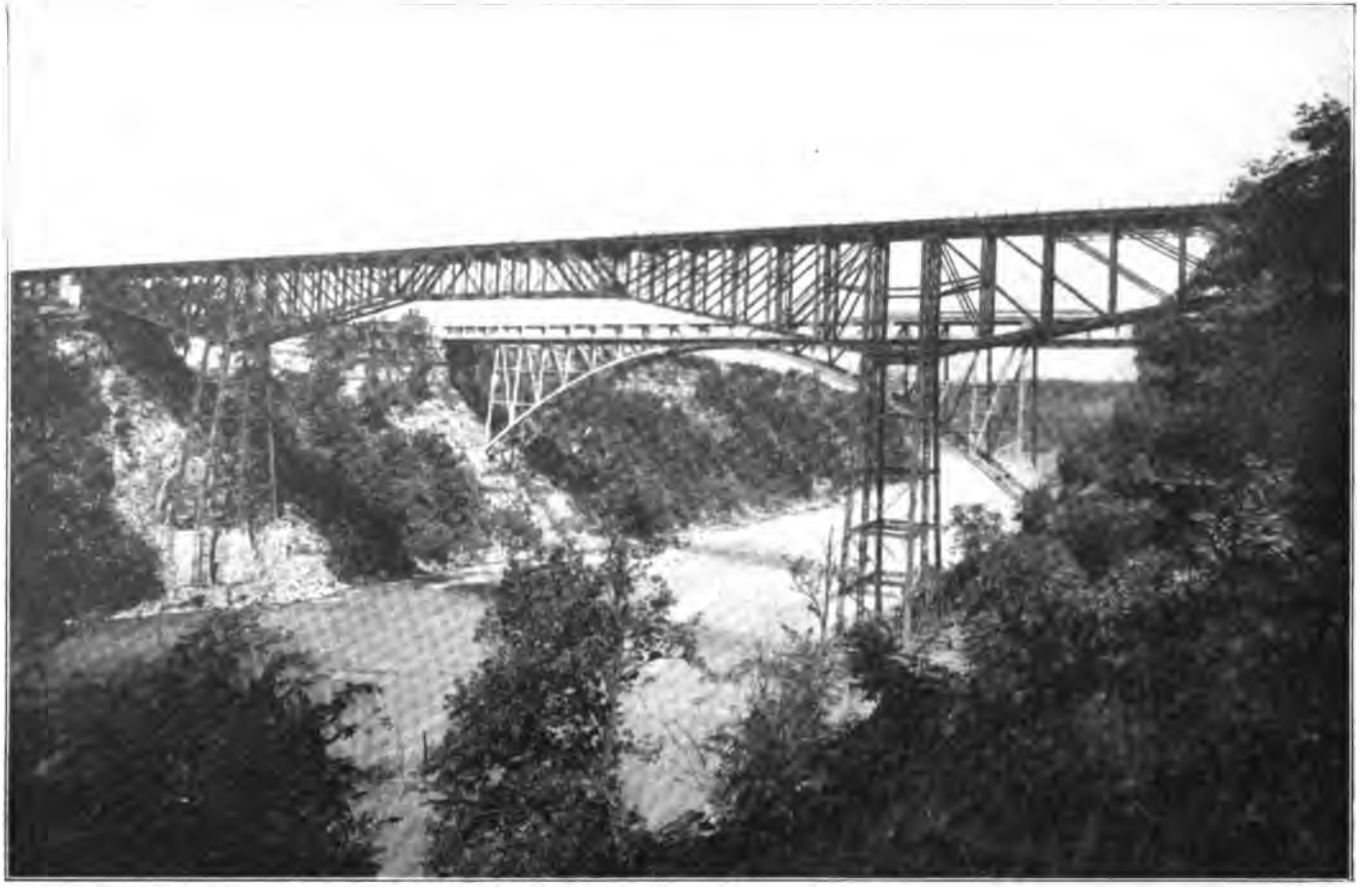
Fig. 180. Arrangement of Plates, Rivets, Pin, etc., at Connection of Intermediate Post to Top Chord.

beam is equal to the vertical component of the diagonal in the panel ahead of the post in question. In this case it is the vertical component of the stress in U_1L_2, and is equal to 242 000 pounds, and this requires a total bearing area of $\frac{242\,000}{24\,000}$ = 10.1 square inches, and a total thickness of $\frac{10.1}{2 \times 5}$ = 1.01 inches on each side, the pin being 5 inches in diameter. From this total thickness must be subtracted the thickness of the web of the channel, and this leaves 1.01 − 0.513 = 0.497 inch as the total thickness of the pin-plates required. This shows that we must use one ½-inch plate. The total thickness of the bearing area is now 0.513 + 0.50 = 1.013 inches.

Each plate takes a total stress of $\frac{0.50}{1.013} \times \frac{242\,000}{2}$ = 59 700



AN UNUSUALLY FINE EXAMPLE OF RUSTIC BRIDGE BUILDING
In Rock Creek Park, Washington, D. C.



CANTILEVER BRIDGE OVER THE NIAGARA RIVER AT NIAGARA FALLS

On line of Michigan Central Railroad. Farther downstream is shown the single-arch steel bridge of the Grand Trunk Railroad.

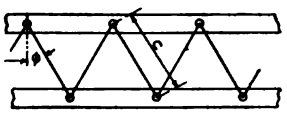
Courtesy of American Bridge Company.

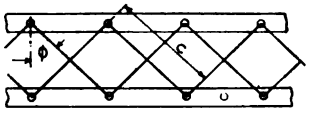
pounds; and the joint being weak in shear, the number of rivets required will be $\frac{59\,700}{7\,220} = 9$ rivets in single shear. The detail will be similar to that in Fig. 180.

The distance, back to back of the channels in this post, will probably not be greater than 12 inches, and this will make the distance between rivet lines about 9 inches. According to (44), the end tie-plates must be at least 9 inches long and of course 12 inches wide. The thickness cannot be less than $\frac{9}{50} = 0.18$ inch, but they will be made $\frac{3}{8}$ inch (36). Between the tie-plates the channels will be connected by means of lattices. The Specifications (45) require that they should not be less than $2\frac{1}{4}$ inches in width and $(1.414 \times 9)\,\frac{1}{40} = 0.318$ (say $\frac{3}{8}$) inch in thickness. Table XXV gives the thickness of lacing bars for any distance between rivets.

TABLE XXV

Thickness of Lacing Bars





Single Lacing $\left(t = \frac{c}{40};\ \phi = 30°\right)$		Double Lacing $\left(t = \frac{c}{60};\ \phi = 45°\right)$	
t	c	t	c
$\frac{1}{4}$ in.	0 ft. 10 in.	$\frac{1}{4}$ in.	1 ft. 3 in.
$\frac{5}{16}$ in.	1 ft. $0\frac{1}{2}$ in.	$\frac{5}{16}$ in.	1 ft. $6\frac{3}{4}$ in.
$\frac{3}{8}$ in.	1 ft. 3 in.	$\frac{3}{8}$ in.	1 ft. $10\frac{1}{2}$ in.
$\frac{7}{16}$ in.	1 ft. $5\frac{1}{2}$ in.	$\frac{7}{16}$ in.	2 ft. $2\frac{1}{4}$ in.
$\frac{1}{2}$ in.	1 ft. 8 in.	$\frac{1}{2}$ in.	2 ft. 6 in.
$\frac{9}{16}$ in.	1 ft. $10\frac{1}{2}$ in.	$\frac{9}{16}$ in.	2 ft. $9\frac{3}{4}$ in.
$\frac{5}{8}$ in.	2 ft. 1 in.	$\frac{5}{8}$ in.	3 ft. $1\frac{1}{2}$ in.

A width of $2\frac{1}{4}$ inches is chosen above, since according to Carnegie Handbook, p. 183, a $\frac{3}{4}$-inch rivet is the largest which can be used in the channel flange.

The post U_3L_3 must be designed for a total stress of 87 000 pounds. It will be assumed that two 10-inch 20-pound channels

with a radius of gyration 3.66 and an area of 5.80 square inches each will be sufficient. The length, as before, is 30.1 feet, and the unit-stress is:

$$P = 16\,000 - 70 \times \frac{30.1 \times 12}{3.66} = 9\,080 \text{ pounds.}$$

The required area is $\frac{87\,000}{9\,080} = 9.60$ square inches. Since the total area of the two channels is 11.76 square inches, and the required area is 9.6 square inches, it is seen that they do not coincide very closely. These channels, however, will be used, since the thickness of the web is the thinnest allowed by the Specifications, and the width of the channels is the smallest that can be used and still give sufficient room to make the connections with the end connection angles of the floor-beams.

The lower end of this post also has a diaphragm which must transfer half of the stress to the outer channel of the post. The sides of the diaphragm are the same as in the posts previously designed; and the number of rivets required is computed in a similar manner and found to be as indicated in Fig. 181, which shows the cross-section of this post.

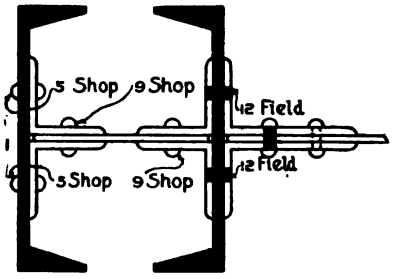


Fig. 181. Cross-Section of Intermediate Post.

At the upper end the bearing area required on one channel is $\frac{87\,100}{2 \times 24\,000} = 1.814$ square inches, and the thickness required is $\frac{1.814}{5} = 0.363$ inch, a 5-inch pin being used. As the web of the channel is 0.382 inch thick, it will give sufficient bearing area without pin-plates.

At the lower end, the vertical component of U_2L_3 is 157 500 pounds. The bearing area required on each side of the post is $\frac{157\,500}{2 \times 24\,000} = 3.28$ square inches, and the thickness is $\frac{3.28}{5} = 0.66$ inch. The thickness of the channel web being 0.382 inch leaves 0.660 − 0.382 = 0.278 inch as the required thickness of the pin-plate;

but $\frac{3}{8}$ inch must be used, making a total thickness of $0.382 + 0.375 = 0.757$ inch. The plate will carry $\frac{0.375}{0.757} \times \frac{157\,500}{2} = 39\,000$ pounds, and this requires $\frac{39\,000}{7\,220} = 6$ shop rivets in single shear.

The distance, back to back of channels, will be the same as in

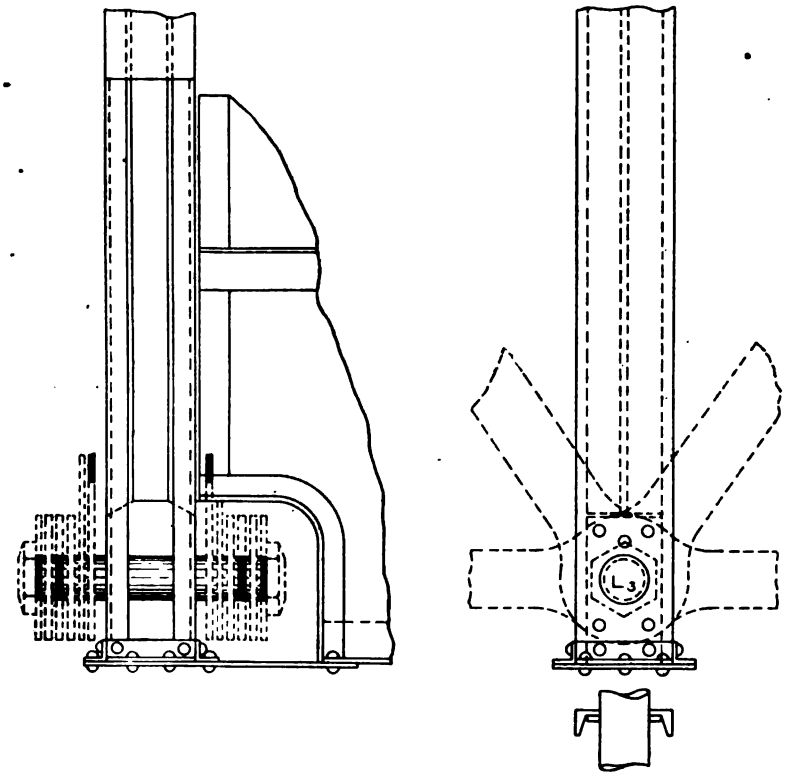


Fig. 182. End and Side Elevations Showing Detail of Construction at Lower End of Intermediate Post.

U_2L_2, and therefore the tie-plates and lacing bars will be the same. Fig. 182 gives a detail of the lower end of U_3L_3.

86. **The Top Chord.** The top chords of small railway bridges may be made of two channels laced on their top and bottom sides. This is not very good practice, since it leaves the tops of the channels open and lets in the rain and snow, which tends to deteriorate the joints. It is better to add a small cover-plate, even if this does give

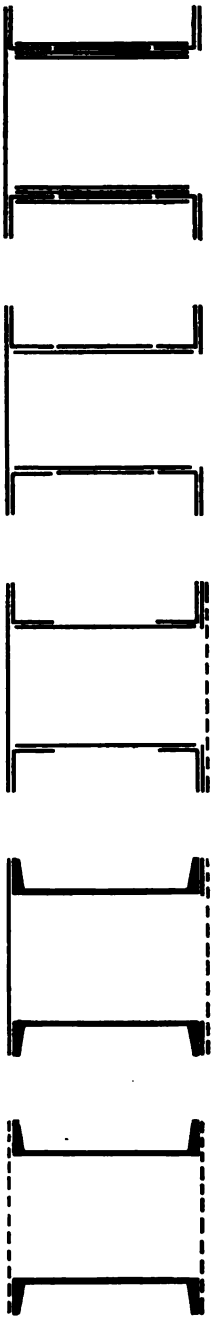


Fig. 183. Chord Sections, Showing Various Types of Construction.

an excessive section. In case of stress such as is demanded, the chords may consist of two channels and a cover-plate. In this case it is necessary to place small pieces called *flats* upon the lower flanges of the channel, in order to lower the center of gravity of the section and to bring it near the center of the web. This section makes a very economical section in that it saves much riveting. On account of channels being made only up to 15 inches in depth, the use of this section is quite limited owing to the fact that it is not deep enough to allow the I-bar heads sufficient clearance, for the I-bar heads in bridges of even ordinary span will exceed this amount.

The most common section is that which consists of two side plates, four angles, and one cover-plate. Sometimes this section has flats placed upon the lower angle in order to lower the center of gravity, as explained above. According to (33), the section should be as symmetrical as possible, and the center of gravity should lie as near the center of the web as is consistent with economy.

In case the stress is great enough to demand a heavier section than that above described, additional plates are added upon the sides of the original plates, and heavier and larger cover-plates and angles are used. Fig. 183 shows different types of chord sections.

In addition to the cover-plate being designed to withstand the total stress, close attention must be paid to (42). This clause has been inserted on account of practical considerations, since it has been found out

that if plates are made much thinner than the proportions here required, they will crumple up and fail long before the allowable unit of stress as computed from the formula has been reached. In some cases—especially where the stress is light—the proportions laid down in (42) and (36) will govern the design of the section, instead of the required net area as determined by the formula for the allowable unit compressive stress.

The design of the first section of the top chord will now be made. Here, as in the case of the first sections of the lower chord, the diameter of the head of the greatest I-bar determines the width of the plates in the section The head of the 7-inch I-bar which constitutes the member U_1L_2 is $17\frac{1}{2}$ inches, and, allowing a clearance of $\frac{1}{2}$ inch on either side of the head, the total depth inside the chord should be $18\frac{1}{2}$ inches. As in the case of the lower chord, plates 18 inches wide may be used.

The size of the angles to be chosen is a matter of judgment. Usually any size should be chosen at first, and the preliminary design will indicate at once what size should have been taken. For this case, $3\frac{1}{2}$ by $3\frac{1}{2}$ by $\frac{3}{8}$-inch will be assumed at first.

For sections of this character, the radius of gyration is approximately equal to $0.4h$, in which h is the height, or rather the width, of the side plate. The approximate radius of gyration is $r = 0.4 \times 18 = 7.2$ inches, and the length is equal to one panel length, or 21 feet. The allowable unit of stress (16) is:

$$P = 16\,000 - 70 \times \frac{21 \times 12}{7.2} = 13\,550 \text{ pounds.}$$

The required area is $\frac{449\,500}{13\,550} = 33.2$ square inches. The correct proportion for sections of this character is that 0.4 of the total area should be taken up by the web. The area of the web would then be $0.4 \times 33.2 = 13.28$ square inches, and the thickness would be $\frac{13.28}{2 \times 18} = 0.37$ inch. According to this, a $\frac{3}{8}$-inch plate should be used, but (42) requires that it shall be $\frac{14.5}{30} = 0.483$ inch or thicker. Therefore an 18 by $\frac{1}{2}$-inch plate must be used for the web.

The correct proportion for sections of this character is that the width between plates should be about $\frac{7}{8}$ the width of the side plates.

This will give the required width between plates equal to $\frac{7}{8} \times 18 = 15.75$ inches. The cover-plate (42) must not be thinner than $\frac{1}{40}$ the distance between the connecting rivet lines. The rivet lines are, in this case, $15.75 + 2 \times 2 = 19.75$ inches apart, and therefore the thickness of the cover-plate cannot be less than $\frac{19.75}{40} = 0.494$ inch. The cover-plate will therefore be taken as $\frac{1}{2}$ inch thick. The width of the cover-plate (see Fig. 184) must be about $15.75 + 2 \times 3\frac{1}{2} + \frac{1}{2} = 23\frac{1}{4}$ inches (say 23 inches). The cover-plate will be taken 23 by $\frac{1}{2}$-inch.

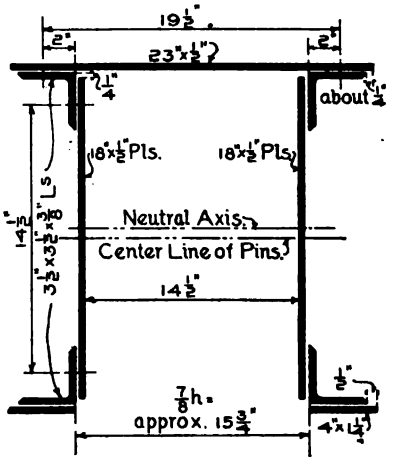


Fig. 184. Section of Top Chord.

The center line of pins will be taken at the center line of the web, and the center of gravity of the section will be assumed as $\frac{1}{2}$ inch above this. In order that the center of gravity may be near that assumed, the moment of the cover-plate about the assumed center of gravity axis should be about equal to the moment of the flats about the same axis. The moment of the cover-plate about the assumed axis is:

$$23 \times \tfrac{1}{2}(9.0 - 0.5 + 0.25 + 0.25) = \frac{23 \times 9}{2};$$

and the moment of the flats about the same axis is:

$$A\,(9.0 + 0.5 + 0.25 + 0.5) = 10.25\,A,$$

in which A is the area in square inches of both of the flats. Equating these two expressions, and solving for A, there results:

$$A = \frac{23 \times 9}{2 \times 10.25} = 10.1 \text{ square inches.}$$

Assuming the flats to be 4 inches wide, the thickness on each side will be 1.25 inches. As this is too thick to punch, the flats on each side will be composed of two 4 by $\frac{5}{8}$-inch plates.

The total area is:

One cover-plate = $23 \times \frac{1}{2}$	= 11.5	sq. in.
Two web plates = $2 \times 18 \times \frac{1}{2}$	= 18.0	" "
Two flats $4 \times 1\frac{1}{4}$	= 10.0	" "
Total	39.5	sq. in.

But the required area is 32.2 square inches, which is considerably less than the area above given, and which does not include the angles and hence we can use the smallest size angles, which are those previously assumed. The area of each of these angles is 2.48 square inches, thus making the total area of the section $39.5 + 4 \times 2.48 = 49.42$ square inches. This is considerably in excess of the area as required according to the formula for compression; but it is the least allowed by the Specifications. Note that this is the case where (42), instead of the formula for compressive stress, is the ruling factor in the determination of the section.

The center of gravity of the approximate section must now be determined, the moment of inertia and the radius of gyration about the neutral axis must be computed, and the required area must be determined by using this radius of gyration as computed. If the required area as determined with the actual radius of gyration is less than the approximate area, then the thickness of the angles or the plates must be increased and the section then examined for its radius of gyration and required area. If the area is sufficient, the section is used; if not, another recomputation is in order.

In the determination of the center of gravity of the section, the moment is taken about the top of the cover-plate. The moments are computed as follows:

Cover-plate $(23 \times \frac{1}{2}) \times \frac{1}{4}$	2.88
Webs $2\,(18 \times \frac{1}{2}) \times (9 + \frac{3}{4})$	175.60
Top angles $2\,(2.48) \times (1.01 + \frac{1}{2})$	7.50
Lower angles $2\,(2.48) \times (\frac{1}{2} + \frac{1}{4} + 18 + \frac{1}{4} - 1.01)$	89.30
Flats $2\,(4 \times 1\frac{1}{4}) \times 19\frac{3}{8}$	196.25
Total	471.53

The center of gravity is now found to be $\dfrac{471.53}{39.50 + 4 \times 2.48} = 9.55$ inches from the top of the cover-plate. The distance from the top of the cover-plate to the middle line of the web is $9 + \frac{1}{4} + \frac{1}{2} = 9.75$ inches, and this leaves a distance of $9.75 - 9.55 = 0.2$ inch from the center line of the web to the neutral axis. This distance is gen-

erally represented by the letter e, and it is known as the *eccentricity of the section.*

The moment of inertia about this axis must now be computed. The relation used is that the moment of inertia about any axis is equal to the moment of inertia about some other axis, *plus* the product of the square of the distance between the two axes by the area of the section whose moment of inertia is desired. The moments of inertia of the various parts of the section (see "Steel Construction," Part IV, pp. 292 and 293) are computed and are as follows:

Cover-plates	955.26
Webs	486.72
Top angles	325.74
Lower angles	359.74
Flats	1 017.37
Total	3 184.83

The radius of gyration is equal to the square root of the quotient obtained by dividing the moment of inertia by the area. It is

$$r = \sqrt{\frac{3\ 184.83}{49.42}} = 8.04.$$

Using this value of the radius of gyration in the formula for the compressive stress, there is obtained 13 800 pounds as the unit allowable stress in compression, and this requires an area of $\frac{449\ 500}{13\ 800} = 32.5$ square inches. Since this is considerably less than the actual area of the section, the section will not be changed but will be taken as first assumed.

In order that the section should be safe about both axes, the moment of inertia about the axis perpendicular to the cover-plate should be equal to or greater than that as above computed. By computing the moment of inertia about the axis perpendicular to the cover-plate, it is found to be 3 256.3, which gives a radius of gyration of 8.11; and since both of these are greater than those first computed, it is seen that the section is safer about the axis perpendicular to the cover-plate than it is about an axis perpendicular to the web plates.

There are small stresses in this member due to its own weight and to the fact that the pins are not placed directly upon the neutral axis (see "Strength of Materials," p. 82). These stresses are seldom more than 1 000 pounds per square inch in the extreme fibre; and

since the section has such an excess of area, they will not be computed, as it is evident that there is sufficient strength in the member to withstand them.

The section just designed is that for the top chord having the greatest stress; and since this is the minimum section allowed by the Specifications, it must be used in all the sections of the top chord.

The section as finally designed is:

One cover plate, 23 by $\frac{1}{2}$ inch;
Two webs, 18 by $\frac{1}{2}$ inch;
Four angles, $3\frac{1}{2}$ by $3\frac{1}{2}$ by $\frac{3}{8}$-inch
Four flats, 4 by $\frac{3}{8}$-inch.

A pin $6\frac{1}{4}$ inches in diameter will be used at the point U_1. The stress in the member U_1U_2 is 378 200 pounds, and the bearing area required is $\frac{378\,200}{24\,000} = 15.75$ square inches, or 7.875 for each side. This makes a total required thickness of $\frac{7.875}{6.25} = 1.265$ inches for one side. Since the thickness of the web plate is $\frac{1}{2}$ inch, it will be necessary to provide pin-plates whose total thickness must be $1.265 - 0.5 = 0.765$ inch. Two $\frac{3}{8}$-inch plates will give a thickness of 0.75 inch; and since this is less than the required thickness by an amount not over $2\frac{1}{2}$ per cent, they may be used. The total thickness of the bearing area is now 1.265 inches. The stress transferred to the two $\frac{3}{8}$-inch plates is:

$$s = \frac{2 \times \frac{3}{8}}{1.25} \times 189\,100 = 113\,500 \text{ pounds.}$$

The rivets required to keep the outer plate from shearing off the other are $\frac{113\,500}{2 \times 7\,220} = 8$ shop rivets, and the rivets required to keep both of the $\frac{3}{8}$-inch plates from shearing off the web of the chord section are $\frac{113\,500}{7\,220} = 16$ shop rivets in single shear. The bearing of a $\frac{7}{8}$-inch shop rivet on a $\frac{1}{2}$-inch plate is 10 500 pounds, and therefore the number of rivets required to keep these pin-plates from tearing the rivets out of the $\frac{1}{2}$-inch web plates is $\frac{113\,500}{10\,500} = 11$ shop rivets in bearing. Fig. 185 shows the detail of this end of the top chord section. The pin-plates should extend well back on the member, and at least one pin-

plate should go over the angle, and enough rivets, as computed above, should go through the angles and this pin-plate. Experiments on full-sized bridge members go to show that unless the pin-plates cover the angles and extend well down on the member, the member will fail before the unit-stress reaches that value computed by the formula for compression.

Since the ends of the chord are milled at the splices, and therefore butt up against each other and allow the stress to be transmitted

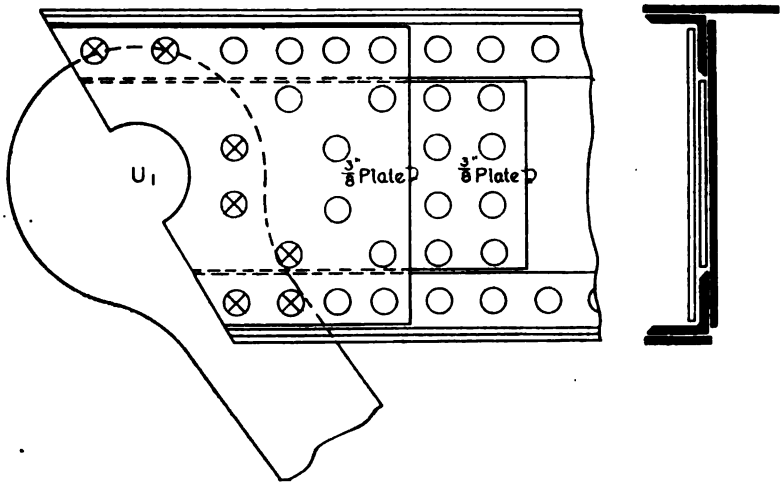


Fig. 185. Detail of Top Chord Section at Point U_1.

directly, only sufficient rivets need be placed in the splice to keep the top chord sections in line (55).

At the point U_2, it is not necessary to put in a pin-plate to take the stress in the upper chord; but it is only necessary to provide a pin-plate to take up the difference in stress between the two chord sections. This difference in stress is equal to the horizontal component of the maximum stress in the member U_2L_3. This is 110 000 pounds, and the area required on each side for bearing is 2.3 square inches; and as a 5-inch pin is used here, the thickness of the bearing area is $\frac{2.3}{5} = 0.46$ inch. As this thickness is less than the thickness of the web plate, no pin-plates will be required.

At the point U_3, a bearing area will be required to withstand the horizontal component of the member U_3L_4. This is 56 300, and the

bearing area required on each side is $\frac{56\,300}{24\,000 \times 2} = 1.18$ inches. The required thickness of the bearing area is $\frac{1.18}{5} = 0.24$ inch, as a 5-inch pin is used here also. As this thickness is less than the thickness of the web plate, no pin-plate will be required.

The under parts of these members must be stiffened by tie or batten plates, and these plates (44) must be equal in length to the distance between rivet lines. This is $19\frac{1}{2}$ inches. They will be made 20 inches long and 23 inches wide. The thickness of these plates (44) must be $\frac{19.5}{50} = 0.39$ inch (say $\frac{7}{16}$ inch). The size of the tie-plates will then be 20 in. by $\frac{7}{16}$ in. by 1 ft. 11 in.

Since the distance between the rivet lines is greater than 15 inches, double latticing must be used (47); and according to Table XXV the lacing must be $\frac{1}{2}$ inch thick; also, according to (45), it must be $2\frac{1}{2}$ inches wide, as the rivets used are $\frac{7}{8}$ inch in diameter. The lattices will then be $2\frac{1}{2}$ by $\frac{1}{2}$-in.

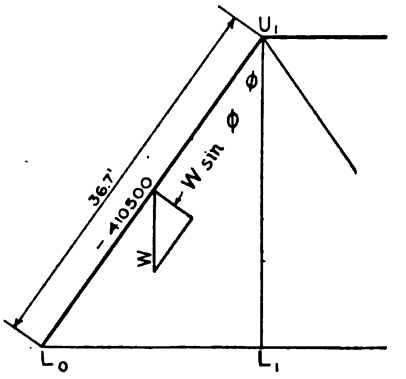


Fig. 186. Calculation of End-Post.

87. **The End-Post.** Since the minimum section as chosen for the top chord is about 50 per cent in excess of that required by the compression formula, it will be assumed to be sufficient for the section of the end-post, and it will now be investigated to see if it is safe.

In addition to the stress due to direct compression, the end-post is stressed by its own weight, by eccentric loading due to the pin being in the center of the web instead of at the center of gravity of the section, and to a bending moment at the place where the portal brace joins it. This is due to the bending action of the wind on the top chord. These different stresses will now be computed; and since the post is in all cases stressed by a combination of bending and compressive stresses, this fact should be considered in the design. In determining the stress in the end-post due to its own weight, the entire

weight must not be used in computing the bending action, but only that component of it which is perpendicular to the end-post. The length of the end-post is readily computed, and is as shown in Fig. 186. The general formula for accurately computing stresses due to bending when the member is also subjected to compression, is:

$$S = -\frac{My_1}{I - \frac{Pl^2}{10E}},$$

in which,

S = Stress in pounds per square inch in the extreme upper fibre of the beam;

M = Exterior moment causing the stress, and is considered positive if it bends the beam downward, and negative if it bends the beam upward;

y_1 = Distance from the neutral axis to the extreme upper fibre;

I = Moment of inertia of the section;

P = Direct compressive stress, in pounds;

l = Total length, in inches;

E = Modulus of elasticity of steel, which is usually taken as 28 000 000 pounds per square inch.

In this case the force causing the bending is that component of the weight perpendicular to the end-post. This is $Wl \sin\phi$, in which W is the weight of the steel in the end-post; and this is computed and is as follows:

Cover-plate	1 435 lbs.
Web plates	2 245 "
Angles	1 250 "
Flats	1 245 "
	6 175 lbs.
Add 25 per cent for details	1 544 "
Total	7 719 lbs.

Substituting in the above formula the various values, there results:

$$S = -\frac{\frac{1}{8} \times 7\,719 \times 36.7 \times 0.572 \times 12 \times 9.55}{3\,185 - \frac{410\,500 \times (36.7 \times 12)^2}{10 \times 28\,000\,000}}$$

= 800 pounds per square inch compression in the upper fibre due to bending.

In the above equation, the stress in the member is 410 500 pounds; the distance y_1 is the distance from the neutral axis to the top of the cover-plate, and the coefficient of elasticity of steel is taken as 28 000 000.

In computing the stress due to the eccentric loading, the moment is equal to the product of the total stress in the member by the distance from the neutral axis to the center of gravity axis causing a negative moment. Substituting in the above formula for combined stresses, there results:

$$S = -\frac{-410\,500 \times 0.2 \times 9.55}{3\,185 - \dfrac{410\,500 \times (36.7 \times 12)^2}{10 \times 28\,000\,000}}$$

$$= 270 \text{ pounds per square inch tension in the upper fibre.}$$

In order to find the compression in the lower fibre, it is only necessary to notice that the stresses are proportional to the distances from the neutral axis. Accordingly (see Fig. 187), the stress in the lower fibre due to the weight is 895 pounds tension, and the stress in the lower fibre due to the eccentric loading is 302 pounds compression.

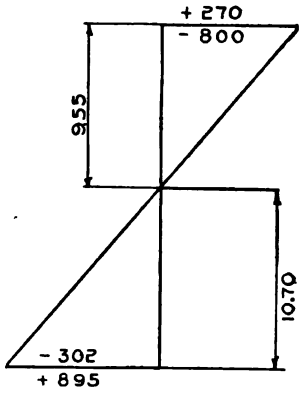


Fig. 187. Calculation of Stress in Chord.

Before computing the stress due to the bending moment caused by the wind on the upper chord, it is necessary to investigate the post to see if it is fixed or hinged at its lower end. This is very important, since, if the post is found to be hinged, the bending moment will be one-half of that which will occur when the post is not hinged.

An end-post is considered hinged when the product of one-half of the total stress *times* the distance between the web plates is greater than the product of the wind load acting at the hip, or joint U_1, *times* the length of the end-post. In this case the first value is $\frac{410\,500}{2} \times 15 = 3\,075\,000$; and the product of the latter (see Article 29) is $12\,600 \times 36.7 \times 12 = 5\,550\,000$. Since the latter is greater than the former, the post is hinged, and the bending moment at the foot of the portal strut, which joins the end-post 28.2 feet from the end, is $6\,300 \times 28.2 \times 12 = 2\,130\,000$ pound-inches. The stress in the extreme fibre due to this bending moment is:

$$S = \frac{2\,130\,000 \times 11.5}{3\,256.3 - \dfrac{410\,500 \times (36.7 \times 12)^2}{10 \times 28\,000\,000}}$$

$= 8\,250$ pounds per square inch tension or compression.

In computing this stress due to the wind moment, care must be taken to take y_1 equal to one-half the width of the cover-plate, and to take the moment of inertia as that about the axis perpendicular to the cover-plate.

In computing the total stress on the extreme fibre, it must be noted that the stresses due to weight and eccentric loading do not stress the same extreme fibres as the stress due to wind, the former stressing the extreme fibres on the top and bottom of the post, while the latter stresses those on the inner and outer sides. The total direct unit-stress is $\frac{410\,500}{49.42} = 8\,310$ pounds per square inch; and this, added to the 8 250 pounds per square inch due to the wind, gives a total of 16 560 pounds per square inch on the extreme fibre only.

The allowable unit-stress is $16\,000 - 70 \times \frac{36.7 \times 12}{8.11} = 12\,200$ pounds per square inch when wind is not taken into account, and (23) is $1\frac{1}{4} \times 12\,200 = 15\,250$ pounds per square inch when the wind is taken into account. The difference between this and the actual stress is $16\,560 - 15\,250 = 1\,310$ pounds per square inch, which shows that the section is not strong enough. The section can be increased by widening the cover-plate or by making the plates thicker; but as this excess is due to wind only, the section being amply sufficient under the other stresses, and is fixed to some extent by the floor-beam connection, no change will be made.

The pin at each end of the end-post will be the same—namely, $6\frac{1}{4}$ inches in diameter—and therefore the pin-plates will be the same at each end. The total stress in the post is 410 500 pounds, which makes a required bearing area of $\frac{410\,500}{24\,000} = 17.2$ square inches for both sides, or 8.6 square inches for one side, and the total required thickness of $\frac{8.6}{6.25} = 1.375$ square inches for one side. Since the thickness of the web plates is $\frac{1}{2}$ inch, this leaves a remainder of $1.375 - 0.5 = 0.875$ inch for the thickness of the pin-plates. One plate $\frac{3}{8}$ inch thick and one plate $\frac{1}{2}$ inch thick will be used.

The proportion of the total stress which is taken by the $\frac{3}{8}$-inch

plate is $\frac{0.375}{1.375} \times \frac{410\,500}{2} = 56\,000$ pounds; and that taken by the ½-inch plate is $\frac{0.50}{1.375} \times 205\,250 = 74\,600$ pounds. The number of rivets required to transfer the stress from the ⅜-inch plate to the ½-inch plate is $\frac{56\,000}{7\,220} = 8$ shop rivets in single shear; and the number of rivets required to transfer the stress from both pin-plates to the web is $\frac{56\,000 + 74\,600}{7\,220} = 18$ shop rivets in single shear. As in the case of the top chord, one pin-plate should extend over the angle, and the number of rivets required in that pin-plate should go through the pin-plate and the angles (see Fig. 188). The ⅜-in. hinge plate is used for erection purposes, and is not considered as a pin-plate. It is omitted at L_0.

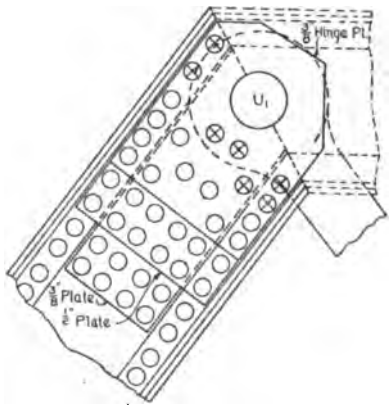


Fig. 188. Plates and Riveting at Upper End of End-Post.

Since this section is the same as that of the top chord, the tie-plates and the lattice bars must be the same size.

88. **The Pins.** The design of the pins requires a simple but quite lengthy computation. Simple Pratt railroad trusses for single-track bridges usually have the same arrangement of tension and compression members; that is, the same tension members occupy relatively the same positions with respect to the compression members. Also, while theoretically a different sized pin will be required at every joint, it is not customary to make them so. In practice the pins at the joints U_1 and L_0 are made of the same diameter, and those at the remainder of the joints are also made in diameter equal to each other but different from those at U_1 and L_0, the pins at U_1 and L_0 usually being larger in diameter. On account of the above conditions and facts, it is unnecessary to design the pins in spans under 200 feet, since usually they are the same for any given span and loading. Table XXVI gives the diameters of pins for spans of 100 up to 200 feet for loading E 50.

TABLE XXVI

Pins for Single-Track Bridges

Loading E 50

Span	Diameter of Pin	
	U_1 and L_0	All Others
100 feet	4½ inches	4 inches
125 "	5½ "	5 "
150 "	6½ "	5¼ "
175 "	6¾ "	5¾ "
200 "	7 "	6 "

For E 40 loading, decrease the above values by ¼ inch; for E 30 loading, decrease them by ⅜ inch. The diameter of pins for spans not given in the table can be interpolated from the given values. No pin should be less than 3½ inches in diameter.

The span of this bridge is 147 (say 150) feet, and the diameter of the pins at U_1 and L_0 is $6\frac{1}{2} - \frac{1}{4} = 6\frac{1}{4}$ inches; and the diameter of the pins at the other panel points is $5\frac{1}{4} - \frac{1}{4} = 5$ inches. It should be noted that no pin is required at point L_1, as the two members which join here are built-up members and are riveted together.

The above table is for single-track bridges only. The diameters of pins for double-track bridges are given in Table XXVII. These values are for E 50 loading; and for E 40 and E 30 loading, deductions must be made as required in the case of Table XXVI.

TABLE XXVII

Pins for Double-Track Bridges

Loading E 50

Span	Diameter of Pin	
	U_1 and L_0	All Others
100 feet	6 inches	5¼ inches
125 "	8 "	6½ "
150 "	9 "	7½ "
175 "	9¼ "	8¼ "
200 "	9½ "	8½ "

No pin in a double-track bridge should be less than 4½ inches in diameter.

Pins for highway bridges are usually much less in diameter than those for railway bridges, except in the case of first-class trusses for heavy interurban traffic or for city bridges carrying paved streets,

where they should be taken equal to those given for E 30 loading. Table XXVIII gives the diameters of pins for different length spans of simple highway bridges designed for 16-ton road-rollers or farm wagons and 100 pounds per square foot of roadway.

TABLE XXVIII

Pins for Country Highway Bridges

Span	Diameter of Pin	
	U_1 and Lower Chord	Upper Chord
50 feet	2½ inches	2 inches
100 "	3 "	2½ "
150 "	3¼ "	2¾ "
200 "	4 "	3 "

89. **The Portal.** In order to have a clearance of 21 feet (2) above the top of rail, it is necessary that the portal be as shown in Fig. 189. The stresses are found by methods of Article 54, Part I, the wind load being computed according to (10). It must be remembered that the column is hinged.

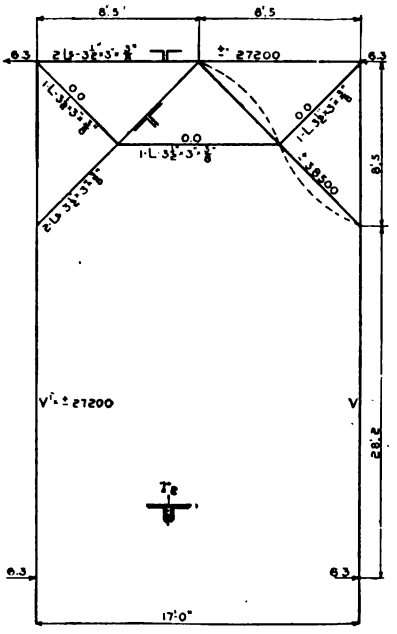


Fig. 189. Portal Dimension and Stress Diagram.

In case the members of the portal braces bend about one axis, their length will be equal to the distance from one end to the other. In case they bend about the other axis as indicated by the broken line in Fig. 189, their length will be one-half of what it was in the first case.

The portal struts or diagonals will be designed first. Their length is $8.5 \times 1.414 =$ 12 feet, or 144 inches. This is the total length. Although the Specifications do not mention it, the ratio of the length to the radius of gyration should not exceed 120. This means that the radius of gyration in this

case should be greater than $\frac{144}{120} = 1.2$. The section of the strut will be composed of two angles placed back to back.

Two angles $3\frac{1}{2}$ by 3 by $\frac{3}{8}$-inch, with an area of 4.6 square inches and r_2 equal to 1.72—see Carnegie Handbook, p. 146, and (72)—will be assumed to be sufficient to take the stress, and they must now be examined to see if the assumption is correct.

The allowable unit-stress (23) is 25 per cent greater than in the case of live or dead loads. This makes the unit-stress as computed from the formula:

$$P = \left(16\,000 - 70 \times \frac{144}{1.72}\right) 1\tfrac{1}{4} = 12\,680 \text{ pounds per square inch.}$$

The required area is $\frac{38\,500}{12\,680} = 3.05$ square inches; and since this is less than the given area, the angle will be amply sufficient. The required area is over one square inch less than the given area, but this angle must be used, since it is the smallest angle allowed by the Specifications. Note that unequal legged angles should be used, as this will make the radius of gyration about one axis larger than about the other; and this will prove economical, since, when one axis is considered, the length of the member is greater than when the other is considered.

The above angle should also be examined for tension, it being considered that one rivet-hole is taken out of the section of each angle. The net section of the two angles will now be $4.60 - 2\,(\frac{7}{8} + \frac{1}{8}) \times \frac{3}{8}$ $= 3.85$ square inches; and the area required for tension is $\frac{38\,500}{16\,000 \times 1.25}$ $= 1.93$ square inches, which shows that the angle is amply sufficient. It should be noted that these Specifications do not require that only one leg of the angle shall be efficient unless both legs are connected. In case this strut had been designed according to Cooper's Specifications, two angles 5 by 3 by $\frac{1}{2}$-inch would have been required, and the 5-inch leg would have been placed vertically and the angle connected by this leg alone. While it is not within the province of this work to discuss the question of connecting angles by one or by both legs, yet it might be said that tests made on angles connected with one leg only, seem to indicate that the ultimate strength in tension is about 60 per cent of that obtained from the same angle when tested with both legs connected.

While according to (20) the alternate strains in the wind bracing do not have to be considered, since they do not occur very closely together, yet in framing the connections it is required that the sum of both positive and negative stresses shall be added. In this case the stress for which the connections must be designed is $2 \times 38\,500 = 77\,000$. It must be remembered that in this case also, the unit-stresses are increased 25 per cent over those allowed for live and dead loads.

The number of rivets required in the end connections will be governed by bearing in the connection plates, and these plates are usually made $\frac{3}{8}$-inch thick. The number of rivets required is $\frac{77\,000}{7\,880 \times 1\frac{1}{4}} = 8$ shop rivets, or $\frac{77\,000}{6\,560 \times 1\frac{1}{4}} = 10$ field rivets.

The portal bracing is riveted up in the shop and brought to the bridge site, where it is connected to the trusses by field-riveted connections at its end. Therefore the end of the portal struts which connect with the top piece will have 8 shop rivets, and the other end which connects with the end-post will have 10 field rivets. Since the angles are small, all the above rivets must go in one line, and this will cause the connection plate to be quite large. It will probably be better to connect both legs of the angle by means of clip angles and thus reduce the size of the connection plates.

The top part of the portal bracing will consist of two angles. Two angles $3\frac{1}{2}$ by 3 by $\frac{3}{8}$-inch will be assumed and examined to determine if the area is sufficient. The length of this strut is the distance center to center of trusses, and is equal to $17 \times 12 = 204$ inches. The least radius of gyration is therefore $\frac{204}{120} = 1.70$. The radius of gyration of the two angles assumed is 1.72 when referred to an axis parallel to the shorter leg when the two angles are placed back to back and one-half inch apart. The unit-stress is now computed:

$$P = \left(16\,000 - 70 \times \frac{204}{1.72}\right) 1\tfrac{1}{4} = 9\,625 \text{ pounds per square inch.}$$

The required area is $\frac{27\,200}{9\,625} = 2.825$ square inches. This is considerably less than the area given by the two angles; but as these are the minimum angles allowable, they must be used. Since the stress in this case is less than in the previous case, and since the angles

used are the same, it is evident that these angles are safe in tension.

The number of rivets is determined by the bearing in the $\frac{3}{8}$-inch connection plates, and is:

$$\frac{2 \times 27\,200}{7\,880 \times 1.25} = \text{6 shop rivets, and}$$

$$\frac{2 \times 27\,200}{6\,560 \times 1.25} = \text{10 field rivets.}$$

As in the case of the lateral strut, this member should be connected by both legs of the angle in order to reduce the size of the connection plates. Fig. 190 gives the details of the portal bracing and its method of connection to the end-post. The full circles represent shop rivets, and the blackened circles represent field rivets. Some engineers connect the portal bracing to the top cover-plate of the end-post. This produces an excessive eccentricity in the end-post and is bad practice.

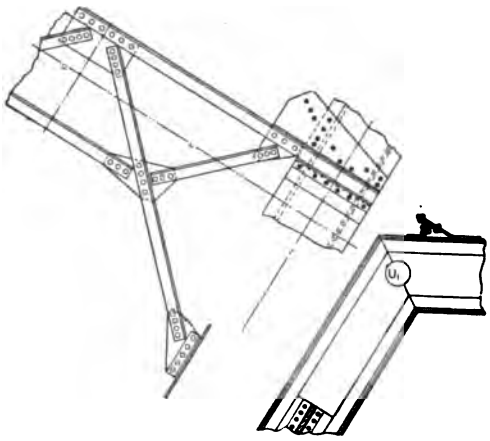


Fig. 190. Details of Portal Bracing and Connection to End-Post.

Those members of the portal bracing which do not take any stress will be made of single angles, and the size of these angles will be taken $3\frac{1}{2}$ by 3 by $\frac{3}{8}$-inch.

90. **The Transverse Bracing.** This bracing will be the same general style as the portal bracing, except that the top member will consist of two angles placed at a distance apart equal to the depth of the top chord, and these angles will be joined together by lacing. As in the case of portal bracing, those members which do not take stress will be made of one angle $3\frac{1}{2}$ by 3 by $\frac{3}{8}$-inch.

The general outline is shown in Fig. 191, and the stresses are computed from (10) and by the methods of Article 54, Part I. In designing this top member, the top angle only is supposed to take the stress. The length in this case is 204 inches. Two $3\frac{1}{2}$ by 3 by $\frac{3}{8}$-inch angles will be assumed as sufficient and will be examined. These angles

give a total area of 4.60 square inches. In examining these it will be found that they are amply sufficient, in fact so much so that it will be better to see if one single angle at the top will not be better. According to the length, the smallest radius of gyration which can be used is 1.7. In looking over the tables of angles, it is seen that the

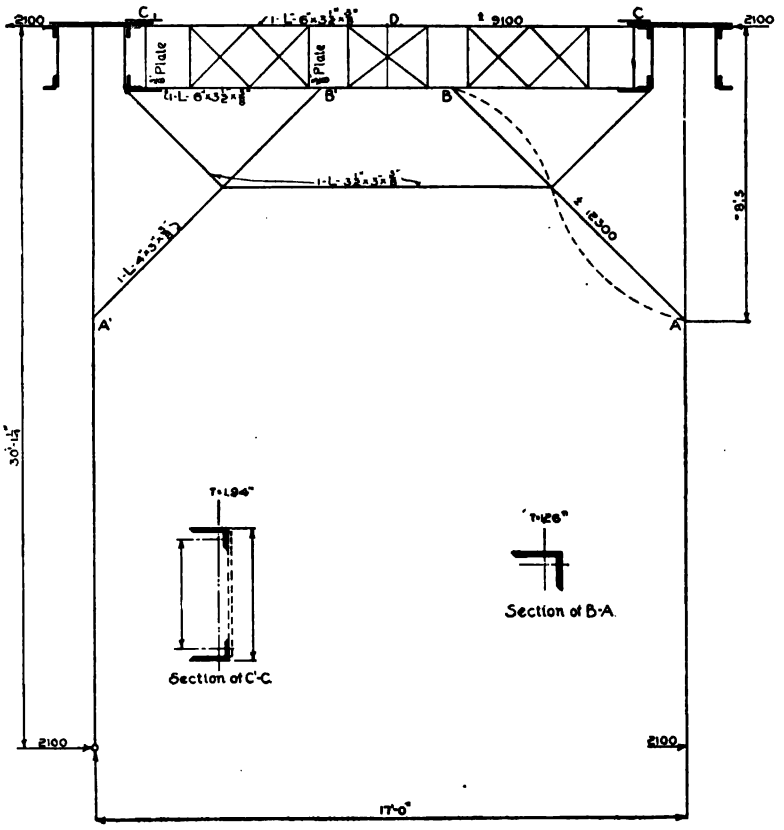


Fig. 191. General Outline of Transverse Bracing.

first angle to fulfil this condition is a 6 by $3\frac{1}{2}$ by $\frac{3}{8}$-inch, and it has a radius of gyration of 1.94. The allowable unit-stress is computed as follows:

$$P = \left(16\,000 - 70 \times \frac{204}{1.94}\right)1\tfrac{1}{4} = 10\,780 \text{ pounds per square inch;}$$

and the required area is $\frac{9\,100}{10\,780} = 0.85$ square inch. This is con-

siderably smaller than the area of the angle, which is 3.97 square inches; but since this is the smallest possible angle which will fulfil the conditions of the Specifications, and since it is much smaller than the two angles as first assumed, it will be used. Fig. 192 gives a cross-section of this member. Since this angle is joined to the cover-plate by one leg, the joints will be weak in single shear, and the number of rivets required will be:

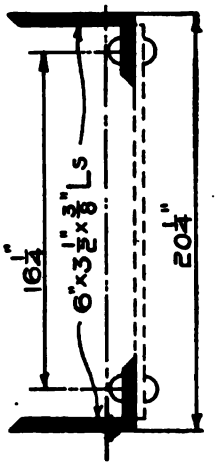


Fig. 192. Cross-Section of Top Member of Transverse Bracing.

$$\frac{2 \times 9\,100}{7\,220 \times 1\frac{1}{4}} = 2 \text{ shop rivets, or}$$

$$\frac{2 \times 9\,100}{6\,013 \times 1\frac{1}{4}} = 3 \text{ field rivets.}$$

According to (45), the width of the latticing must be $2\frac{1}{2}$ inches; and according to Table XXV, the thickness must be $\frac{7}{16}$ inch, the distance c being 1 foot 11 inches.

The length of the knee-bracing is 144 inches; but on account of the small stress, one angle will be used. One 4 by 3 by $\frac{3}{8}$-inch angle, with an area of 2.48 square inches and a radius of gyration 1.26, will be assumed as sufficient. The radius of gyration is greater than the minimum allowable, which is 1.2. The allowable unit-stress is:

$$P = \left(16\,000 - 70 \times \frac{144}{1.26}\right) 1\tfrac{1}{4} = 10\,000 \text{ pounds per square inch.}$$

The required area is $\frac{12\,300}{10\,000} = 1.23$ square inches. The required area is much less than the given area; but this angle must be used, since it is the only one allowed on account of its radius of gyration. Two of the minimum sized angles might have been used; but their total area, 4.60 square inches, is much in excess of that of the angle used.

This angle must be examined for tension. The net area is $2.48 - (\frac{7}{8} + \frac{1}{8}) \times \frac{3}{8} = 2.1$ square inches. The required net area in tension is $\frac{12\,300}{16\,000 \times 1\frac{1}{4}} = 0.615$ square inch, which shows this angle to be amply sufficient.

The number of rivets required will be governed by the shear, since the angle is connected by one leg only; and it is:

$$\frac{2 \times 12\,300}{7\,220 \times 1\frac{1}{4}} = 3 \text{ shop rivets, and}$$

$$\frac{2 \times 12\,300}{6\,013 \times 1\frac{1}{4}} = 4 \text{ field rivets.}$$

91. **The Lateral Systems.** The stresses in these systems must be computed according to (10) and Article 54, Part I. They are given on the stress sheet, Plate III (p. 251). Since according to (68) these members must be constructed of rigid shapes, it is customary, in computing the stresses, to assume that one-half the shear is taken by each of the diagonals in any given panel; that is, one diagonal is in tension, and the other diagonal is in compression. The stresses given on the stress sheet are computed by making this assumption. Also, since both diagonals in each panel are considered as acting at the same time, the stresses in all the verticals are zero.

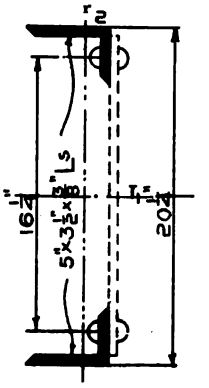


Fig. 193. Section of Upper Lateral Member.

The section of the upper lateral members will be made up of two angles placed apart a distance equal to the depth of the top chord. Fig. 193 shows the section. The radius of gyration about the axis parallel to the long leg will be considerably larger than that about an axis parallel to the shorter leg. In fact, it is so much greater that the strut will not need to be examined with respect to this axis. The diagram of the first panel is given in Fig. 194. The radius of gyration is to be taken about the horizontal axis if the entire length is to be taken; and the radius of gyration is to be taken about the vertical axis if one-half the length is taken, in which case it will bend as shown by the broken line in Fig. 194. The members are designed for the latter conditions only, since they are amply safe in regard to the first condition if they satisfy the latter. The length in this latter condition is 13.5 feet, which requires a radius of gyration not less than $\frac{13.5 \times 12}{120} = 1.35$.

Two angles 5 by 3 by ⅜-inch, with a total area of 5.72 square

inches and a radius of gyration equal to 1.61, will be assumed and investigated to determine if they are sufficient.

The unit-stress is computed to be $P = (16\,000 - 70 \times \frac{13.5 \times 12}{1.61}) \times 1\frac{1}{4} = 11\,000$ pounds per square inch, and the required area is $\frac{6\,700}{11\,000} = 0.61$ square inch. The required area is very much less than the given area; but the angle chosen must be used, since this is the smallest one which conforms to the requirements of the Specifications.

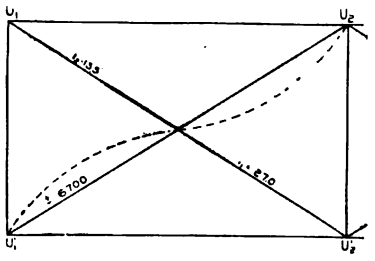


Fig. 194. Outline Diagram of First Panel in Upper Lateral System.

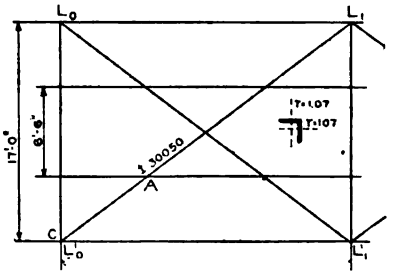


Fig. 195. Outline Diagram of First Panel in Lower Lateral System.

The width of the lattices (46) must be $2\frac{1}{2}$ inches; and according to Table XXV, the thickness must be $\frac{7}{16}$ inch, the distance c being 23 inches.

Single shear governs the number of rivets required. In accordance with (20) and (23), their number is $\frac{2 \times 6\,700}{6\,013 \times 1\frac{1}{4}} = 2$ rivets. Field rivets 3 in number are used in all places, since the lateral system is riveted up after the trusses are swung into place.

Since this is the minimum sized angle which will give a radius of gyration greater than 1.35, it must be used in the remainder of the panels of the top chord. Four angles of the minimum size might have been used, and would have been satisfactory, except that the area would have been excessive.

The stresses in the lower lateral system are computed according to (10), a similar assumption to that for the upper lateral system being made—namely, that both diagonals in each panel are stressed at the same time, one taking tension and the other taking compression. Fig. 195 shows the first panel of the lower lateral system. These

diagonals are connected to the stringers wherever they cross them, and also to each other where they cross in the center. This reduces the length which must be used in computing the cross-section of the member. In this case it is the distance C–A, and is equal to 90 inches.

Since the angle is free to move about either axis, angles with even legs should preferably be employed, since this will give greater economy. The radius of gyration must be greater than $\frac{90}{120} = 0.75$.

One angle $3\frac{1}{2}$ by $3\frac{1}{2}$ by $\frac{3}{8}$-inch, with an area of 2.48 square inches and a radius of gyration of 1.07, will be assumed and investigated. The allowable unit-stress is $P = (16\,000 - 70 \times \frac{90}{1.07})\ 1\frac{1}{4} = 12\,650$ pounds per square inch, and the required area is $\frac{30\,500}{12\,650} = 2.38$ square inches. This is nearly equal to the given area, and therefore the angle chosen will be taken for the section.

This angle must now be investigated for tension, one rivet-hole being taken out of the section. The net area is $2.48 - (\frac{7}{8} + \frac{1}{8})\ \frac{3}{8} = 2.10$ square inches. The required net area is $\frac{30\,500}{16\,000 \times 1\frac{1}{4}} = 1.53$ square inches, which shows the angle to be sufficiently strong.

Single shear determines the number of rivets to be required. These are:

$$\frac{2 \times 30\,500}{6\,013 \times 1\frac{1}{4}} = 8 \text{ field rivets.}$$

All rivets in the lower lateral system are field rivets, since this system also must be riveted up in the field after the trusses are swung into place.

The total stress in the second panel is 21 500 pounds, and a $3\frac{1}{2}$ by 3 by $\frac{3}{8}$-inch angle, with an area of 2.30 square inches and a least radius of gyration of 0.90, will be assumed and examined. The allowable unit-stress in compression is $1\frac{1}{4}\ (16\,000 - 70 \times \frac{90}{0.90}) = 11\,250$ pounds per square inch, and the required area is $\frac{21\,500}{11\,250} = 1.91$ square inches. Since this is less than the given area, and since the size of the angle (72) is the smallest allowable, this angle must be used.

It is required that this member shall have a net area of $\frac{21\,500}{16\,000 \times 1\frac{1}{4}} = 1.08$ square inches in tension. The net area of the angle, one rivet-hole being taken out, is 1.92 square inches, which shows the angle to be safe in tension.

The number of rivets required is determined by single shear,

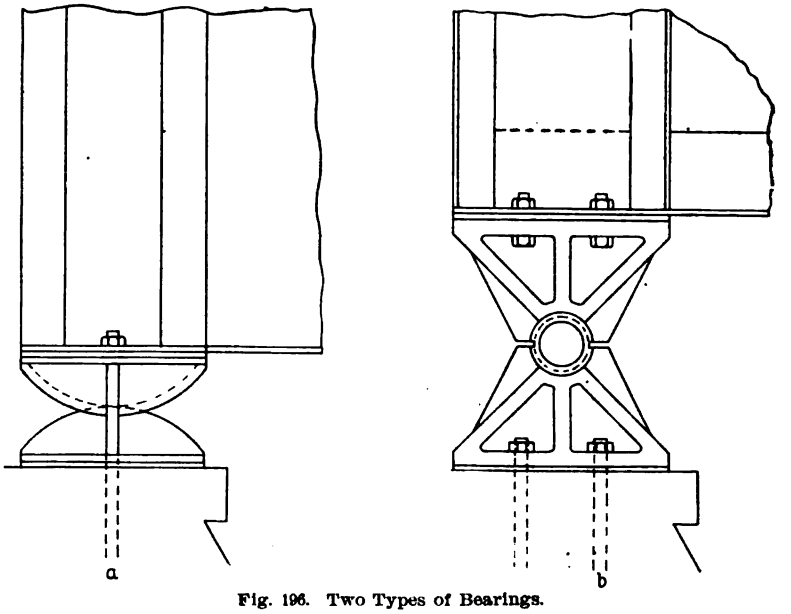


Fig. 196. Two Types of Bearings.

since they tend to shear off between the member itself and the connecting plates. The number required is $\frac{2 \times 21\,500}{6\,013 \times 1\frac{1}{4}} = 6$ field rivets.

Since the above angle is the smallest that can be used, and since the remaining angles of the panel of the lateral bracing have smaller stresses than the one just designed, it is evident that this size angle must be used in all panels of the lower lateral system other than the first.

92. **The Shoes and Roller Nests.** For bridges of short spans and for plate-girders whose spans require rocker bearings to be provided (80), several different classes of bearings are in use. Two such bearings are shown in Fig. 196 (*a* and *b*). The type illustrated by *a* is seldom used on any spans except plate-girders. That shown in *b*

may be used on either plate-girders or small truss spans; it is the invention of Mr. F. E. Schall, Bridge Engineer of the Lehigh Valley Railroad, who uses it on plate-girders. It has given very great satisfaction; and for simplicity of design and also for economy it is to be recommended. Some railroads have used a bearing which consisted of a lens-shaped disc of phosphor-bronze, the faces of which fitted into corresponding indentations in both the masonry and the bearing plates. One advantage of this bearing is that it allows movements due to the deflection of the girder, and also lateral deflection of the floor-beam. It is claimed to have given satisfaction.

A bearing which is used on both short-span and long-span bridges

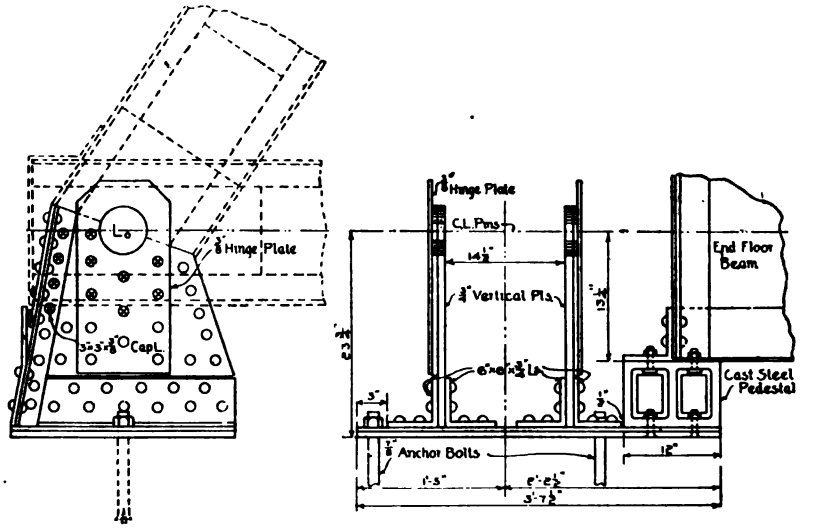


Fig. 197. Bearing Adapted to Bridges of Both Short and Long Span.

is shown in Fig. 197. This class of bearing will be used. The end reaction of the bridge proper is equal to the vertical component of the stress in the end-post, and is $\frac{30.1}{36.7} \times 410\,500 = 336\,500$ pounds, which requires a bearing area (19) on the masonry, of $\frac{336\,500}{600} = 561$ square inches. According to the table on page 193, the masonry plate will be 28 inches long.

The total bearing area for one of the vertical plates is:

$$\frac{336\,500}{2 \times 24\,000} = 7.0 \text{ square inches;}$$

and the total required thickness is:

$$\frac{7.0}{6.25} = 1.12 \text{ inches},$$

a $6\frac{1}{4}$-inch pin being used at L_0. Since the vertical plates will be made $\frac{3}{4}$ inch thick, this leaves a remainder of $\frac{3}{8}$ inch to be made up of pin-plates.

The amount of stress which is carried by the $\frac{3}{8}$-inch pin-plate is $\frac{0.375}{1.125} \times \frac{336\,500}{2} = 56\,100$ pounds. These plates will tend to shear off the rivets at a plane between the plates, and therefore $\frac{56\,100}{7\,220} = 8$ shop rivets will be required to fasten them to the vertical plate.

Since the length of the masonry plate is 28 inches, and the total area required is 561 square inches, the required width is $\frac{561}{28} = 20$ inches. The actual width will be greater than this, since it must be sufficient to allow for the connecting angles and also for the bearings of the end floor-beam. The connecting angles should be $\frac{3}{4}$ inch thick, and should not be less than 6 by 6 inches; and the plates to which they are connected should not be less than $\frac{3}{4}$ inch in thickness, and likewise they should not be greater, on account of the punching. The bottom plate should extend outward about 3 inches, in order to allow sufficient room for the anchor bolts, which should be $\frac{7}{8}$ inch in diameter and should extend into the masonry at least 8 inches.

In addition to the reaction of the bridge proper, the masonry plate must be of sufficient area to give bearing for the end reaction of the end floor-beam. The maximum end reaction (see Article 83, p. 197) is 104 740 pounds. The bearing area required on the masonry is $\frac{104\,740}{600} = 175$ square inches; and assuming that the base of the bearing will be 12 inches long (see Fig. 197), the required length will be 14.6 inches. Usually, however, the bearing is extended the entire length of the masonry plate, which is 28 inches in this case.

The distance from the center of the pin to the top of the masonry will be the same for both the fixed and the roller end. This distance should be such that the angles of the shoe will clear the bottom chord member and allow the floor-beam to rest upon the plate as shown. Since the first section of the bottom chord is $18\frac{1}{2}$ inches deep, the top of the angles of the two must be at least $9\frac{1}{4}$ inches from the center line

of pins. This requires that the distance from the center line of the pin to the base of the angle shall be at least $(9\frac{1}{4} + 6) = 15\frac{1}{4}$ inches, or more.

The tops of all floor-beams are at the same height, and the bottoms of the intermediate floor-beams must be on a level with the bottom of the first section of the lower chord (see Fig. 174). This requires that the bottom of the intermediate floor-beams shall be $9\frac{1}{4}$ inches below the center line of pins, and this brings the top of the floor-beams $(48\frac{1}{4} - 9\frac{1}{4}) = 39$ inches above the center line of the pins. Since the end floor-beam is $52\frac{1}{4}$ inches deep, back to back of angles, the

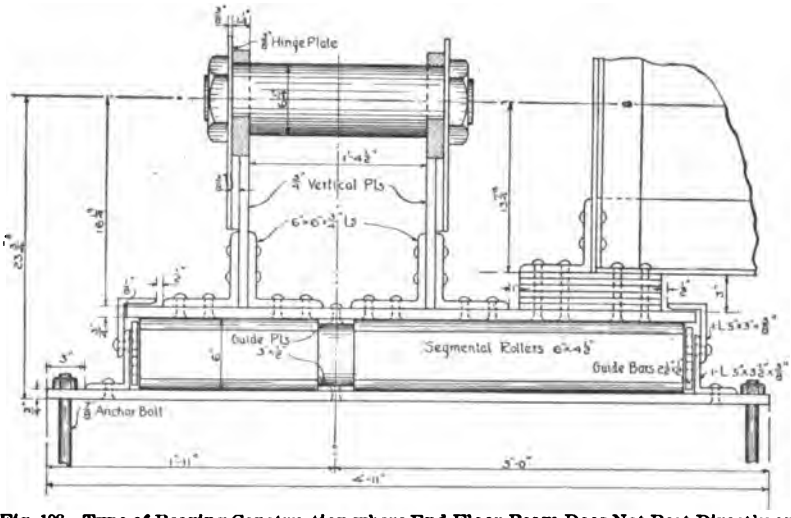


Fig. 198. Type of Bearing Construction where End Floor-Beam Does Not Rest Directly on Bearing or Masonry Plate. Grillage of Iron Bars Used Instead of Cast-Steel Pedestal.

lower flange will be $(52\frac{1}{4} - 39) = 13\frac{1}{4}$ inches below the center line of pins. In case the end floor-beam does not rest directly upon the bearing plate or the masonry plate, the intervening space is filled out with a grillage of iron bars or a cast-steel pedestal, as shown in Figs. 197 and 198.

The small plates upon the side of the shoe, going entirely around the pin, are called the *shoe hinge-plates*. These do not take any stress, and require only sufficient rivets to hold them in position. They are used during erection to keep the end-post in line; and after erection their function is to keep the end-post on the shoe, and to prevent it from having any upward motion due to the vibration of the structure.

The rivets through the vertical legs of the shoe angles are in double bearing in the $\frac{3}{4}$-inch angles, in single bearing in the vertical plate, and in double shear. A rivet in double shear has a less value than in bearing in the plates. This value is 14 440 pounds, and therefore the number of shop rivets required through the vertical legs of the angles is:

$$\frac{336\,500}{2 \times 14\,440} = 12 \text{ rivets.}$$

The rivets which go through the horizontal leg of the angle and through the cap plate and cap angles, do not take stress. The number of rivets put in is that demanded by the detailing, the rivets in the horizontal legs of the angles usually staggering with those in the vertical legs. The cap plate tends to keep the vertical plates in line, and to keep out the dust and dirt and other deteriorating influences of the elements.

Wherever the rivet-heads tend to interfere with other members or project beyond surfaces which are required to be flat—as, for example, the bottom of the masonry or bearing plates—they must be countersunk (see Carnegie Handbook, p. 191, and "Steel Construction," Part III, p. 192).

The space for the anchor bolts, that for the connection angles, and that for the bearing of the end floor-beam, require that the total width of the masonry plate for the fixed end shall be $2 \times \frac{3}{4} + 14\frac{1}{2} + 2 \times 6 + \frac{1}{2} + 3 + 12 = 3$ feet $7\frac{1}{2}$ inches.

The design of the roller end requires that the length of the masonry bearing, the size of the vertical plates and angles, and also the number of rivets shall be the same as that for the fixed end. The width of the masonry plate is determined by the length of the rollers and their connections at the end.

The rollers (60) are required to be 6 inches in diameter, and the unit-stress (19) per linear inch is $6 \times 600 = 3\,600$ pounds, which requires:

$$\frac{336\,500}{3\,600} = 93.5 \text{ linear inches.}$$

This is for the reaction of the bridge alone; and in addition to this, there are required for the floor-beam reaction:

$$\frac{104\,740}{3\,600} = 29.0 \text{ linear inches.}$$

The total number of linear inches is $93.5 + 29.0 = 122.5$; and if 5 rollers are used, they must be at least $\frac{122.5}{5} = 24.5$ inches long. The masonry plate is only 28 inches long, and therefore cylindrical rollers cannot be used, since they would occupy a space 30 inches or over. Segmental rollers (see Fig. 199) must be used.

The determination of the sizes of the angles which go at the end of the rollers, and also of the guide-plates, is a matter of judgment and experience. Those sizes indicated in Fig. 198, represent good engineering practice, and will be used.

The distance from the center line of pins to the top of the masonry can now be determined, and is $16\frac{1}{4} + \frac{3}{4} + 6 + \frac{3}{4} = 23\frac{3}{4}$ inches.

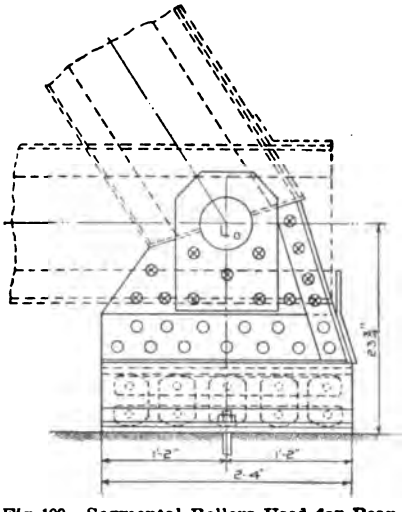


Fig. 199. Segmental Rollers Used for Bearings in Space under 30 Inches.

On account of putting in sufficient connections and angles as shown in Fig. 198, the masonry plate must be considerably wider than that theoretically determined. According to Fig. 198, the total width must be as follows, and the width should be computed in two parts, as the plate is not symmetrical about the center line of the truss:

From center line to outer edge:

$$\frac{14\frac{1}{2}}{2} + \frac{3}{4} + 6 + \frac{1}{2} + (3 - \frac{3}{8} = 2\frac{5}{8}) + (3\frac{1}{2} - \frac{3}{8} = 3\frac{1}{8}) + 3$$
$$= 1 \text{ ft. } 11\tfrac{1}{4} \text{ in., (say, 1 ft. 11 in.).}$$

From center line to inner edge:

$$\frac{14\frac{1}{2}}{2} + \frac{3}{4} + 6 + 1 + 12 + \frac{1}{2} + 2\frac{5}{8} + 3\frac{1}{8} + 3 = 3 \text{ ft. } 0\tfrac{1}{4} \text{ in., say, 3 ft. 0 in.}$$

Total width 4 ft. 11 in.

Allowing guide-plates and guide-bars of dimensions as shown in Fig. 198, and assuming $\frac{1}{8}$ inch as clearance at the ends, the total length of the rollers is:

$$(4 \text{ ft. } 11 \text{ in.}) - 2\,(3 + 3\tfrac{1}{2} + \tfrac{1}{8} + \tfrac{1}{8} + \tfrac{1}{2}) = 44.5 \text{ inches.}$$

This shows them to be amply long enough, as only 22 inches is

theoretically required. Here, as in most cases for single-track spans up to 200 feet in length, the width of the masonry plate is determined by the detail and not by the unit bearing stress.

The guide-plates are small bars riveted to the top of the bottom plates, and serve to keep the rollers in line. The guide-bars are connected to rollers at their ends, and serve to keep the rollers equidistant, therefore causing them to roll easier and keeping them from becoming worn by contact with each other.

The expansion (57) must be allowed for at the rate of $\frac{1}{8}$ inch for every 10 feet in length of span. This makes a total allowed for temperature of expansion of $\frac{147}{10} \times \frac{1}{8} = 1\frac{3}{4}$ (say 2) inches. No slotted holes are to be provided for the anchor bolts, since they do not go

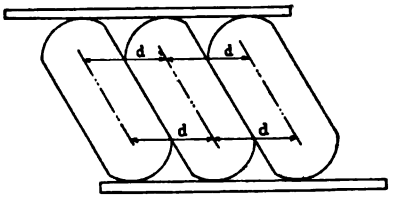


Fig. 200. Binding of Insufficiently Spaced Segmental Rollers.

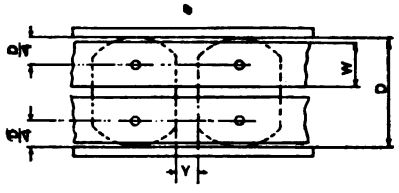


Fig. 201. Computation of Spacing for Segmental Rollers.

through that part of the bridge which slides. The shoe slides over the rollers, and is kept in place by the angles at the end, which are riveted to the masonry plate (see Fig. 198).

Unless sufficient room is allowed between the segmental rollers, they will tend to bind when the bridge has reached the extreme position for expansion or contraction (see Fig. 200). This distance can be computed from proportions as indicated in Fig. 201, and from the following formulæ*:

$$y = \frac{1}{\cos \phi};$$

$$\phi = \frac{e}{4} \times \frac{360}{3.14\ D},$$

in which e is the amount allowed for the change of temperature, and D is the diameter of the rollers, both being taken in inches. The angle ϕ is in degrees. In the present case, e is 2 inches; D is 6 inches;

* Derived by Prof. Frank B. McKibben of Lehigh University, and published in *Engineering News*, December, 1896.

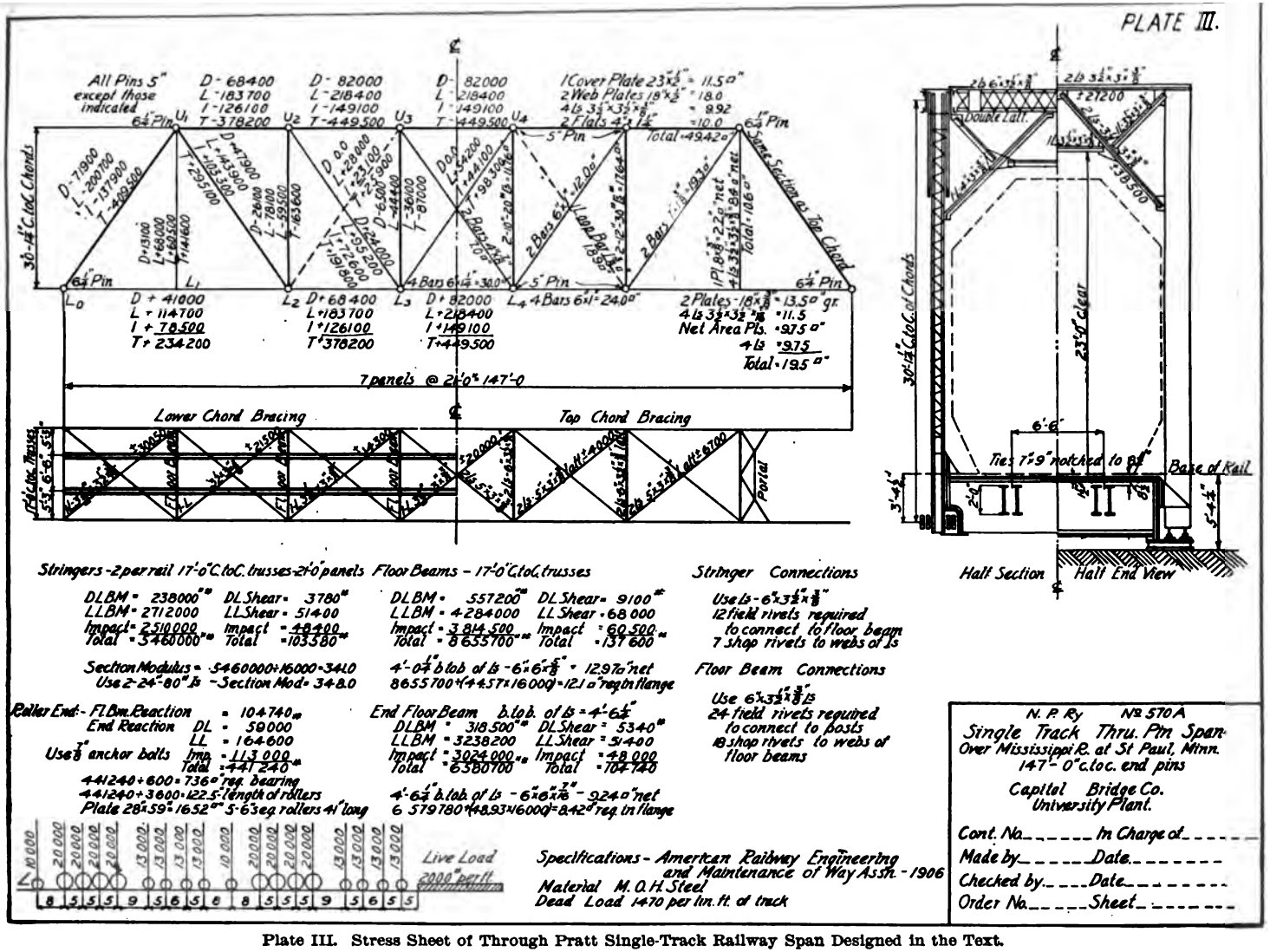


Plate III. Stress Sheet of Through Pratt Single-Track Railway Span Designed in the Text.

and ϕ, computed from the above formula, is 9° 30′. Substituting in the equation giving the value for y, there is obtained 1.02 inches (say $1\frac{1}{4}$ inches) for the distance between rollers. Rollers must not be less in thickness than the total expansion allowed for temperature.

Since there are 5 rollers, there are 4 spaces between them. Also, since the rollers must occupy a space of 28 inches, the length of the masonry plate, each roller must be:

$$\frac{28 - 4 \times 1\frac{1}{4}}{5} = 4.6 \text{ inches (say } 4\tfrac{1}{2} \text{ inches) wide.}$$

The width of the guide-bars must be such as to allow freedom

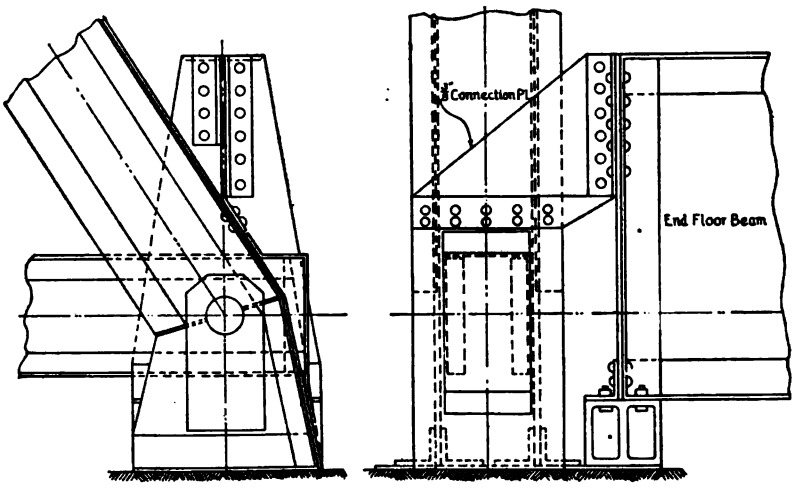


Fig. 202. Details of End Floor-Beam Connections.

of motion for the rollers. The maximum width allowable is given by the formula:*

$$W = \frac{D}{2} \cos \phi,$$

in which ϕ and D are indicated above. This requires the bar to be.

$$W = \frac{6}{2} \times 0.985 = 2.96 \text{ (say } 2\tfrac{1}{2}\text{) inches wide.}$$

93. **The Stress Sheet.** Plate III shows the stress sheet of the bridge which has been designed in the preceding articles. This sheet represents the best current practice among the larger bridge

* Derived by Prof. Frank B. McKibben of Lehigh University, and published in *Engineering News*, December, 1896.

corporations. It will be noted that very few details are given upon the sheet; also that few rivets are noted, and that sketches showing the manner in which the parts go together are entirely wanting. The shears and moments for the stringers and floor-beams, as well as the reactions and the number of rollers required, are given. This is to save the draftsman the trouble of recomputing values which have necessarily been determined by the designer.

The details of the various members, and also the manner in which the different members are connected, are left to the draftsman, who is under the direct supervision of the engineer in charge of the drafting room, upon whom rests the responsibility for good details. The figures given in the text indicate the best current practice. Figs.

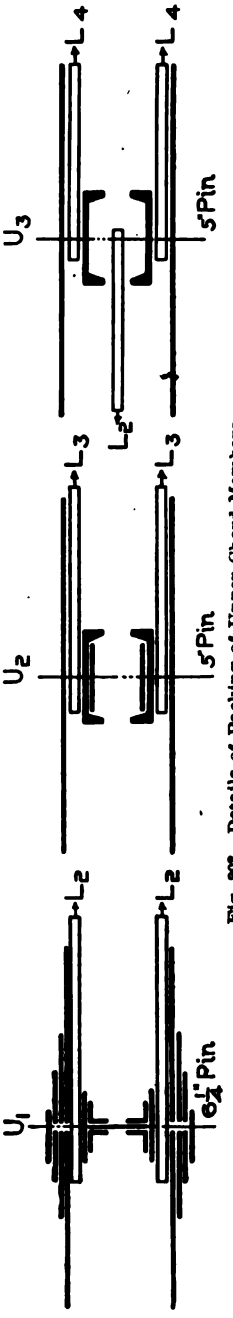


Fig. 203. Details of Packing of Upper Chord Members.

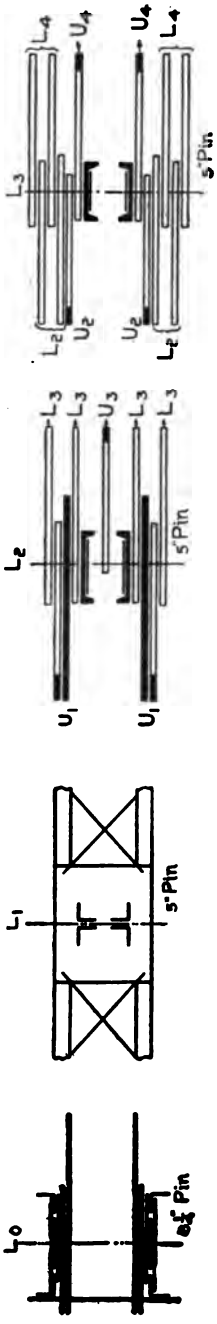


Fig. 204. Details of Packing of Lower Chord Members.

202 to 204 show details of the end floor-beam connections, and also the packing of the members of the upper and the lower chord. The arrangement here given may be said to be standard for single-track Pratt truss spans up to 200 feet in length.

BIBLIOGRAPHY

The following books are recommended to the student in case it is desired to pursue further the study of the subjects of Bridge Analysis and Bridge Design:

The Theory and Practice of Modern Frame Structures. JOHNSON, BRYAN, and TURNEAURE. John Wiley & Sons, New York, N. Y.

Roofs and Bridges. MERRIMAN and JACOBY. John Wiley & Sons, New York, N. Y.

Design and Construction of Metallic Bridges. BURR and FALK. John Wiley & Sons, New York, N. Y.

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Details of Bridge Construction. FRANK W. SKINNER. McGraw Publishing Company, New York, N. Y.

Steel Mill Buildings. Milo S. Ketchum. Engineering News Publishing Company, New York, N. Y.

Stresses in Indeterminate Stresses. HIROI. Engineering News Publishing Company, New York, N. Y.

Stresses in Frame Structures. A. JAY DU BOIS. John Wiley & Sons, New York, N. Y.

The Tasseldrüfte und Nebenspannungen eiserner Fachwerkbrücken. FR. [illegible]. Julius Springer, Berlin, Germany.

Bridge Drafting. WAGNER and WONG. Engineering News Publishing Company, New York, N. Y.

It must not be presumed that the above is a complete list of the books which have been published relating to the theory and practice of Bridge Engineering; neither must it be presumed that the obtaining of information relative to bridges is limited to textbooks on the subject. One of the best sources of information is found in the current engineering periodicals and the "Proceedings" of the various technical societies. The great advantage of these sources is that they give the most up-to-date information, and usually they are very profusely illustrated.

SUSPENSION BRIDGE OVER THE NIAGARA RIVER AT LEWISTON, N. Y.

Built in 1899. Span length of stiffening truss, 800 ft.: span between main towers, 1,040 ft.; distance between anchorages, 1,200 ft.

Courtesy of American Bridge Company.

202 to 204 show details of the end floor-beam connections, and also the packing of the members of the upper and the lower chord. The arrangement here given may be said to be standard for single-track Pratt truss spans up to 200 feet in length.

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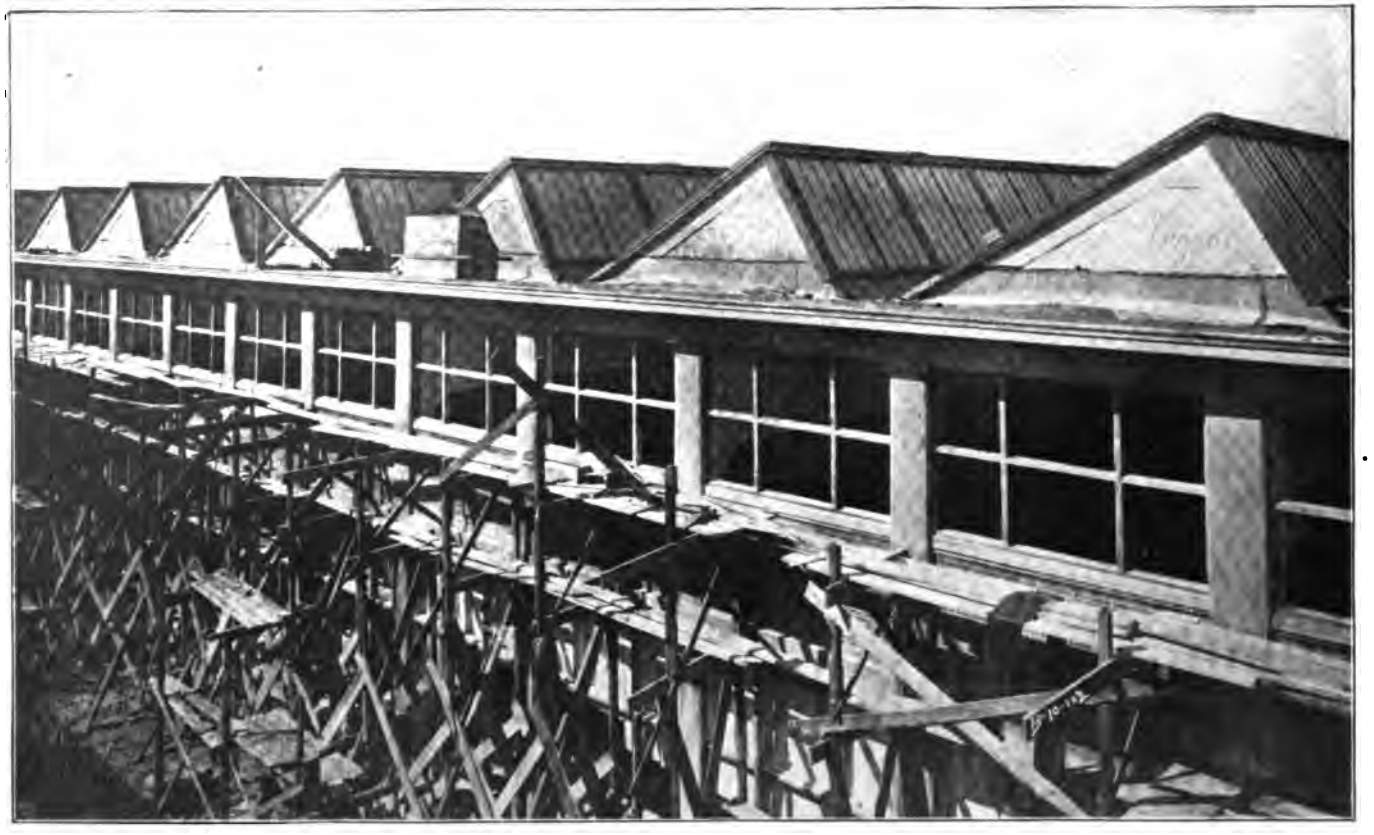
Courtesy of American Bridge Company.



PART OF SAWTOOTH ROOF OF ASSEMBLY BUILDING OF THE GEORGE N. PIERCE COMPANY, BUFFALO, N. Y.

Giving a partial view of the 61-foot girders. Kahn System of Reinforced Concrete.

Courtesy of Trussed Concrete Steel Co., Detroit, Mich.

ROOF TRUSSES

1. **Classes of Roof Trusses.** Roof trusses may be divided into three classes according to the shape of their upper chord. These three classes are:

(1) Triangular roof trusses;
(2) Crescent roof trusses;
(3) Roof trusses other than these.

Fig. 1 shows various forms of triangular roof trusses. The *Pratt* and *Howe* trusses are shown respectively by *a* and *b*. These trusses obtain their name on account of their web bracing being of the Pratt or Howe type. The triangular truss in most common use is the *Fink*, next to which is the *Saw-tooth*. The Fink truss is built in a variety of forms, as shown in Fig. 1 (*c*, *d*, *e*, and *f*), *c* being for spans up to 60 feet; *e* for spans up to 70 feet, and *d* and *f* for spans up to 80 feet and over. The great advantage of this style of truss is that many of its members have the same stress, and therefore it can be constructed more cheaply on account of the fact that a large amount of the same sized material can be purchased at once.

When the top chord of a roof truss becomes bent as shown in Fig. 2, the truss is called a *crescent* roof truss. The bracing in the crescent roof trusses is not of any particular form, being as a usual thing built of members which can take either tension or compression. This is made necessary by the fact that the curved upper chord may cause either tension or compression in the webbing, according to the angle of its inclination with the horizontal.

Roof trusses which do not come under either of the above classes may be regarded in a class of their own. To this class belong those trusses which are somewhat like a bridge truss in that the two chords are horizontal or nearly so. The ends of these trusses may be rectangular or not. For various types of this class of truss, see Fig. 3.

In addition to the above classification, which is based on the form of the chords, roof trusses may be divided according to the manner in which their members are connected. This classification

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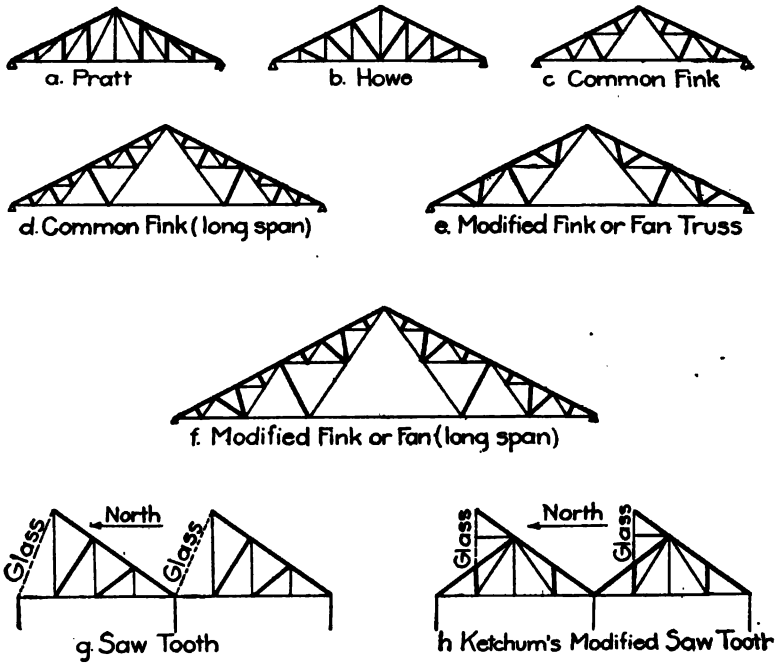


Fig. 1. Triangular Roof Trusses.

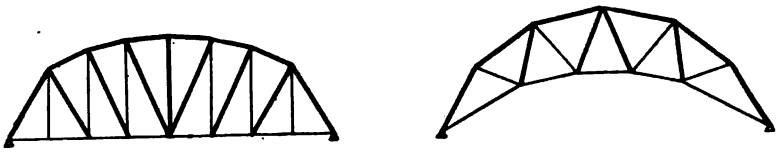


Fig. 2. Crescent Roof Trusses.

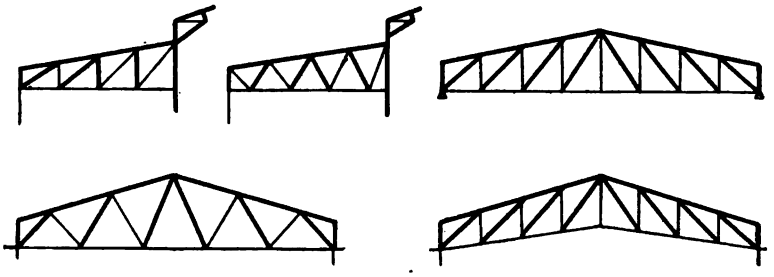


Fig. 3. Trusses with Chords Almost Parallel.

is that of *pin-connected* and *riveted.* For a definition of this, and for figures showing such joints, see "Statics," pp. 22 and 23.

Trusses are seldom built as pin-connected unless they are of long span, since roof trusses are comparatively light, and pin-connected trusses, unless of considerable weight, do not give very great stiffness.

Riveted roof trusses are used for nearly all practical purposes, since they give great rigidity under the action of wind and of moving loads, such as cranes, which may be attached to them.

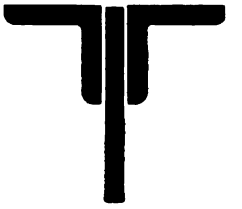


Fig. 4. Chord Section for Heavy Stresses.

2. **Physical Analysis of Roof Trusses.** In pin-connected roof trusses, the tension members consist of I-bars or rods; and the compression members are made of channels or angles and plates, either plain or latticed. In riveted trusses, both tension and compression members are made up of angles and plates or a combination of the two. The top chords of roof trusses of medium span usually consist of two angles placed back to back. If the stress becomes too great to be taken up by two angles larger than 5 by 3½ inches, then two angles and a plate are used (see Fig. 4). In case the roof truss is of great size and

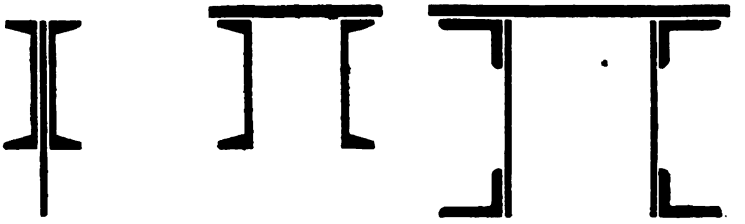


Fig. 5. Fig. 6

Chord Sections for Trusses of Long Span.

the stresses are exceedingly large, the chord member may be built up in a manner somewhat similar to a bridge truss, being constructed of two channels and a plate, or four angles and three plates. Figs. 5 and 6 show cross-sections of chords for long-span riveted trusses. These cross-sections may also be used for pin-connected trusses.

The web members of a truss usually consist of one angle; and if this is insufficient, two angles back to back are used. Fig. 80, page 65, gives a diagram of a roof, and shows not only the roof trusses

but also various other parts which will be referred to in the succeeding articles.

3. **Wind Pressure and Snow Loads.** The wind pressure on a flat surface varies, of course, with the velocity of the wind, and is very closely given by the formula:

$$P = 0.004V^2$$

By substituting in this formula, the values shown in Table I are determined for given velocities in miles per hour.

TABLE I

Wind Pressure at Various Velocities

Velocity (Miles per hour)	Pressure (Lbs. per square foot)	Remarks
10	0.4	Breeze
20	1.6	Strong breeze
30	3.6	Strong wind
40	6.4	High wind
50	10.0	Storm
60	14.4	Heavy storm
70	19.6	Hurricane
80	25.6	"
100	40.0	"

The pressures indicated in Table I are perpendicular to the direction of the wind. When the wind blows on an inclined surface, the wind is assumed to be acting horizontally, and the normal component on the inclined surface is determined. This component is not equal to the horizontal pressure times the sine of the angle of inclination, as one would suppose (see Fig. 7), but is greater by a small amount. Roofs are usually figured on a basis of 40 pounds pressure on a vertical surface. The value of the normal component for a horizontal wind pressure of 40 pounds per square foot, is given on page 24 of "Statics," and is here, for convenience, reduced to the normal pressure for any given pitch.

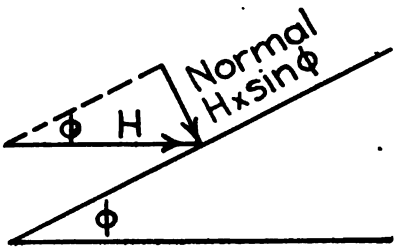


Fig. 7. Theoretical Determination of Normal Component.

Pitch	Normal Wind Pressure
$\frac{1}{2}$	34 pounds per square foot.
30°	32 " " " "
$\frac{1}{4}$	30 " " " "
$\frac{1}{5}$	26 " " " "
$\frac{1}{6}$	22 " " " "

If the normal pressure on a roof making any other angle with the horizontal is desired, see "Statics," p. 24.

The determination of these values is based for the most part on data obtained by experiment. In the computations relative to the design of buildings, the wind is usually assumed to exert a pressure on the walls of 30 pounds per square foot.

The snowfall varies with the locality. The heaviest snow loads which come upon a roof are not always in the locality of the heaviest snowfall, since a comparatively light snowfall may occur, and if this is followed by wind and sleet, the result will be a load greatly in excess of the snowfall itself. The snow load per square foot of roof surface varies with the pitch of the roof, and will be greater the smaller the pitch. The ice and sleet will be comparatively constant. Fig. 8* gives values of snow and sleet loads which are recommended for use. It is customary to figure the snow load by taking it as so much per square foot of horizontal projection.

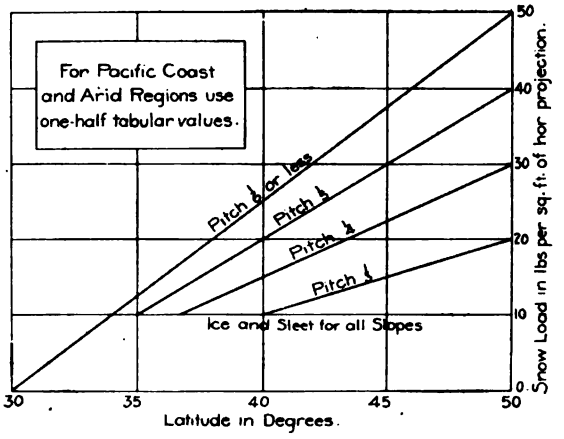


Fig. 8. Unit Snow Loads.

EXAMPLES FOR PRACTICE

1. Compute the wind panel load on a roof whose pitch is $\frac{1}{4}$, and whose panel length is 15 feet, the distance between trusses being 16 feet.

2. Compute the snow panel load for the truss of Problem 1, above.

*Ketchum's "Steel Mill Buildings," p. 11.

4. **Weights of Roof Trusses.** The weight of a roof truss varies with the material of which it is constructed, the span, the distance between trusses, the pitch, and the capacity of the truss. The actual weight, of course, cannot be determined until after the truss is designed; but an approximate weight may be obtained from any of the empirical formulæ which are now in use. Table II gives the most common and best of the empirical formulæ, together with the names of their authors.

TABLE II

Formulæ Giving Weights of Roof Trusses

Formula	Author
$W = \frac{3}{4} al \left(1 + \frac{l}{10}\right)$	Mansfield Merriman
$W = al \left(1 + \frac{l}{25}\right)$	E. R. Maurer, (p. 23, "Statics")
$W = al^2 \left(\frac{1}{25} + \frac{l}{6000}\right)$, wooden trusses.	N. C. Ricker
$W = \frac{Pal}{45} \left(1 + \frac{l}{5\sqrt{a}}\right)$	Milo S. Ketchum*
$W = 2a\left(4 + \frac{l}{25}\right) \sqrt{l^2 + r^2}$	C. W. Bryan
$W = al\,(0.06\,l + 0.6)$ for heavy loads $W = al\,(0.04\,l + 0.4)$ " light "	C. E. Fowler

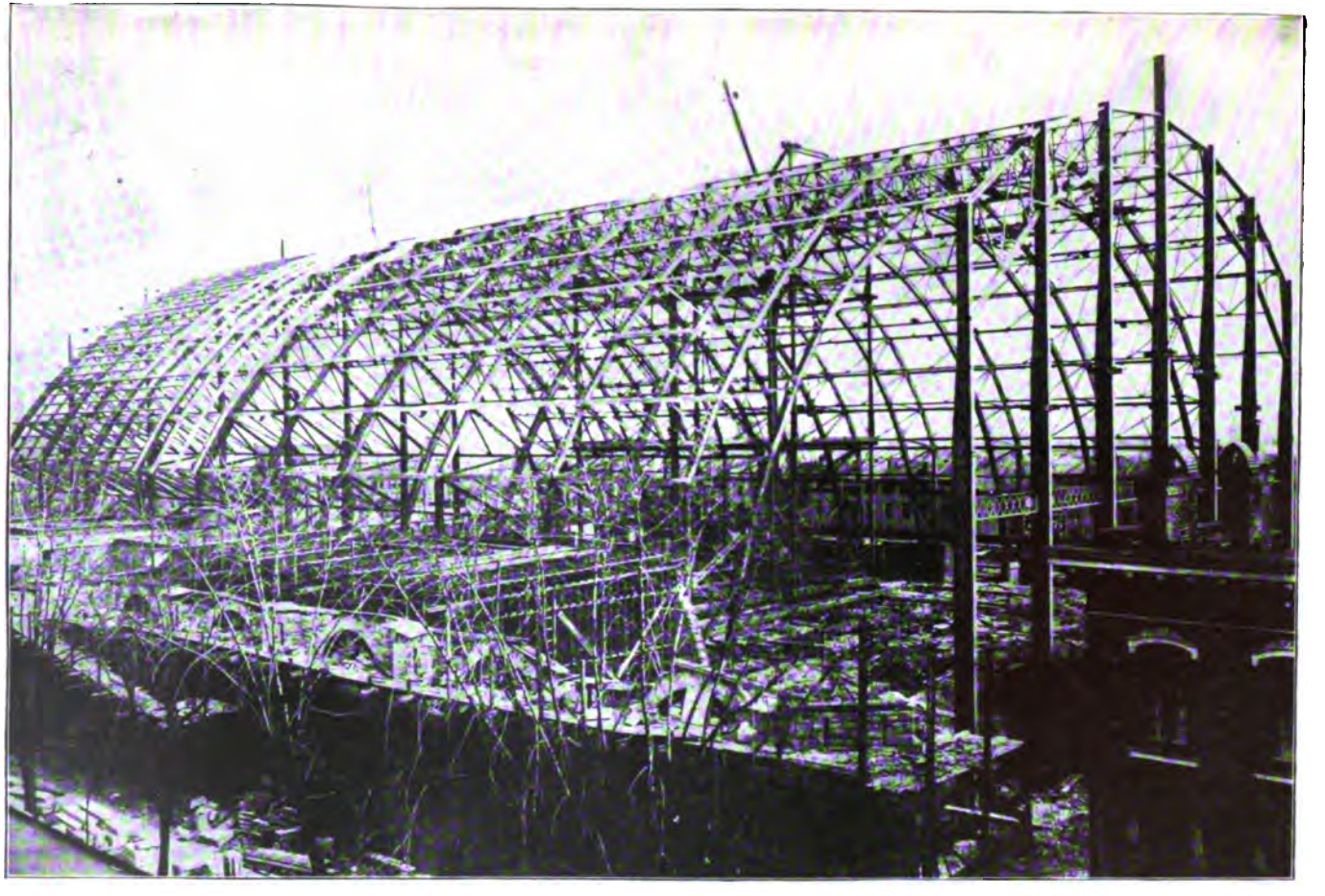
In the above formulæ,

W = Weight of steel in truss, in pounds;
P = Capacity of truss in pounds per square foot of horizontal projection of roof;
r = Rise of peak, in feet;
a = Distance center to center of trusses, in feet;
l = Span of truss, in feet.

ROOF COVERINGS

5. The roof is covered with some material which will protect the interior of the building from the action of the elements. This covering may consist of any one or more of the materials which, together with their weights per square foot, are indicated in Table III. The weights here given for materials which must be laid upon sheathing, do not include the weight of the sheathing, which is given separately. A short description, together with necessary information for use in estimates, will now be given.

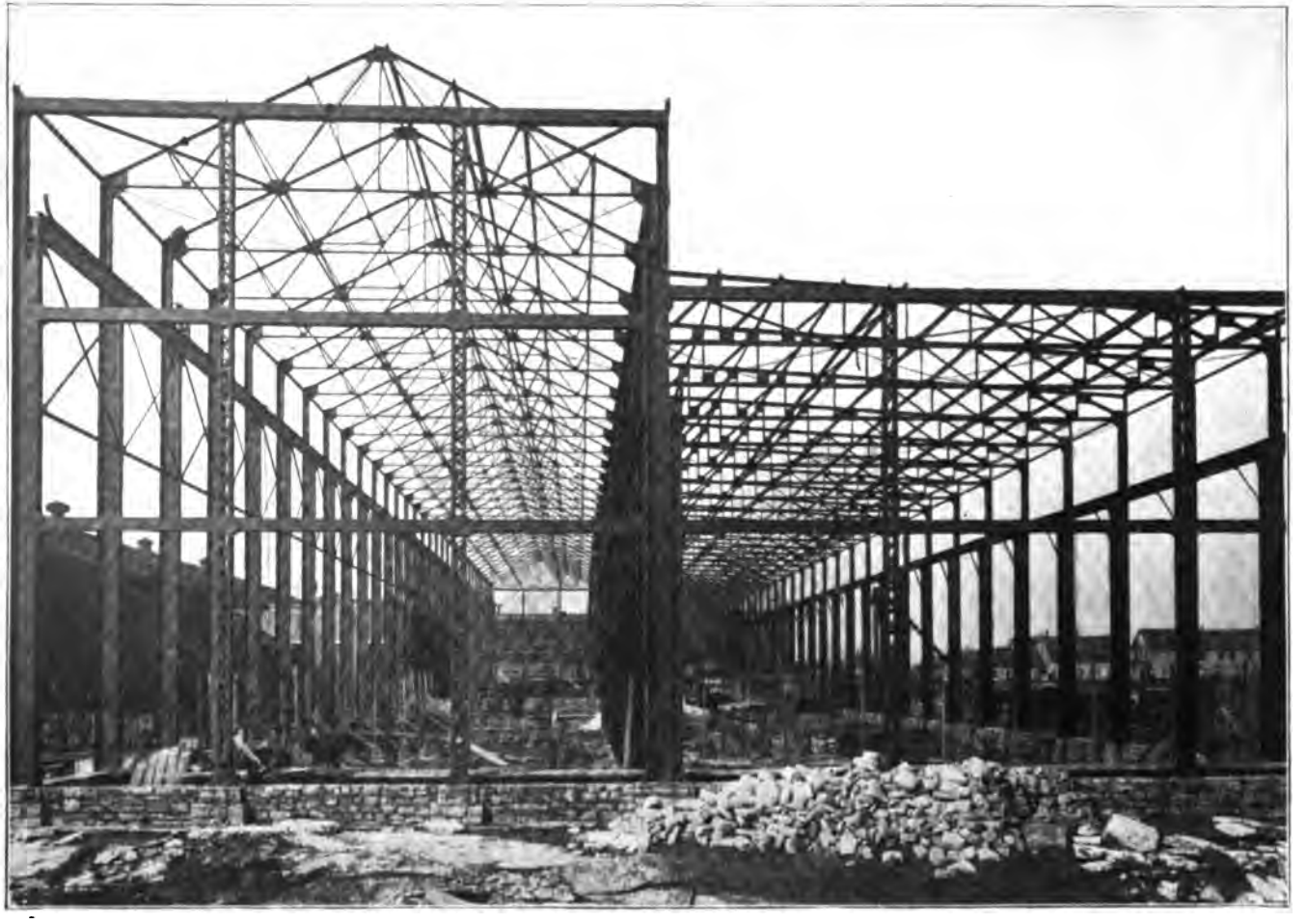
* "Steel Mill Buildings," p. 5.



STEEL FRAME OF BUILDING FOR FIFTH REGIMENT ARMORY, BALTIMORE, MARYLAND

Span, 200 ft., centers; length of structure, 300 ft.; total weight of steel work, 1,928,000 lbs.

Courtesy of American Bridge Company.



MACHINE SHOP FOR SNOW PLANT OF INTERNATIONAL STEAM PUMP COMPANY, BUFFALO, N. Y.

Length of building, 513 ft.; two bays, one 50 ft. wide by 60 ft. high; the other 50 ft. 9 in. wide by 36 ft. 9 in. high; with crane runways. Total weight of steel work, 1,300,000 lbs.

Courtesy of American Bridge Company.

TABLE III

Approximate Weights of Roof Coverings

Material	Weight per Square Foot
White pine sheathing 1 inch thick	3 lbs.
Yellow pine sheathing 1 inch thick	4 "
Batten sheathing, 4-in. by 1-in.	2½ "
Slate	10 "
Skylight glass, including frame	10 "
Tin	1 "
Shingles	3 "
Corrugated steel	2 "
Flat tiles	12 to 25 "
Corrugated tiles	10 "
Concrete slabs	35 to 50 "
Felt, asphalt and gravel	10 "
Felt and gravel	10 "
Patent roofings	½ to 1¼ "
Sheet steel	1½ "
Non-condensing base	1 "

Sheathing. Sheathing is generally laid directly upon the purlins (see Article 6); and upon this are laid the shingles, slate, tin, or tile. Sheathing is usually made of a single thickness of planks, 1 to 2½ inches thick, laid close together. In some cases, however, when batten sheathing is used, it is spaced from 2 to 4 inches apart. This has the advantage of being cheap and at the same time allowing good circulation of air beneath the roof covering, and consequently dampness due to any cause will soon dry out. Batten sheathing is much used where the roof covering consists of slate, shingles, or tile.

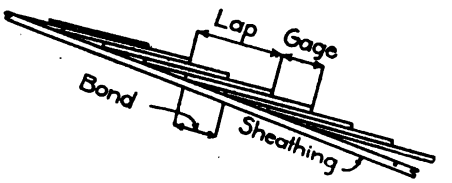


Fig. 9. Method of Laying Slate.

Slate. Roofing slate is generally of the characteristic slaty color, but may be obtained in nature in greens, purple, reds, and other colors. It is made in thicknesses of from ⅛ to ¼ inch, in widths from 6 to 24 inches, and in lengths from 12 to 44 inches. The 12 by 18 by $\frac{3}{16}$-in. slate is probably the most commonly used. Slate should be laid as shown in Fig. 9, and the pitch of the roof should not be less than ¼. If the pitch of the roof is less than this, the lap should be made greater than 3 inches, as is shown in Fig. 9. The lap should be increased at least ½ inch for every $\frac{1}{100}$ in pitch; and the minimum

pitch should never be less than $\frac{1}{6}$, since it is practically impossible to prevent roofs with a smaller pitch than this from leaking, especially if a strong wind is blowing.

The number of different sizes of slate required to lay 100 square feet of surface, and also their weight, are given in the handbooks of the various slate companies. With a 3-inch lap, it takes 160 of the size and thickness mentioned above to lay 100 square feet, and the total weight of this square is 650 pounds. Slate is one of the most durable of roofing materials. Its first cost is high, being from 5 to 8 dollars per hundred square feet; but the cost of maintenance is almost nothing, since it is affected neither by the elements nor by the action of gases or acids. In case the roof would be subjected to the action of gases or acids, it is advisable to use copper slating nails.

Skylight Glass. Skylights usually consist of glass about $\frac{3}{16}$ to

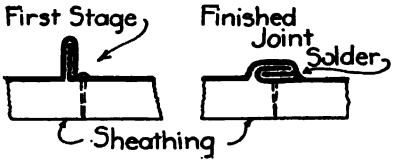


Fig. 10. Tin Laid with Flat Seam.

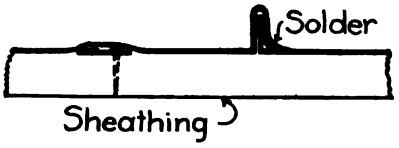


Fig. 11. Tin Laid with Standing Seam.

$\frac{3}{8}$ inch in thickness, supported on light members of iron or galvanized iron which act as a framework. The actual weight of glass of different kinds can be accurately obtained from manufacturers' catalogues, and the student is referred to these; they may be had by addressing the manufacturers (see Figs. 73 and 74).

Tin. This is made by coating thin, flat sheets of iron or steel, either with tin alone or with a mixture of tin and lead. In the first case the product is called *bright tinplate*, and in the second case *terne plate*. Terne plate must not be used where it will be subjected to the action of acids or corrosive gases, since the lead coating is rapidly destroyed, and then of course the iron also.

Tin plates come in various sizes and thicknesses; but usually 112 come in one box. The most commonly used is a sheet 20 by 28 inches, and of sheet iron of No. 27 gauge, which weighs 10 ounces to the square foot. This is marked "IX." If the box were marked "IC," it would indicate that the sheets were of No. 29 gauge metal, which weighs 8 ounces to the square foot. The value of the roofing

depends to a great extent upon the amount of tin used in the coating. This will vary from 8 to 50 pounds for a box of the 20 by 28-inch sheets.

A tin roof is formed by fixing together a number of these sheets. The sheets may be connected as shown in Fig. 10, or as shown in Fig. 11. In the first case, they are said to be laid with a *flat seam*, and in the second case they are said to be laid with a *standing seam*. Tin roofs rot out very readily unless they are kept painted. If a new coat

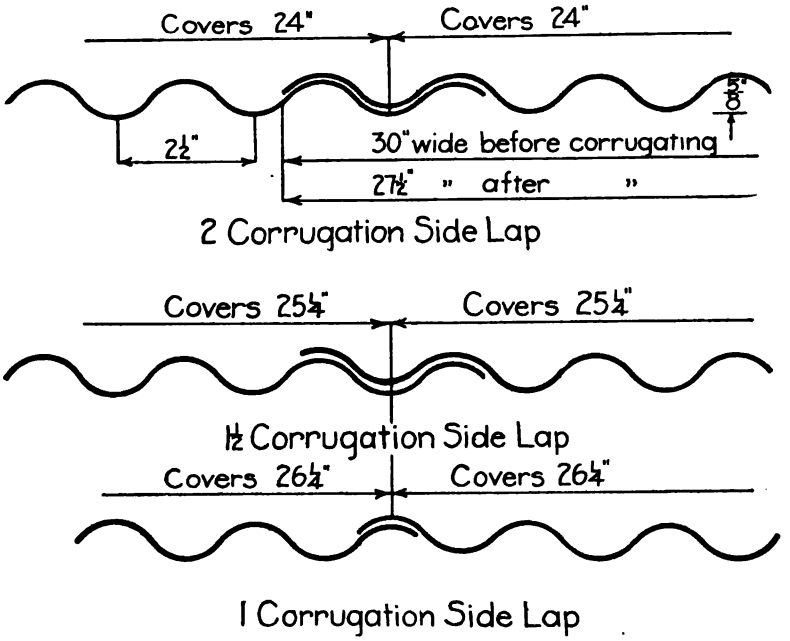


Fig. 12. Lapping of Corrugated Steel.

is given them every couple of years, they will last from twenty to thirty years.

Tin can be laid on roofs whose pitch is very small, say $\frac{1}{10}$. The first cost is about as much as that of slate, but the cost of maintenance is very high.

Shingles. Shingle roofs are very seldom used on buildings for manufacturing purposes, for the reason that they take fire quite readily, leak quite easily, and require renewal quite often. Shingles are from 18 to 24 inches long, and usually run from 2 to 8 inches in

width, although they can be obtained of a uniform width of from 4 to 6 inches. They are laid like slate, the lap being made 4 inches or more. They should never be laid on roofs whose pitch is less than $\frac{1}{3}$. It takes about from 800 to 1,000 shingles to lay 100 square feet of roof. The cost is about $5.00 per 100 square feet; but under the best conditions, the life of shingles is only about ten years.

Corrugated Steel. Corrugated steel is made from flat sheets of standard gauges, and may be either galvanized or left as it comes from the rolls. The corrugations are of different sizes and widths; the total width of the plates runs from 24 to 28 inches, and their length from 5 to 10 feet, varying by $\frac{1}{2}$ foot. The sheet most used for roof construction has $2\frac{1}{2}$-inch corrugations; is 26 inches wide after corrugating; and will cover a width of 24 inches with one lap and $21\frac{1}{2}$ inches with two laps. This roofing should be laid with a pitch of not less than $\frac{1}{4}$, and should have from 6 to 8 inches lap at the ends. For further information regarding the method of lapping and the width covered, see Fig. 12.

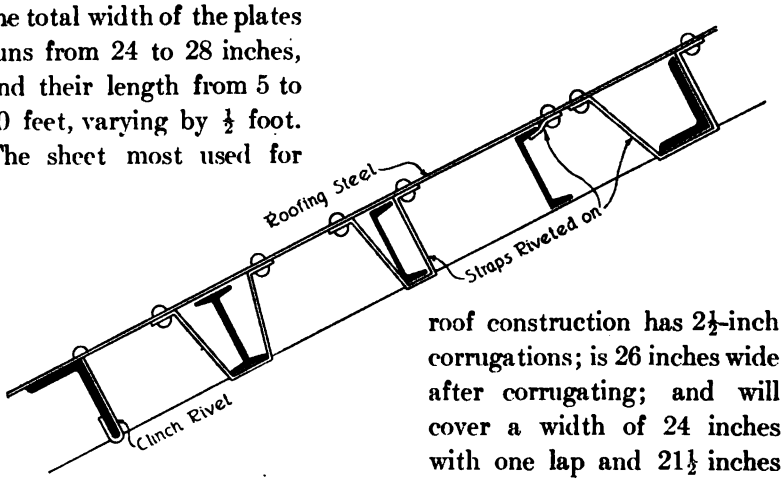


Fig. 13. Showing How Steel Roofing is Fastened to Purlins.

Corrugated steel is fastened either directly to wooden purlins by means of nails, or directly to iron purlins either by means of a bolt and clip or by a clinch nail (see Fig. 13).

It is often advisable to know the strength of corrugated steel when supported at certain distances apart by supports perpendicular to the corrugations. This unsupported length determines in many cases the spacing of the purlins. The load in pounds per square foot which can be carried by a plate of span l, parallel to the corrugation, is given by the formula:

$$W = \frac{330\,S\,d\,t}{l^2},$$

in which,

l = Unsupported length of sheet, in inches;
t = Thickness of sheet, in inches;
S = Allowable unit-stress;
d = Depth of corrugation, in inches.

Table IV, giving data relative to corrugated sheets, is taken from page 172 of the Pocket Companion of the Carnegie Steel Company (edition of 1902), where also other valuable information is given.

TABLE IV

Corrugated Steel Data

No. by Birmingham Gauge	Thickness (Inches)	Weight in lbs. per 100 Sq. Ft. of Roof when Laid with 6-in. End Lap and One Corrugation, 2½-in., Side Lap, and Length of:					
		5 ft.	6 ft.	7 ft.	8 ft.	9 ft.	10 ft.
16	0.065 in.	365	358	353	350	348	346
18	0.049 "	275	270	267	264	262	261
20	0.035 "	196	192	190	188	186	185
22	0.028 "	156	154	152	150	149	148
24	0.022 "	123	121	119	118	117	117
26	0.018 "	101	99	97	97	96	95

Tiles. One of the most common sizes of plain roofing tile is 10½ inches long by 6¼ inches wide and ⅝ inch thick. Tile of this size weigh about 2¼ pounds each. They are laid with a lap equal to one-half their length. They may be laid directly upon plank sheathing in a manner similar to shingles or slate, or they may be laid directly upon purlins (see Figs. 14, 15, and 16). In the first case they are nailed directly to the sheathing, and in the second case they are connected with the purlins either with copper wire or clinch nails. Flat tiles are usually laid in cement; corrugated tiles are made so as to interlock, and consequently in most cases require no cement. One convenience of the tile roof is that the skylights may be formed by laying glass tile in place of the other.

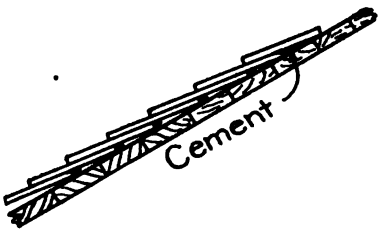


Fig. 14. Method of Laying Flat Tile on Plank Sheathing.

Tile roofs are very substantial; but are very costly, in regard not only to the tiles themselves, but also in regard to the additional

weight required in the trusses by reason of the great weight to be supported. Tile weigh from 700 to 1,000 pounds per 100 square feet of roof surface. They cost from $12.00 to $40.00 per 100 square feet on the roof.

Concrete Slabs. These are usually moulded directly in place by suspending forms from the roof trusses. They may or may not be reinforced, and in any case are usually not over 4 inches in thickness. Their weight is about 50 pounds per square foot. Their cost is from $16.00 to $30.00 per hundred square feet of roof surface. They are expensive, not only on their own account, but also from the fact that the weight of the roof trusses must be increased in order to carry the

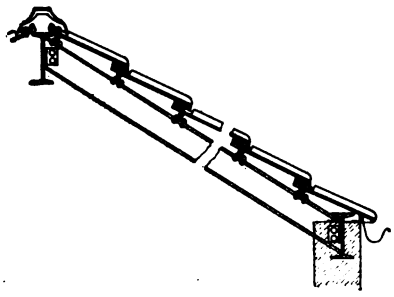


Fig. 15. Ludowici Tile on Steel Purlins.

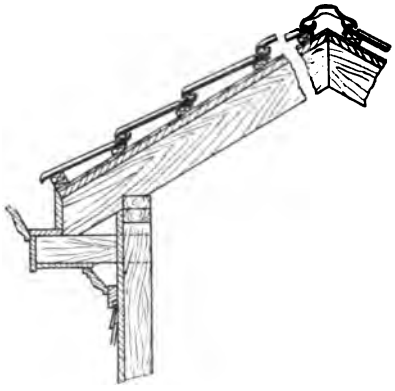


Fig. 16. Ludowici Tile on Sheathing.

weight of the slabs. Concrete roofing may be used on roofs which are practically flat, $\frac{1}{2}$ inch to 1 foot being sufficient pitch.

Felt and Asphalt. This roofing is laid upon shingles, and consists of one thickness of dry felt, three or four thicknesses of roofing felt well cemented together with asphalt cement, and laid with good laps where they join, and a coating of from 100 to 200 pounds of asphalt per 100 square feet of roof surface. Upon this asphalt, while hot, gravel screened through a $\frac{5}{8}$-inch mesh is spread in the quantity of about $\frac{1}{5}$ of a cubic yard per 100 square feet. This class of roofing should never be laid on roofs whose pitch is greater than $\frac{1}{6}$, since, when heated by the rays of the sun, the asphalt will run and destroy the surface. It gives good satisfaction on roofs whose pitch is $\frac{1}{12}$. This class of roofing can be bought in rolls, and in this case the gravel is exceedingly fine, being screened through a $\frac{1}{8}$-inch mesh.

Felt and Gravel Roofing. This roofing is similar to the above; only, in this case, tar instead of asphalt is used for the cementing constituent. This roofing does well on roofs of flat pitch, and should never be used on roofs whose pitch exceeds $\frac{1}{6}$. It can also be bought in rolls ready for laying, in which case the gravel is screened through a $\frac{1}{8}$-inch mesh. The prepared roofings are cheaper than those laid by hand; but they do not give good service unless great care is taken to fasten them down securely. In economy of first cost and maintenance, they are equal to or better than tin.

Sheet Steel. This should not be laid on a pitch less than $\frac{1}{4}$, unless the ends are cemented together where they lap. It comes in sheets 28 inches wide and from 4 to 12 feet long, or it may be purchased in rolls 26 inches wide and about 50 feet long. When used in sheets, it may be had with standing crimped edges, in which case it is laid as

Fig. 17. Method of Laying Sheet Steel with Crimped Edges.

Fig. 18. Method of Laying Roofing on an Anti-Condensing Base.

shown in Fig. 17. In case it comes in rolls, it may be laid in the same manner as tin, with either standing seams or horizontal flat ones as shown in Figs. 10 and 11. Like corrugated steel, it comes in different gauges, No. 28 being that most commonly used. It can be laid cheaper than tin, on account of the long lengths obtainable.

Patent roofings of many kinds are on the market. These come in rolls usually from 2 feet to 3 feet wide, and cover about 200 square feet of roof surface. The basis of most of these covers is asbestos, felt, magnesia, or rubber; and this is treated with either asphalt, tar, or some other preparation, and in some cases is covered with fine gravel.

Non-Condensing Roofing. In cases where a metal, slate, or tile roof is used without sheathing, moisture is liable to condense upon the under side and drip on the floor beneath. This can be prevented by laying the material upon an anti-condensing base consisting of a layer of wire netting on top of which are placed one or more sheets of asbestos paper about $\frac{1}{16}$ inch thick (see Fig. 18).

6. **Rafters and Purlins.** Roof trusses are usually connected by beams running from one to the other. These beams are called *purlins*. In case the purlins are spaced too far apart to lay the roof covering directly upon them, beams are placed upon the purlins, and on these beams the roof covering is placed. These beams are called *rafters*. Rafters are usually made of wood, while purlins are made of channels, I-beams, Z-bars, and, if the trusses are spaced sufficiently close together, *tees* or *angles*. Figs. 19 and 20 show how rafters and purlins are placed. Fig. 21 illustrates the use of purlins made of tees. As purlins are more rigid about an axis perpendicular to their webs, they are liable to sag toward the eaves at their center. In this case, *sag rods* are used, as shown in Fig. 22.

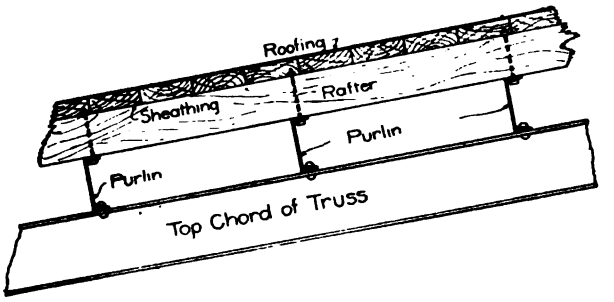


Fig. 19. Roof Construction in which Rafters Supporting the Sheathing are Laid on the Purlins which Connect the Trusses.

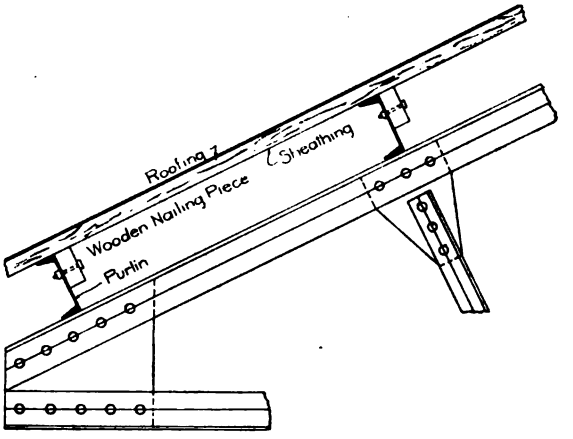


Fig. 20. Sheathing Laid Directly on Purlins.

EXAMPLES FOR PRACTICE

1. Compute the roof rafters if the purlins are spaced 10 feet apart, the roof covering weighs 10 pounds, the sheathing 4 pounds, and the snow load per square foot of roof surface 12 pounds.

This problem may be solved, either by assuming the size of the rafters and computing their spacing, or by assuming the spacing and computing the size of the rafters. The latter method is the one most commonly used. The spacing of rafters is from 18 inches to 4 feet.

The common spacing is 2 feet. The weight of the rafter itself is neglected in its design.

The total weight per square foot which comes on the rafter is $12 + 10 + 4 = 26$ pounds. Since each rafter carries a portion of the roof 10 by 2 feet, the total weight on one rafter is $10 \times 2 \times 26 = 520$ pounds. The moment created by this weight is $(520 \times 10 \times 12) \div 8 = 7\,800$ pound-inches. This should be equated to $\frac{S\,I}{c}$. Allowing 1 000 pounds to the square inch as the unit-stress on the extreme fibre, and noting that $I \div C = \frac{b\,d^3}{12} \div \frac{d}{2} = \frac{b\,d^2}{6}$ there results:

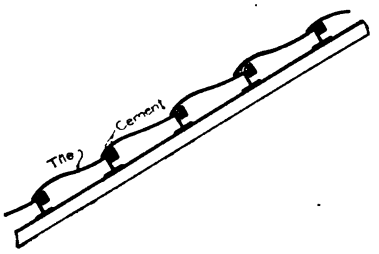


Fig. 21. Use of Purlins Made of Tees.

$$\frac{1\,000\,bd^2}{6} = 7\,800$$

$$d = \sqrt{46.8 \div b}$$

The market widths of rafters are $1\frac{1}{2}$, 2, 3, and 4 inches, 2 inches being the size usually employed. Substituting in the above formula, we have:

$$d = \sqrt{46.8 \div 2} = 4.8 \text{ inches.}$$

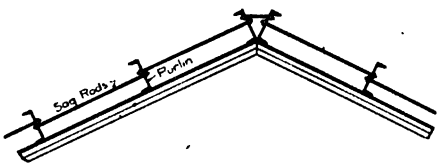


Fig. 22. Use of Sag Rods to Prevent Sagging of Purlins at their Center.

The rafters will be made 2 by 6 inches, since this is the nearest market size. If a 3-foot spacing of rafters was used, the required depth would be 5.92 inches, and a 2 by 6-inch would still be used. This spacing and this size of rafter would be the one to employ in the solution of the above problem.

2. Design the purlin for the roof of Problem 1, above, if the trusses are spaced 16 feet apart.

The rafters are spaced so close together that their own weight, the weight of the roof covering, and the snow load may be considered as uniformly distributed over the purlin. The total weight which comes upon one purlin is the weight of snow and roof covering on a space 16 feet long and 10 feet wide. This weight is:

Snow load = 10 × 16 × 12	1 920 pounds
Roof covering = 10 × 16 × 14	2 240 "
16 rafters 6 by 2-in., 10 feet long, at 3 lbs. per 144 cu. in.	480 "
Total	4 640 pounds

The moment caused by this weight is:

$$(4\,640 \times 16 \times 12) \div 8 = 111\,360 \text{ pound-inches.}$$

The determination of the beam which will be used to withstand this bending moment is made by means of its section modulus. The formula $\frac{M}{S} = \frac{I}{c}$ is used in the design of beams. The values of I and c are constant for any given beam, and therefore the value of $I \div c$ for any particular beam is a constant, and this constant is called the *section modulus*. It is therefore evident that if we have a certain bending moment and a certain allowable unit-stress, we can obtain the value of the section modulus by dividing the moment by the allowable unit-stress. Then, looking into one of the steel handbooks, the beam can be determined which will have a section modulus equal to or slightly in excess of the value that has been obtained by dividing the bending moment by the unit-stress. This beam will be the beam which, with a unit-stress equal to the one assumed, will withstand the bending moment under consideration.

The handbooks issued by many of the steel companies are indispensable to the intelligent design of structural steel. That issued by the Carnegie Steel Company (edition of 1903) is one of the most convenient; and since it will be frequently referred to in this text, its purchase by the student is desired. This book may be obtained by addressing the Carnegie Steel Company at its offices in any of the larger cities. The cost to students has usually been 50 cents; to others, $2.00.

Assuming an allowable unit-stress of 18 000 pounds per square inch on the extreme fibre, the section modulus required to withstand the bending moment of 111 360 pound-inches is:

$$\frac{111\,360}{18\,000} = 6.19.$$

Looking in the Carnegie Handbook at column 11 on page 100, column 11 on page 102, and column 9 on page 104, it will be seen that any one of the following shapes will be sufficient:

One 5-inch 14.75-pound I-beam;
One 7-inch 9.75 " channel;
One $4\frac{1}{8}$ by $3\frac{3}{16}$ by $\frac{9}{16}$-inch Z-bar weighing 17.9 pounds per linear foot.

Instead of the 5-inch I-beam as given above, a 6-inch 12.25-pound I-beam with a section modulus of 7.3 could be used, and would be more economical, since it is less in weight; and it would also be stiffer, since its depth is greater and its section modulus is greater. A comparison of the above weights shows the channel to be the most economical, since its weight is considerably less than either of the other two shapes. Channels usually make the most economical purlins; and for this reason no other shapes are usually inspected, the channels being used in the first case without being compared with other sections. Inspection of column 11, page 110, Carnegie Handbook, shows that a 6 by 4 by $\frac{3}{4}$-inch angle could have been used for the purlin, since it gives a section modulus of 6.25. The weight of this angle, 23.6 pounds per linear foot, shows it to be far too uneconomical to employ.

EXAMPLES FOR PRACTICE

1. Design the rafters when the total weight of the snow and roof covering is 30 pounds per square foot, and the purlins are spaced 15 feet apart. Use 1 000 pounds per square inch as the allowable unit-stress.

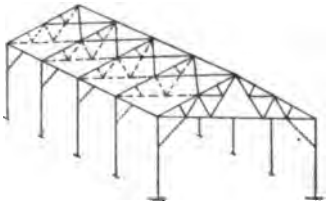
2. Design the purlins if the trusses are 12 feet center to center; the purlins are spaced 8 feet apart; the roof covering, which weighs 6 pounds per square foot, is laid upon 1-inch yellow pine sheathing resting directly upon the purlins; and the snow load is 10 pounds to the square foot of roof surface. Use 18 000 pounds per square inch as the allowable unit-stress, and use a channel for the purlin section.

7. **Bracing.** In order to keep the roof trusses erect, bracing is employed to join together their top chords and also their bottom chords. This bracing may consist either of small round or square rods, or it may consist of angles. The latter is the best practice, since it gives great rigidity to the structure; and in fact it should be used in all cases where machinery of any kind is attached to the trusses. One disadvantage of the rod bracing is that good connections with the trusses are usually difficult. The bracing between the lower chords is lighter than that between the top chords, since its office is merely to prevent vibration, while that between the upper chords must take up the stresses caused by the wind blowing upon the ends of the building. The stresses in each of these classes of bracing can only be approximately determined; and for that reason it has become customary to determine their section by judgment rather than by computation. For lower chord bracing, single angles 3 by

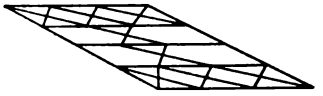
2 by $\frac{5}{16}$-inch are recommended; and for upper chord bracing, 3 by 3 by $\frac{5}{16}$-inch angles should be used.

It is not customary to place bracing between each pair of trusses, but to place them between each alternate pair or between every third pair of trusses. Fig. 23 shows several ways in which the bracing may be inserted.

8. **Economical Spacing and Pitch of Trusses.** The term pitch which has been used in the preceding pages is the fraction obtained by dividing the span into the height of the truss at the center of the span. For example, if a truss has a span of 60 feet, and a rise of 12 feet at the center, it would be said to have a pitch of $\frac{1}{5}$; if the rise were 15 feet, the pitch would be $\frac{1}{4}$; and if the rise were 20 feet, the pitch would be $\frac{1}{3}$. The pitch of a truss is seldom expressed in degrees by giving the angle that the top chord makes with the horizontal. One exception is very common. It is to use the 30° pitch. This has the advantage of making the height of the center equal to one-half the length of one side of the top chord—a fact which lends itself to ease in making the shop drawings.



Upper Chord Bracing



Looking Upward - The Lower Chord Bracing

Fig. 23. Methods of Inserting Bracing between Trusses.

The maximum or minimum allowable pitch for any given roof depends to a great extent upon the class of roof covering employed. For pitches required for any given class of roof covering, see Article 5, p. 6. It might be noted that most of the patent roofings, or any roofing in which tar or asphalt is an ingredient, should not be laid upon roofs with a pitch greater than $\frac{1}{5}$ or $\frac{1}{6}$; while most of the coverings which consist of steel or clay products require pitches of $\frac{1}{5}$ or over.

Pitches varying from $\frac{1}{6}$ to $\frac{1}{3}$ have very little effect upon the weight of the trusses. This is true only for trusses with horizontal lower chords. If the lower chord is *cambered*—that is, raised above the horizontal position—it greatly increases the stresses in the truss, and consequently the weight of the truss. The greater the camber, the greater the weight of the truss, the pitch remaining the same. If the

camber is constant, then the greater the stresses (and consequently the weight of the truss), the smaller the pitch. It is advisable not to camber the lower chord unless it is positively necessary. A camber of 5 per cent of the span will increase the weight of the truss from 10 to 40 per cent, according to the pitch.

Taking all things into consideration, a pitch of $\frac{1}{3}$ or $\frac{1}{4}$ is to be preferred over that of $\frac{1}{5}$ or less, since, after the pitch becomes less than $\frac{1}{5}$, the weight of the truss increases quite rapidly, the span being constant.

For any given roof, there is an economical spacing of the trusses. As the spacing of the trusses increases, the weight of the purlins and bracing per square foot of area increases, while the weights of the trusses, the columns that support them, and the girts, or members which run from one column to the other and on which the siding of the building is placed, decrease. The most economical spacing of the trusses is such as will make the *cost* of the above quantities a minimum. It is evident that this spacing for trusses which rest upon masonry supports will be different from the spacing in case they rest upon steel columns. Attention is called to the statement that the sum of the costs, instead of the sum of the weights of the above-mentioned quantities, should be a minimum. This is due to the fact that the unit-cost of the purlins is considerably less than that of the trusses, it being in some cases only about one-half.

The spacing of trusses is sometimes governed by local conditions, such as the placing of the machinery in the building and the probable position of future additions. Considering the spacing from a purely economical standpoint, it is probably well to space trusses about as indicated in Table V.

TABLE V

Spacing of Trusses

Span, in Feet	Spacing, in Feet
10 to 30	12
30 to 60	15
60 to 75	20
75 to 150	21 to 25

The spacing indicated in Table V is for triangular roof trusses of equal size and span. For other conditions—such as when the main roof consists of

one span, and the side roofs consist of different spans and different classes of trusses—the economical spacing may be somewhat different, and is usually less.

The best method of determining the economical spacing is either to make a comparative design or to consult the back volumes of *The Engineering Record, Engineering News,* or some other good engineering periodicals. Designs of buildings which have been constructed are frequently given in these periodicals; and from these the student may, in addition to the spacing of the trusses, obtain much other valuable information regarding roof construction.

Bulletin No. 16 of the University of Illinois Experiment Station gives a systematic study of roof trusses, and shows the effect on the variation in the weights of rafters and purlins due to a variation in the length of span. This bulletin, which can be had free for the asking, should be in the hands of the student. It may be had by addressing "The Director," Engineering Experiment Station, University of Illinois, Urbana, Illinois. A most valuable book giving a systematic and extensive study of roof trusses and mill buildings, is "Steel Mill Buildings," by M. S. Ketchum, Engineering News Publishing Company, New York, N. Y.

9. **Stresses in Roof Trusses, and Sizes of Members.** Stresses in roof trusses of any form can be computed by the methods of "Statics" (pp. 23 to 73). On account of the ease and economy of manufacture, some form of truss is usually used in which there are many members with equal stresses. The Fink truss, or some modification of it, is almost universally used (see Fig. 1, *c, d, e, f*). On pages 21 and 22 are shown some forms of trusses, together with the pitches which are commonly used.

The stresses in the various members due to a vertical panel load of one pound are given. To obtain the stress in that member due to any other vertical panel load, multiply the stress here given by the vertical panel load.

For example, if the stresses in U_2L_2 (Fig. 24) or L_0L_1 (Fig. 31) due to a panel load of 3 000 pounds, were required, they would be determined as follows:

$U_2 L_2$ (Fig. 24) $3\,000 \times -1.73 = -5\,190$ pounds.
$L_0 L_1$ (Fig 31) $3\,000 \times +5.00 = +15\,000$ pounds

These diagrams are especially useful, since it is the custom of many engineers not to compute the stresses due to wind, snow, and

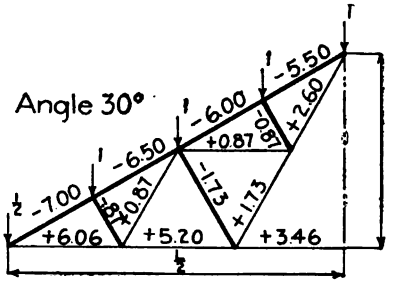


Fig. 24.

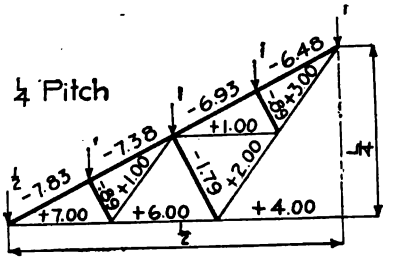


Fig. 25.

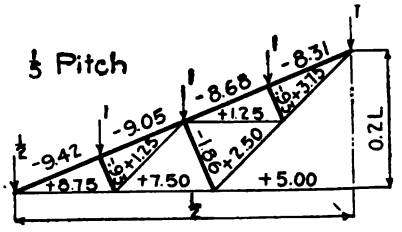


Fig. 26.

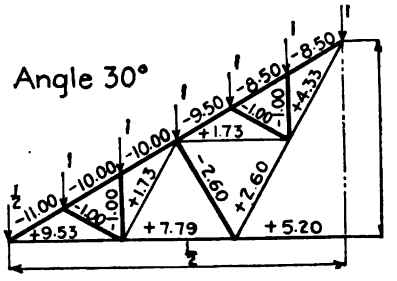


Fig. 27.

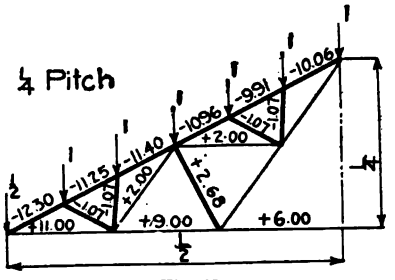


Fig. 28.

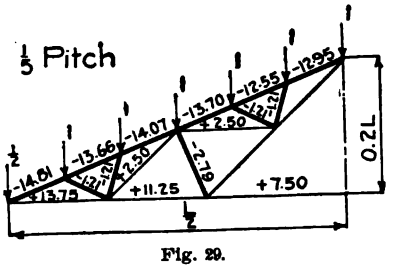


Fig. 29.

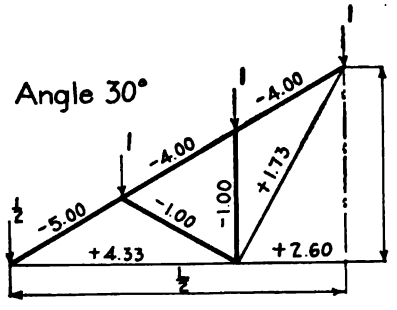


Fig. 30.

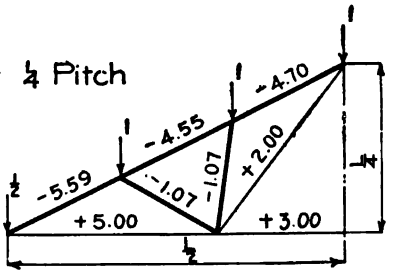


Fig. 31.

Analysis of Stresses in Various Members of Fink Truss Due to Unit-Loads.

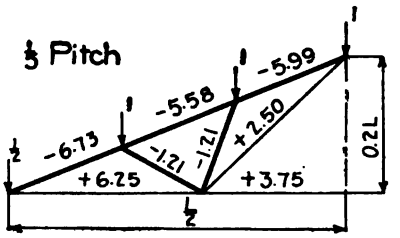


Fig. 32.

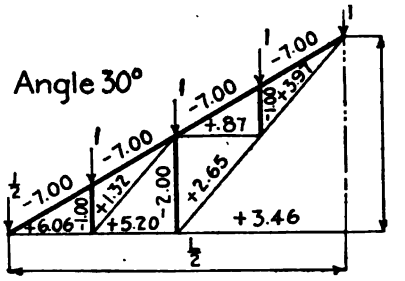


Fig. 33.

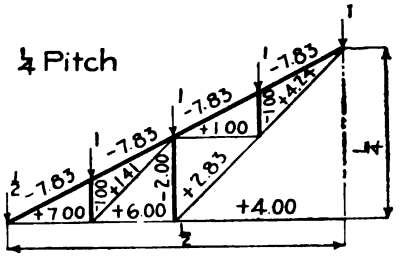


Fig. 34.

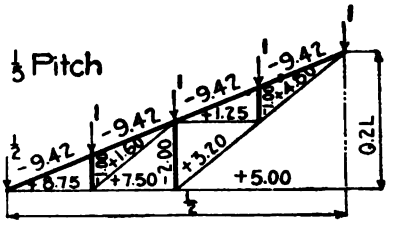


Fig. 35.

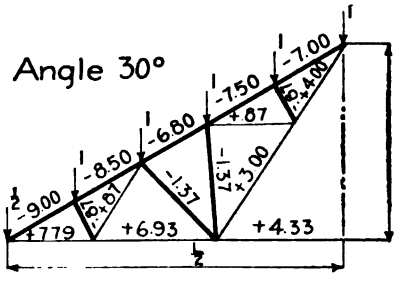


Fig. 36.

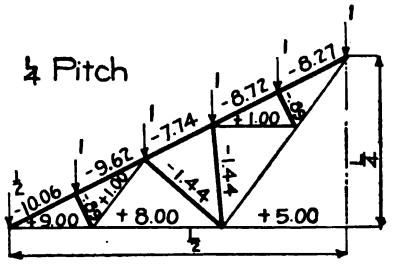


Fig. 37.

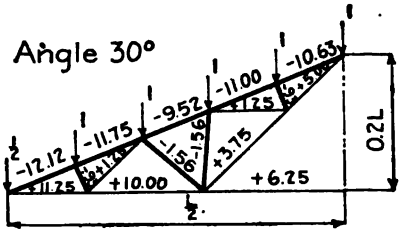


Fig. 38.

Analysis of Stresses in Various Members of Fink Truss Due to Unit-Loads.

dead weight of roof trusses and coverings, but to compute the stresses due to a dead panel load caused by 40 pounds per square foot of horizontal projection. The stresses resulting from this procedure are very nearly equal to those produced by considering the various loads—as snow, dead load, and wind—separately or together. Whenever differences occur, they are on the safe side, except as noted below, and in the next article, in case of the stresses produced by the use of knee-bracing.

The panel load to be used when 40 pounds per square foot of horizontal projection is considered, may be computed from the formula:

$$P = \frac{40 \times a \times l}{n},$$

in which,

a = Distance between trusses, in feet;
l = Span of truss, in feet;
n = Number of panels in top chord of truss.

For example, let it be required to compute the panel load P for the truss of Fig. 24 when the span is 70 feet and the distance between trusses is 16 feet. Here $a = 16$; $l = 70$; and $n = 8$.

$$P = \frac{40 \times 16 \times 70}{8} = 5\,600 \text{ pounds.}$$

The truss would then be computed for a vertical panel load of 5 600 pounds, and the members designed to withstand the stresses thus obtained.

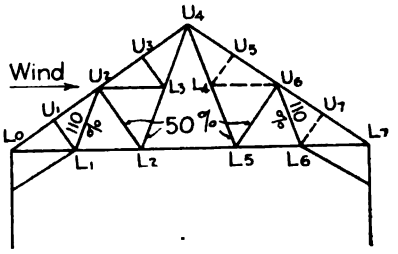


Fig. 39. Allowance for Stresses Due to Wind.

This method is applicable to all spans up to 100 feet when the truss is set on masonry walls (or steel columns built in masonry walls) and the roof covering is of corrugated steel or any of the ordinary materials. Where clay tile or slate are used; 50 pounds should be taken; and in case of concrete slabs, 65 pounds would be about right. It is better practice to compute the stresses due to wind, snow, and dead loads when clay tile, slate, or concrete are used.

In cases where the roof truss is placed on steel columns and is connected with the column by a knee-brace at the first joint (see Fig. 39), stresses caused by the overturning action of the wind take

place in those members shown by heavy lines. In this case the stresses caused by 40 pounds per square foot of horizontal projection are not large enough; but the truss will be safe enough if the stresses as determined by the 40 pounds are increased by the amounts indicated in Fig. 39.

For example, let the truss of Fig. 24 be supported by steel columns and knee-bracing. Let the span be 60 feet, and the distance between trusses 16 feet; and let it be required to compute the stresses in $L_1 U_2$ and $L_3 U_4$. Here $P = (40 \times 16 \times 60) \div 8 = 4\,800$, and the stresses will be:

$$L_1 U_2 \quad (0.87 \times 4\,800) \times 2.10 = +\ 8\,350 \text{ pounds.}$$
$$L_3 U_4 \quad (2.60 \times 4\,800) \times 1.50 = +18\,700 \text{ pounds.}$$

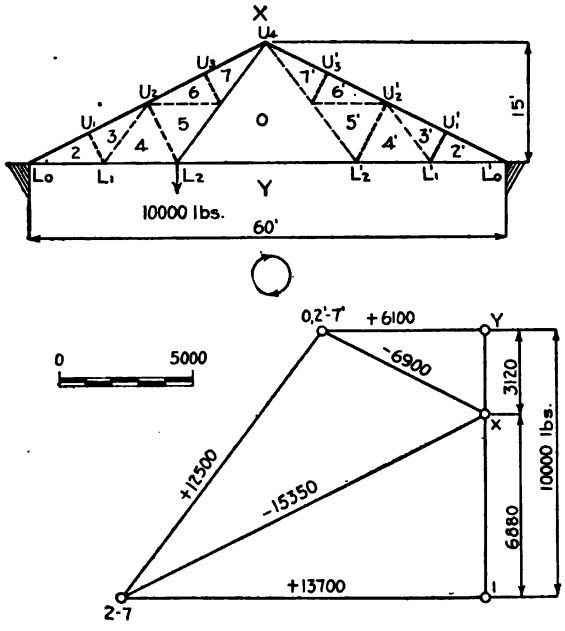


Fig. 40. Fink Truss Loaded with 5-Ton Hoist; also Stress Diagram of Same.

In addition to the above conditions, shafting, heating apparatus, small cranes, and electric wiring and other conductors are often attached to the lower chord of the truss. These cause additional stresses. The case is that of a concentrated load or loads at the lower chord, and the stresses may be computed by the methods given in "Statics."

For example, let a 5-ton hoist be connected as shown in Fig. 40. This hoist runs longitudinally of the shop, or perpendicularly to the plane of the roof truss. The maximum stress in the truss due to this cause will occur when the hoist is directly beneath the truss. The stresses will be those caused by a load of 10 000 pounds at the second panel point of the lower chord. Fig. 40 gives the stress diagram for

TABLE VI

Hoist Stresses in Fink Truss

Member	Stress	Member	Stress
$L_0 U_4$	−15 350	$L_2 U_4$	+12 500
$L_0 L_2$	+13 700	$U_4 L'_0$	− 6 900
$L'_2 L'_0$	+ 6 100	All Others	0

this condition, and Table VI gives the stress record. From this it is seen that the hoist does not affect all members of the truss. The stresses due to the hoist should be added to those caused by the 40 pounds per square foot of horizontal projection, and the member designed accordingly. Of course, if the stress caused by the hoist decreases the stress caused by the 40 pounds, the member must be designed for the stress due to the 40 pounds.

Note that concentrated loads, as in the case of the hoist, cause different stresses in symmetrical members on the two sides of the truss. In the final design, the members are made the same, being designed for the greatest stress. This is done for the sake of economy in manufacture; and besides, it might be desirable to change the hoist to the other side of the truss.

For Fink trusses with pitches of from $\frac{1}{6}$ to $\frac{1}{3}$, and spans of less than 100 feet, very light angles are usually required for the members, unless heavy, concentrated loads are placed on the lower chord. The thickness of the connection plates is seldom more than $\frac{3}{8}$ inch, the top chord angle seldom greater than 5 by $3\frac{1}{2}$-inch, the lower chord angle seldom greater than 3 by 3-inch; and the web members are usually composed of angles either 2 by 2-inch or $2\frac{1}{2}$ by $2\frac{1}{2}$inch. It appears to be the rule, in present practice, to make the sizes such that the thickness shall be $\frac{1}{4}$ or $\frac{5}{16}$ inch. Connection plates for spans up to 60 or 70 feet are usually $\frac{1}{4}$ inch thick, except in the case of that at point L_0.

The stresses in knee-braces depend upon the height and also the width of the building. The stresses may be computed according to the methods of the next article, and the knee-bracing should be

designed accordingly. The inspection of a number of plans seems to indicate that the sizes of knee-braces vary from two angles $2\frac{1}{2}$ by $2\frac{1}{2}$ by $\frac{1}{4}$-inch for spans of 30 feet and a height of building of 35 feet, to two angles 4 by 3 by $\frac{5}{16}$-inch for a span of 70 feet and a height of building to the top of the truss of 75 feet.

In case of roof trusses with the chords nearly parallel (see Fig. 3, p. 2), the stresses, on account of the small depth, are usually quite large, and much heavier members than above mentioned are required. In some cases, 6 by 6-inch angles with 8-inch plates are used, and connection plates of $\frac{3}{8}$ to $\frac{1}{2}$ inch are common.

In cases where the trusses are subjected to the action of corrosive gases, the thickness of the members should be made greater than that

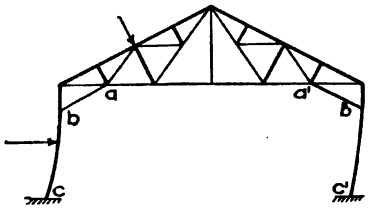


Fig. 41. Bending Tendency, Ends Free.

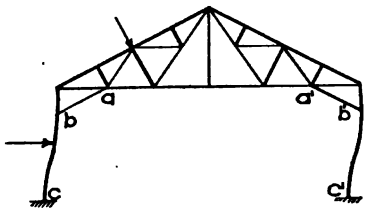


Fig. 42. Bending Tendency, Ends Fixed.

required by the design alone, since corrosion will decrease the section considerably, and this should be allowed for.

10. **The Steel Truss-Bent.** When a truss is connected to steel columns at its ends and by means of knee-bracing (see Fig. 39), it forms what is called a *steel truss-bent.* The stresses in the truss due to the roof covering and snow loads are the same as when it is supported by a masonry wall; but the wind stresses are different. The wind blowing on the roof and also on the sides of the building, causes stresses in the truss. The wind on the building is transferred to the columns, which, by means of the knee-braces, cause stresses in the truss. The whole bent tends to bend as shown in Fig. 41 if the ends of the columns rest on masonry pedestals. If the ends of the columns are securely bolted to heavy masonry pedestals so that the ends of the post will remain vertical, they will tend to bend as shown in Fig. 42. In the first case, the overturning is resisted by the bending of the post as shown at b and b' (Fig. 41); in the second case, by bending as at b, c, b', and c' (Fig. 42). Since the post is the same size throughout, and the bending caused by the wind the same in both cases, the bend-

ing moment in the post at b and b' (Fig. 41) is less than what it is at b and b' (Fig. 42), as in the first case there are only one-half the number of points to withstand the total bending that there are in the second case.

The wind blowing on one side of the building causes a compressive stress in the column on the *leeward* side (the side opposite that on which the wind blows) and a tensile stress in the column on the *windward* side (the same side on which the wind blows). It also creates a bending moment as mentioned above; and this, as well as the direct stresses, must be taken into account when the post is designed. The case is that of a member under direct compression and bending at the same time.

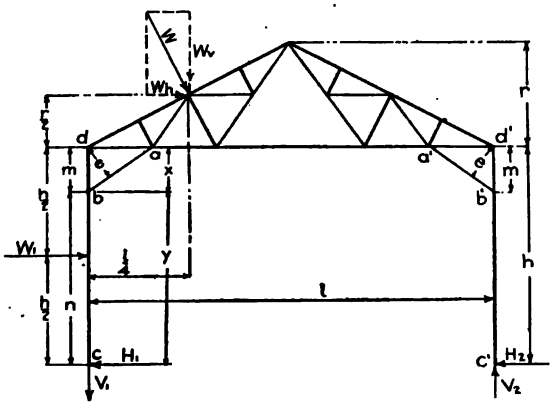


Fig. 43. Notation for Formulæ, Ends Free.

The stresses in the knee-braces and the columns, and the bending in the columns when the ends of the posts are not fixed, may be computed from the following formulæ, in which,

W = Total wind load perpendicular to the roof;
W_h = Horizontal component of W;
W_v = Vertical component of W;
W_1 = Total wind load on the side of the building;
w = Unit wind load normal to the roof;
w_1 = Unit wind load normal to the side of the building;
a = Distance between trusses, in feet.

These and other characters are shown in Fig. 43.

$$W = wa\sqrt{r^2 + \left(\frac{l}{2}\right)^2}$$

$$W_1 = w_1 ah$$

$$H_1 = H_2 = \frac{W_h + W_1}{2}$$

$$S_{b'a'} = -\frac{W_1 \frac{h}{2} + W_h (h + \frac{r}{2}) + W_v \frac{l}{4}}{e} = V_2$$

$$S_{bc} = +\frac{W_1 \frac{h}{2} + W_h (h + \frac{r}{2}) - W_v \frac{3l}{4}}{e} = V_1$$

$$S_{a'b'} = -\frac{H_2 h}{e}$$

$$S_{ab} = +\frac{H_1 h - W_1 \frac{h}{2}}{e}$$

Bending moment at $b = H_1 - W_1 (\frac{h}{2} - m)$
Bending moment at $b' = H_2 n$.

The stresses in the truss caused by the wind are the same as if it were under the action of the normal wind load W, and in addition two concentrated loads equal in intensity and direction to the stresses in the knee-braces and at the same point of application, and two forces E_1 and E_2, which may be computed as follows:

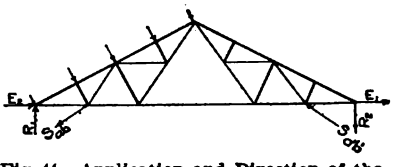


Fig. 44. Application and Direction of the Exterior forces.

$$E_1 = \frac{H_2 n}{m}$$

$$E_2 = \frac{H_1 n - W_1 (\frac{h}{2} - m)}{m}$$

For the points of application for these loads and for their direction, see Fig. 44. The stresses can now be computed by the method of Statics.

The diagram for such a truss-bent is given in Fig. 45. The span is 60 feet, the rise $\frac{1}{4}$, the distance between trusses 16 feet; and the wind pressure is taken as 18 pounds per square foot normal to the roof surface, and 20 pounds per square foot normal to the sides. In this case, $w = 18$ pounds; $a = 16$ feet; $r = 60 \div 4 = 15$ feet; $w = 20$ pounds; $h = 20$ feet; $n = 14$ feet; $m = 6$ feet; and $l = 60$ feet. The length of L_0U_4 is readily computed to be 33.5 feet; $L_0 L_1$, 9.1 feet; and $e = 5$ feet. The values of the quantities and stresses are computed as follows (see Fig. 46):

$W = 18 \times 16 \sqrt{30^2 + 15^2} = 9\,650$ pounds.
$W_1 = 16 \times 20 \times 20 = 6\,400$ pounds.
$W_h = (9\,650 \div 33.5) \times 15 = 4\,320$ pounds.
$W_v = (9\,650 \div 33.5) \times 30 = 8\,650$ pounds.
$H_1 = H_2 = (4\,320 + 6\,400) \div 2 = 5\,360$ pounds.

$$S_{ab} = +\frac{5\,360 \times 20 - 6\,400 \times 10}{5} = +8\,640 \text{ pounds.}$$

$$S_{a'b'} = -\frac{5\,360 \times 20}{5} = -21\,440 \text{ pounds.}$$

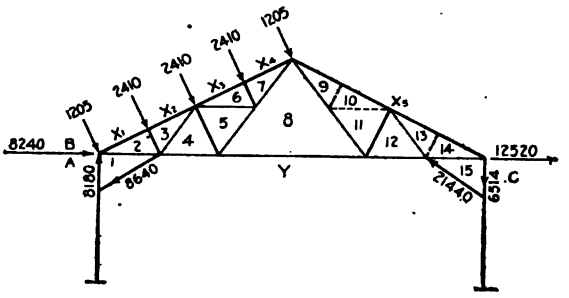
$$E_1 = \frac{5\,360 \times 14}{6} = 12\,520 \text{ pounds.}$$

$$E_2 = \frac{5\,360 \times 14 - 6\,400 \times 4}{6} = 8\,240 \text{ pounds.}$$

The horizontal and vertical components of the stresses in the knee-bracing should now be computed. They are:

For ab: horizontal, 7 240; vertical, 4 760 pounds.

For $a'b'$: horizontal, 17 900; vertical, 11 800 pounds.



As a check upon the computations, the sum of the values of E_1, E_2, and W_h should be equal to the sum of the horizontal components of the knee-braces. By summing up the above values, it will be seen that they check by 80 pounds, which is less than 0.4 of one per cent and is a close enough check (see Figs. 46 and 47).

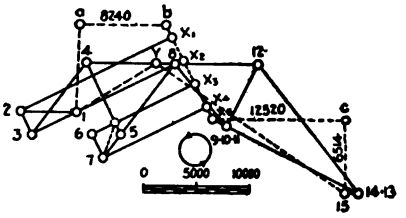


Fig. 45. Stress Diagram of Truss-Bent under Wind Load.

To obtain the vertical reactions, proceed as with a simple truss. For R_2, take the center of moments at L_0 (see Fig. 47). Then:

$$R_2 = \frac{1}{60}\{8\,650 \times 15 + 4\,320 \times 7.5 + 4\,760 \times 9.1 - 11\,800 \times (60 - 9.1)\}$$
$$= -6\,514 \text{ pounds.}$$

The negative sign indicates that the reaction acts downward; that is, the truss must be riveted to the post at L_8, or the end of the post would be lifted off the top of the column.

For R_1, the center of moments is at L_8, and the resulting equation is:

$$R_1 = \frac{1}{60}\{-11\,800 \times 9.1 + 8\,650\,(60 - 15) - 4\,320 \times 7.5 + 4\,760\,(60 - 9.1)\}$$
$$= +8\,180 \text{ pounds.}$$

The bending moment at b is:

$$M_b = 14 \times 5\,360 - 4 \times 6\,400 = 49\,440 \text{ pound-feet;}$$

and the bending moment at b' is:

$$M_{b'} = 5\,360 \times 14 = 75\,040 \text{ pound-feet.}$$

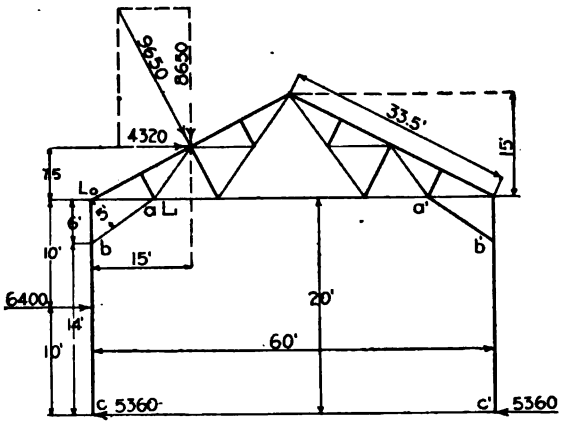


Fig. 46. Position, Direction, and Intensity of Wind Forces, Ends Free.

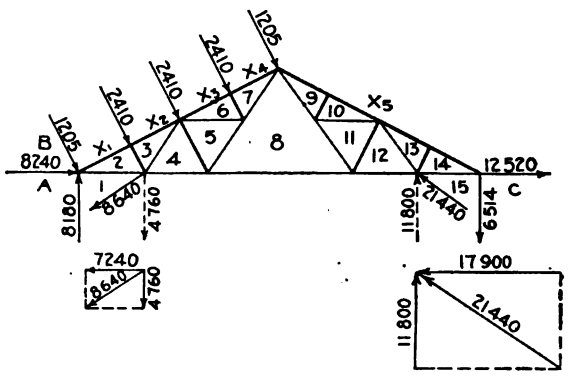


Fig. 47. Position, Direction, and Intensity of Exterior Forces.

The forces in their proper direction are now placed on a diagram of the truss (Fig. 47), and the stresses are solved by the method of Statics. The stress diagram is given in Fig. 45, and the stress record in Table VII.

The above formulæ are for cases when the columns are free at the lower end. When the columns are not free, they are called *fixed*; that is, they are supposed to be so tightly connected that they cannot move when the post bends as shown in Fig. 42. In such cases the result is the same as if the columns were shortened by an amonut $n \div 2$, and the following formulæ result (see Fig. 48):

TABLE VII

Stress Record of Truss-Bent under Wind Load

Member	Stress	Member	Stress
X-2	−15 700	9-10, 11-12	+ 6 500
X-3	−15 700	12-13, 14	−15 300
X-6	−10 700	Y-4	+ 6 500
X-7	−10 700	Y-8	− 1 900
X-9	+ 1 500	Y-12	− 9 600
X-13	+15 400	13, 14-15	− 1 300
1-2	+ 5 200	1-Y	+ 8 640
2-3	− 2 410	15-Y	−21 440
3-4	+ 8 500	A-1	− 8 180
4-5	− 7 500	c-15	+ 6 514
5-6	+ 2 700	b-c	− 3 457
6-7	− 2 410	b'-c'	− 5 193
7-8	+11 000	9-10-11	0
8-9	− 7 600	13-14	0

$$W = wa\sqrt{r^2 + (\frac{l}{2})^2}$$

$$W_1 = w_1 a\,(m + \frac{n}{2})$$

$$H_1 = H_2 = \frac{W_h + W_1}{2}$$

$$S_{ab} = + \frac{H_1(m + \frac{n}{2}) - W_1\left(\frac{m + \frac{n}{2}}{2}\right)}{e}$$

$$S_{a'b'} = - \frac{H_2(\frac{m}{2} + \frac{n}{4})}{e}$$

$$S_{bc} = + \frac{W_1(\frac{m}{2} + \frac{n}{4}) + W_h(m + \frac{n}{2} + \frac{r}{2}) - W_v\frac{3l}{4}}{l}$$

$$S_{b'c'} = + \frac{W_1(\frac{m}{2} + \frac{n}{4}) + W_h(m + \frac{n}{2} + \frac{r}{2}) + W_v\frac{l}{4}}{l}$$

$$E_1 = H_2\frac{n}{2m}$$

$$E_2 = \frac{H_1 \frac{n}{2} - W_1 \left(\frac{n}{4} - \frac{m}{2} \right)}{m}$$

Bending moment at $b = M_b = H_1 \frac{n}{2} - W_1 \left(\frac{n}{4} - \frac{m}{2} \right)$

Bending moment at $b' = M_{b'} = H_2 \frac{n}{2}$.

For the truss-bent of Fig. 45, when the columns are fixed at the base, the stresses are the same as if the columns were shortened by an amount $n \div 2$, as above mentioned. The bent would then appear as in Fig. 49, and the values of the various stresses and the quantities, together with their points of application, are:

$W = 18 \times 16\sqrt{30^2 + 15^2} = 9\,650$ pounds, as before.

$W_1 = 13 \times 16 \times 20 = 4\,160$ pounds.

$H_1 = H_2 = \frac{4\,160 + 4\,320}{2} = 4\,240$ pounds.

$S_{ab} = + \frac{4\,240 \times 13 - 4\,160 \times 6.5}{5} = +5\,616$ pounds.

$S_{a'b'} = - \frac{4\,240 \times 13}{5} = -11\,024$ pounds.

$S_{bc} = + \frac{4\,160 \times 7 + 4\,320 \times 20.5 - 8\,650 \times 45}{60} = -4\,526$ pounds.

$S_{b'c'} = - \frac{4\,160 \times 7 + 4\,320 \times 20.5 + 8\,650 \times 15}{60} = -4\,124$ pounds.

$E_1 = \frac{4\,240 \times 7}{6} = 4\,947$ pounds.

$E_2 = \frac{4\,240 \times 7 - 4\,160 \times 0.5}{6} = 4\,600$ pounds.

$M_b = 4\,240 \times 7 - 4\,160 \times 0.5 = 27\,600$ pound-feet.

$M_{b'} = 4\,240 \times 7 = 29\,680$ pound-feet.

The stresses in the bent are then computed in a manner similar to that used when the columns are fixed, E_1, E_2, and the stresses in the knee-braces being attached to the truss as concentrated loads.

Since in this case, E_1, E_2, and the stresses in the knee-braces are less than they are when the columns are free at the base, the wind stresses throughout the truss will be less when the columns are fixed than when they are free.

On account of the difficulty of fixing the ends rigidly, it is advisable always to consider the ends free and to compute the stresses accordingly.

The student is advised not to take the trouble of determining

the wind stresses in trusses of steel truss-bents by the method given above, but to use the 40 pounds per square foot of horizontal projection and to correct the stresses as previously mentioned (see Fig. 39).

The formulæ of this article giving the stresses in the knee-bracing and the bending moment in the columns, should be used in all cases, and the posts and knee-braces designed according to the stresses so determined.

In cases where the 40 pounds per square foot is used, the direct stress in each column is:

$$S = \frac{40 \times a \times l}{2};$$

and the column should be designed for this stress, together with the stress due to the bending at the point where the knee-brace joins the column. See "Strength of Materials," pp. 85 and 86.

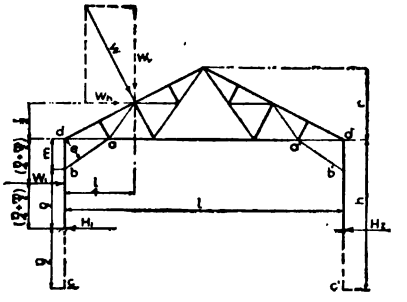


Fig. 48. Notation for Formulæ, Ends Fixed.

In case a crane is attached to either the truss or the column, the stresses due to its action must be considered in the design.

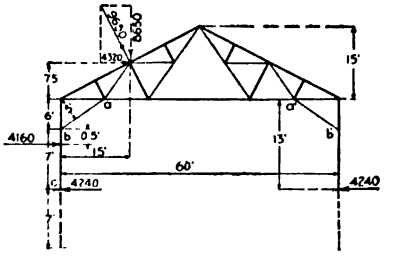


Fig. 49. Position, Direction, and Intensity of Wind Forces, Ends Fixed.

11. **Suspended Loads.** Under this head come any loads which may be suspended from the lower chord of the truss. The load may not be actually suspended from the underneath part, but may be placed above, and the connections so arranged as to bring the weight on the lower chord. This weight should preferably be concentrated at a panel point. In case it cannot be brought directly to the panel point, it may be distributed over a portion or all of the panel. In this case the portions distributed to the adjacent panel points are computed, and they are, for purposes of computation, considered as concentrated loads at the panel points. The sections of the chord over which these loads are distributed are in the condition of direct

tension and bending, and must be designed for such stresses (see "Strength of Materials," pp. 85 and 86).

The suspended loads may consist of small hand cranes; shafting for transmission of power; heating apparatus, such as steam or hot-air pipes; water or compressed-air tanks; or platforms on which stand the operators for the cranes or hydraulic lifts. Figs. 50 and 51 show trusses with various forms of suspended loads attached.

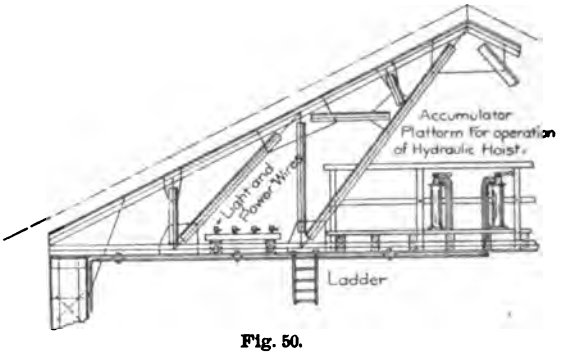


Fig. 50.

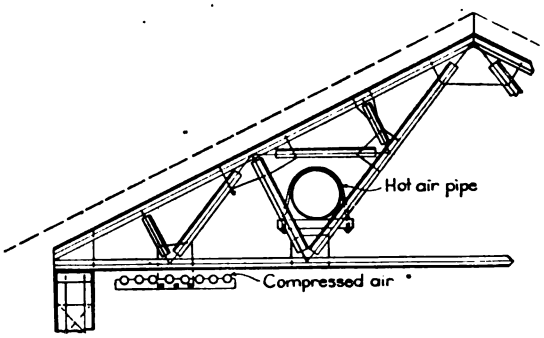


Fig. 51.
Various Kinds of Suspended Loads.

12. **Details of Roof Trusses.** The spans of triangular roof trusses of the Fink type are usually less than 100 feet, and the spans of roof trusses with chords nearly horizontal are seldom greater than 50 feet. For trusses of such spans the details are almost standard. Since these spans and trusses constitute a large majority of those built, only the details of such trusses will be considered in this text.

Where trusses rest on masonry walls or on light columns in masonry walls, provision is made for expansion due to temperature. For trusses up to 75 or 80 feet, slotted holes are placed in the end-bearing, and the bearings rest directly upon another plate. Bolts are fastened to the masonry, and extend upward through the slotted holes and have nuts on their ends. The nuts hold the truss securely to the wall, while the slotted holes allow the bearing to move backward and forward when the temperature falls or rises. *The slotted holes*

should be ⅛ inch in length for every ten feet of span. The bolts should not be less than ½ inch in diameter, and should be buried in the masonry at least 6 inches. Fig. 52 shows details of an expansion bearing of this character. In case the span of the truss is greater than 75 or 80 feet, a roller or a rocker bearing is used. Figs. 53 and 54 show details of this class of bearings.

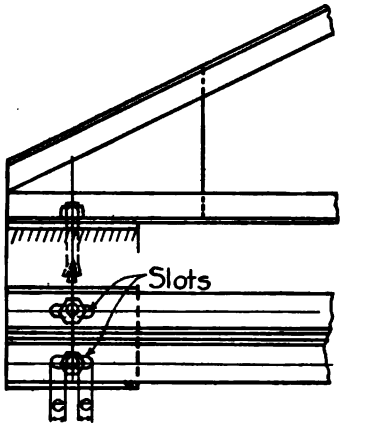


Fig. 52. Slotted-Hole Expansion Bearing.

For convenience in references to the common Fink truss, the following notation will be used: the points in the upper chord are given the letter U, with a subscript corresponding to the number of the joint from the left end. The lower chord and interior joints are given the letter L, with a subscript corresponding to the number of the joint from the left end (see Figs. 24 to 38). The advantage of this system of notation is that it enables one to refer to any particular joint by the use of the letter and its subscript, and its position will at once be apparent to the mind without the use of a figure.

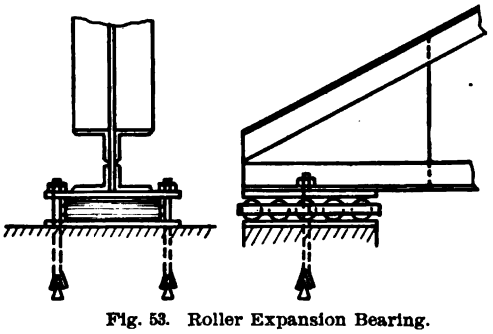


Fig. 53. Roller Expansion Bearing.

If a truss rests on masonry walls, three methods of making the details at L_0 are in common use. These are shown in Figs. 55, 56, and 57. The detail shown in Fig. 55 is the most commonly used; but its use is not advised unless a sufficient number of rivets are placed in the members to take up both the direct stress and that due to the fact that the point of application of the reaction does not coincide with the intersection of the center lines of the chord members.

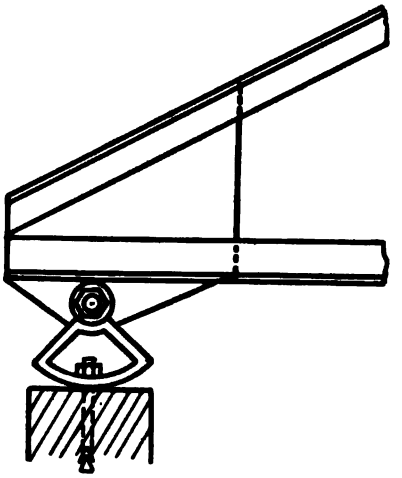


Fig. 54. Rocker Expansion Bearing.

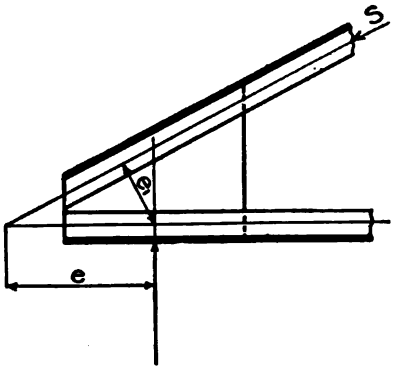


Fig. 55.

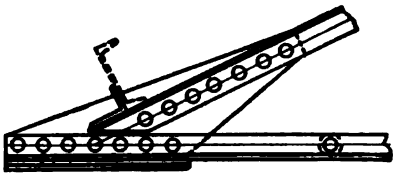


Fig. 56.

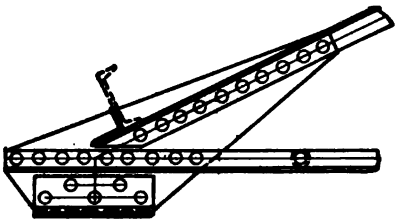


Fig. 57.
Details of Ends of Roof Trusses.

In case the bearing shown in Fig. 55 is used, the number of rivets in $L_0\ L_1$ may be calculated from the equation:

$$n^2v - Rn = \frac{6\,Re}{p},$$

in which,

n = Number of rivets required;
v = Allowable stress on one rivet;
R = Vertical reaction;
p = Rivet spacing, in inches;
e = Distance as shown in Fig. 55.

If the number of rivets in $L_0\ U_1$ is desired, it may be calculated from the equation:

$$n^2v - Sn = \frac{6\,S\,e_1}{p},$$

in which S is the stress in $L_0\ U_1$, e the distance shown in Fig. 55, and the remaining notation as above.

If the point of application of the reaction coincides with the intersection of the center lines of the top and bottom chords, the number of rivets required to withstand the direct stress, which is the only stress would be equal to the stress in that member divided by the allowable stress in one rivet.

In order to illustrate the use of the above equation, and to bring out the fact that more rivets are required when the point of application of the reaction does not coincide with the intersection of the upper and lower chords than when it does coincide, an example will be solved. The stresses, the thickness of the connection plate, and the distance of the point of application of the reaction from the intersection of the chord, are as shown in Fig. 58.

It will be assumed that the chords consist of two angles each; and since this is the case, the allowable unit-stress in one rivet will be

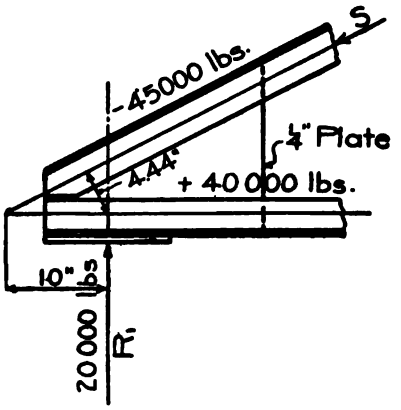


Fig. 58. Data for Example on Page 37.

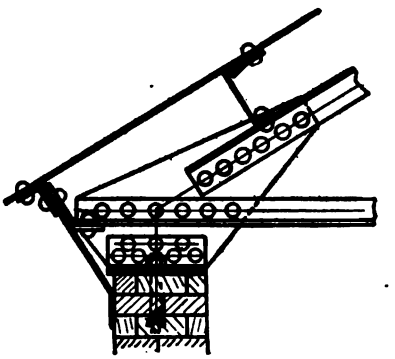


Fig. 59. Eave Detail for Fig. 57.

3 750 pounds, the value of a ¾-inch rivet in bearing in a ¼-inch plate when the allowable unit bearing stress is 20 000 pounds per square inch. If the point of application of the reaction coincides with the intersection of the two chords, the number of rivets required will be:

$$\text{For } L_0\, U_1 \quad \frac{45\,000}{3\,750} = 12.00 \text{ rivets.}$$

$$\text{For } L_0\, L_1 \quad \frac{40\,000}{3\,750} = 10.67 \text{ rivets.}$$

Since the point of application of the reaction does not coincide with the intersection of the chord, the number of rivets required in $L_0\, U_1$ is:

$$3\,750\, n^2 - 45\,000\, n = \frac{6 \times 45\,000 \times 4.44}{3},$$

the spacing being 3 inches; dividing by 3 750, we have:

$$n^2 - 12n = 106.56.$$

Completing the square and solving for n, there is obtained:

$$n = 6 + \sqrt{142.56} = 17.9, \text{ say 18 rivets.}$$

The number of rivets required in $L_0 L_1$ is:

$$3\,750\, n^2 - 20\,000\, n = \frac{6 \times 20\,000 \times 10}{3};$$

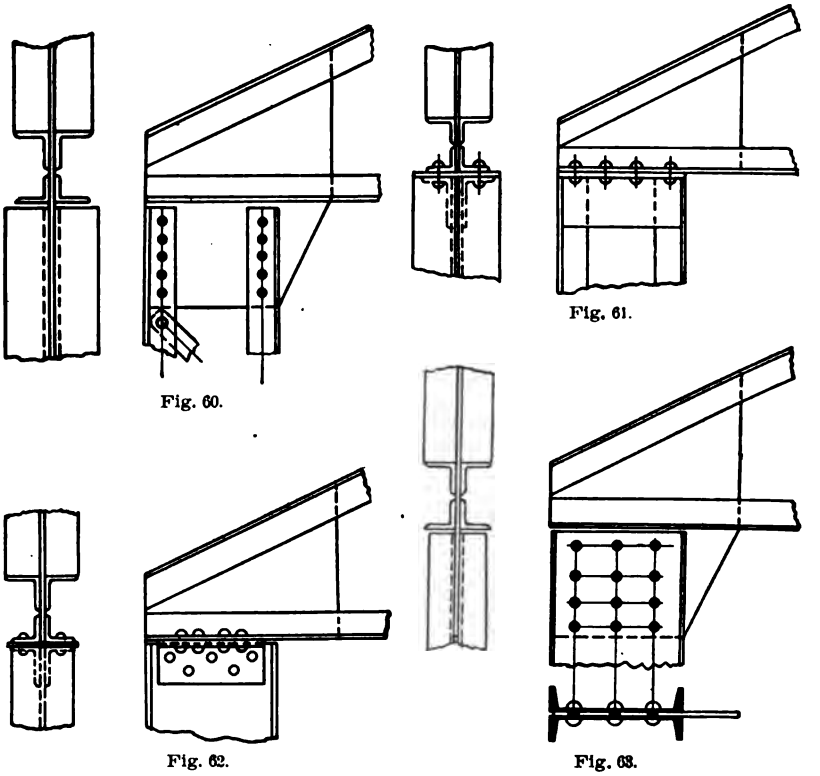


Fig. 60. Fig. 61. Fig. 62. Fig. 63.

Details of Tops of Columns.

and dividing by 3 750 and completing the square, there results:

$$n = 2.67 + \sqrt{113.75} = 13.32, \text{ say 14 rivets.}$$

Inspection of the above results shows that when the point of application of the reaction is placed 10 inches from the intersection of the chords, it requires 6 more rivets in the upper chord and 3 more rivets in the lower chord than would be required if the point of application of the reaction coincided with the intersection of the chords.

The detail just discussed is a very convenient one, and is very commonly used; but in most cases no allowance is made for the additional rivets required because of the fact that the reaction does not coincide with the intersection of the chord members. The student should always compute the rivets by the formulæ given above, since it is very evident that neglect to do so causes the joint to be exceedingly weak, in some cases as much as 50 per cent, as is shown in the case of $L_0 L_1$ in the above problem.

Fig. 56 is excellent, but the length of the bearing plate, which should be as long as the connection plate, is liable to become greater than the width of the wall. In such cases the detail shown in Fig. 57 is to be used. The objection raised to these details is that the end connection plate prevents the placing of a purlin near the end of the roof truss. In case sheathing is used, this objection does not hold good, since the overhanging sheathing will reach to the end of the truss and form a good eave detail, as shown in Fig. 59.

When the roof truss rests on steel columns which are composed of latticed angles, the connections may be made as shown in Figs. 60 and 61. Fig. 60 is preferable, because it gives a more rigid connection than is given by Fig. 61. If the columns consist of two panels placed close together, back to back, the same details may be used. If the column consists of one I-beam or of two channels placed back to back at some distance apart, then details shown in Figs. 62 and 63 may be used.

Where one member is joined to another and makes an angle or is perpendicular to it, then details as shown in Figs. 64 and 65 may be used. It is not good practice to cut the angles as shown at *b* in Fig. 65; *a* is a better detail. No joints should have less than two rivets.

In places where three members meet, and two make the same angle with one of the others, the details should be made as shown in Fig. 66. The leg of the angle which is not joined to the plate should always be upward. This prevents the dust and dirt from becoming mixed with the moisture and running or jarring down into joints at the lower ends of the members.

At L_1 and L_3, square plates (see at left, Fig. 67) should be used where possible. If the stresses are such that more rivets are required in one member than in the other, then the plate should be cut as shown at right in Fig. 67.

At L_2 the splice occurs, since Fink trusses are usually shipped in two parts. In addition to the vertical connection plate, which also acts as a splice plate, the bottom plate is used (see Fig. 68). Rivets shown in black indicate that the holes are left open, the pieces in which they occur are shipped separately, and then are riveted together at the place where the truss is put up.

In some cases where the member L_2 L_5 is long enough to sag

Fig. 64.

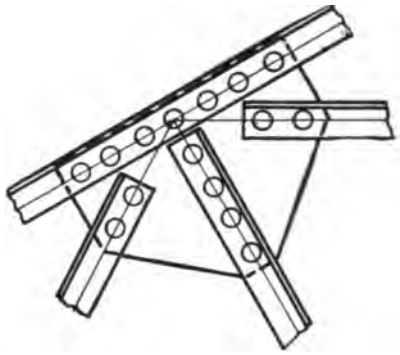


Fig. 66.

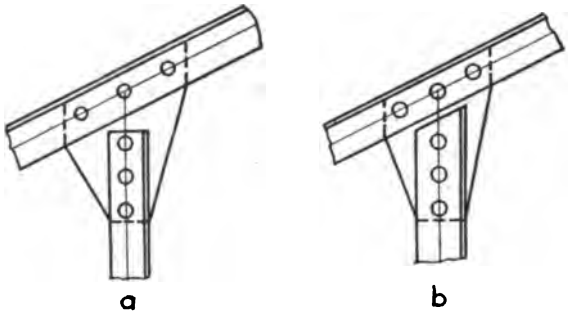


Fig. 65.

Details of Roof Truss Connections. See also Figs. 67 and 68.

considerably, or where it is desired to connect a load (such as a hand hoist) at its center, a vertical U_4 M is run from U_4 and connected to the lower chord. No stress is caused in this member by any load except the load at M, in which case the stress is equal to that load. If a load is at M, it will cause stresses in other members of the truss, the stresses in the truss being the same as if the dead panel load at U_4 were increased by an amount equal to the load at M.

The general details of a Fink truss are shown in Plate I (p. 43), Plate II (p. 60), and Plate III (p. 61).

In case the building is devoted to some purpose wherein no smoke or noxious gases are produced, some form of *patent ventilator* may be used. One very excellent make is shown in Fig. 69—called the *Star* ventilator (Merchant & Co., Philadelphia, Pa.). These ventilators are made from 2 to 60 inches in diameter at the lower portion, where they fit to the ridge of the roof. Fig. 70 shows one of them in position on a roof. The number and size of these ventilators depend of course upon the number of times per hour it is desirable to change the air in the shop.

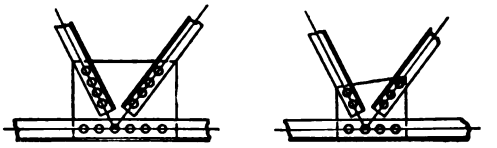


Fig. 67.

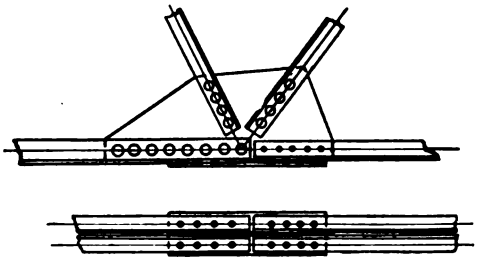


Fig. 68.

Details of Roof Truss Connections.
See also Figs. 64, 65, and 66.

In case the shop is for such purposes that smoke, gases, or noxious fumes of any kind are produced, it is desirable to have some channel for ventilation which is considerably larger than those given by the patent ventilators. In such cases the ventilation is usually obtained by a small house-shaped construction called a *lantern, monitor,* or *ventilator* (see Fig. 71). The sides of these ventilators may be fitted with *louvres* or windows, or left open. Louvres may be made either of wood or of corrugated or plain bars. For details of monitors and louvres, see Figs. 124, 125, and 126.

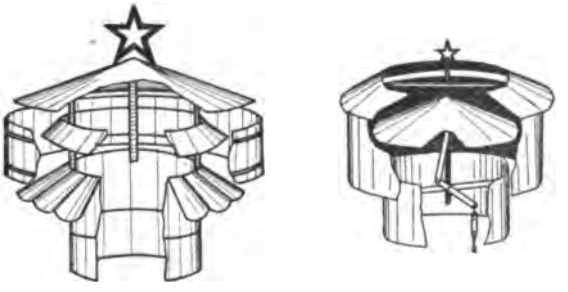


Fig. 69. Details of "Star" Ventilators.

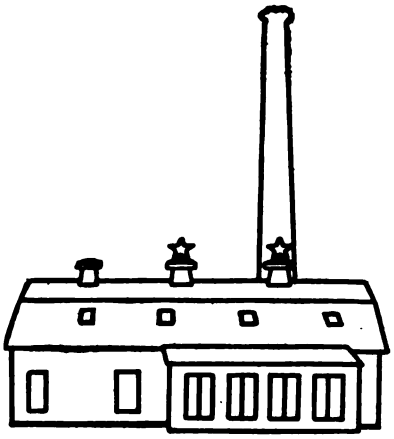


Fig. 70. "Star" Ventilators on a Roof.

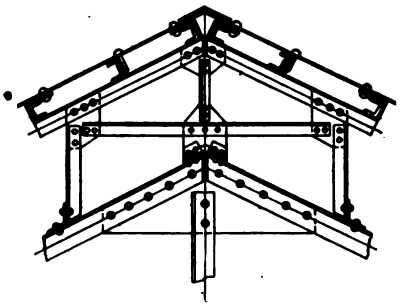


Fig. 71. Detail of a Monitor Ventilator.

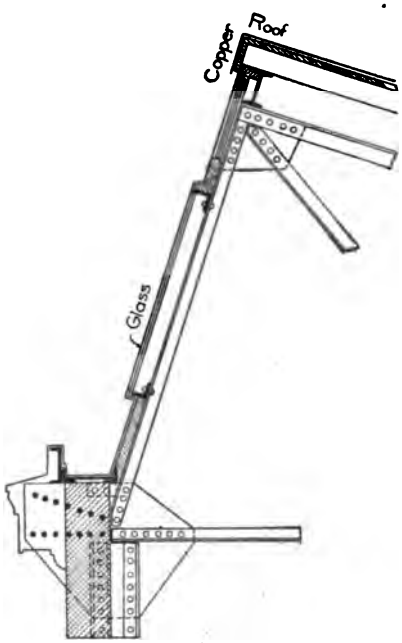
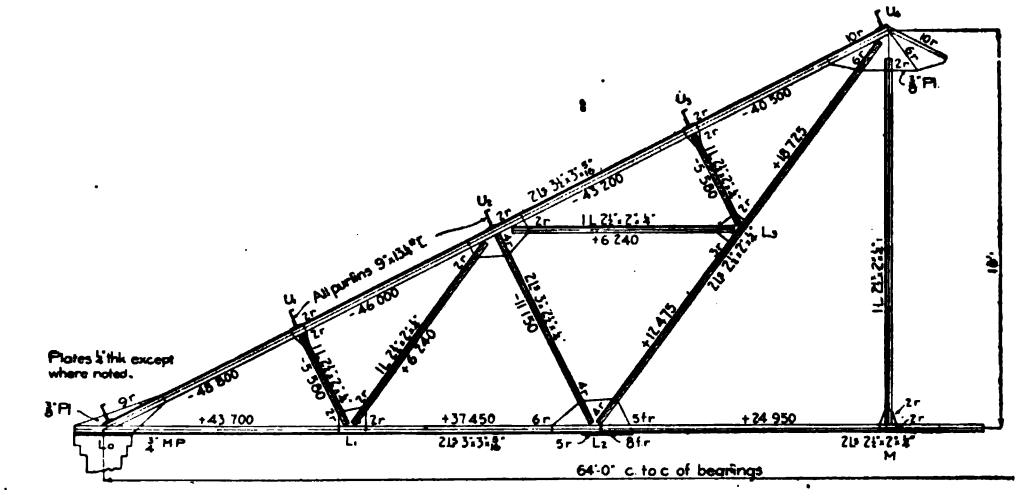
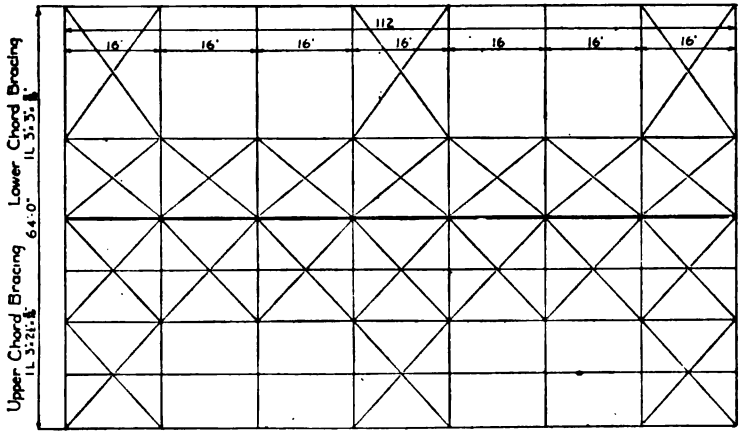


Fig. 72. Detail of Window in Saw-Tooth Roof.

In order to admit sufficient light into the building, part of the roof of buildings over 80 feet wide must be made of glass, since the amount of light admitted from the sides of the building is not sufficient to light up those parts of the shop near the center of the trusses. In some cases the saw-toothed truss is used, in which case the entire surface of the short rafter is covered with glass. In case the ordinary triangular roof truss is used, a portion of the roof covering must be made of glass, so put on as to prevent leakage and also to prevent the moisture which forms on the under side of the glass from dropping in the shop. Fig. 72 shows the glass in place on a saw-toothed roof; and Figs. 73 and 74 give the details of several methods of securing glass on the roof so that no leakage or condensation will get onto the shop floor. The glass area should be from $\frac{1}{10}$ to $\frac{1}{4}$ of the floor area.

13. **Specifications for Roof Trusses and Steel Buildings.** In case of an important structure, special specifications are written, embody-





STRESS SHEET
FOR
FINK TRUSS ROOF

Plate I. Stress Sheet for a Fink Truss Roof.

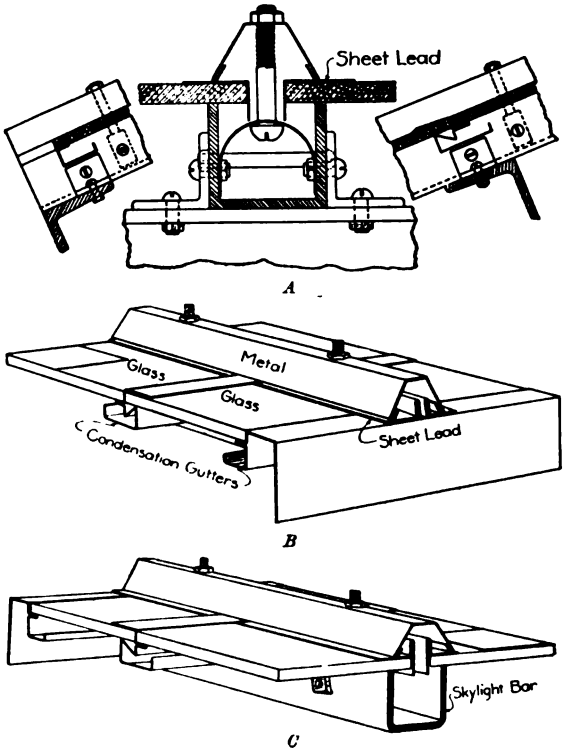


Fig. 73. "Paradigm" Method of Glazing.

ing certain features which the experience of the engineer in charge indicates as necessary. For ordinary structures, however, several very satisfactory specifications are on the market. These consist of from 15 to 20 pages, bound in paper, and may be had for twenty-five cents a copy. Two very satisfactory specifications are those of Charles Evan Fowler and Milo S. Ketchum. Either may be had by addressing the Engineering News Publishing Company, New York City. Fowler's specifications, in addition to giving specifications for load stresses and workmanship, give much valuable information regarding the stresses in different kinds of trusses, besides various details showing the use of corrugated steel.

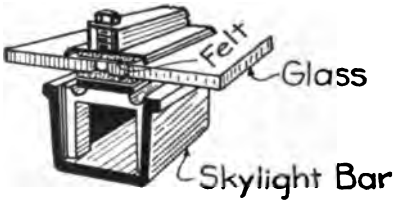


Fig. 74. "Anti-Pluvius" Method of Glazing.

An extended set of specifications is not required for the design of ordinary roof trusses. In addition to the information regarding the weight of trusses, the weight of roof covering, the snow load, and the wind load, the use of Table VIII will be found to be all that is necessary in order to design cross-sections of the various members, once the stresses are determined.

TABLE VIII

Allowable Unit-Stresses, Medium Steel

For Shear	10 000	pounds per square inch.
For Bearing	20 000	" " " "
For Tension	15 000	" " " "
For Bearing of Steel on Masonry	250 to 400	" " " "

For Compression . . . . . $P = 24\,000 - 110\frac{l}{r}$.

In case the stresses are those due to crane loads, the unit-stresses in tension and compression indicated in Table VIII should be reduced $\frac{1}{3}$ and $\frac{1}{2}$ respectively. Members of the lateral bracing and their connections may be allowed an increase of 25 per cent over the unit-stresses there indicated.

In the equation above given "For Compression," l is the length of the member in inches, and r the least radius of gyration. The ratio of $\frac{l}{r}$ should never be greater than 120.

The *gauge line* or *gauge* is the line on the flange of a shape, on which the rivets are placed. In angles and channels it is located by

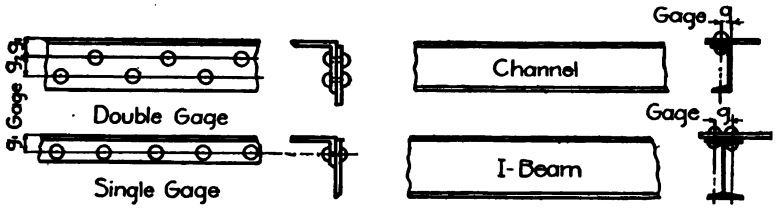


Fig. 75. Gauges for Angles, Channels, and I-Beams.

giving its distance from the back of the shape; in the case of I-beams the distance between two gauge lines on opposite sides of the web is indicated. Some angles have double gauge lines, in which case the rivets are placed first on one and then on the other; this is called *staggering*. Fig. 75 shows gauge lines for various shapes.

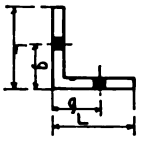
Rivets $\frac{3}{4}$ inch in diameter are generally used in legs of angles 3 to 4 inches long or greater. For the gauge lines and the maximum sizes of rivets to be used in angles, see Table IX. For similar data for channels and I-beams, see Carnegie Handbook, pp. 177–185.

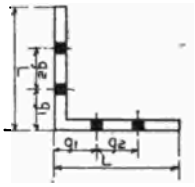
It is often desirable to express the length in feet instead of inches, in which case the formula becomes:

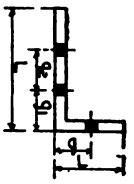
$$P = 24\,000 - 1\,320\,\frac{L}{r}.$$

TABLE IX

Gauges and Maximum Allowable Rivets for Angles







L	g	Maximum Rivet or Bolt	L	g	Maximum Rivet or Bolt	L	g	Maximum Rivet or Bolt
8	$4\frac{1}{2}$	$\frac{7}{8}$	$3\frac{1}{2}$	2	$\frac{7}{8}$	2	$1\frac{1}{8}$	$\frac{1}{2}$
7	4	$\frac{7}{8}$	3	$1\frac{3}{4}$	$\frac{7}{8}$	$1\frac{3}{4}$	1	$\frac{1}{2}$
6	$3\frac{1}{2}$	$\frac{7}{8}$	$2\frac{3}{4}$	$1\frac{5}{8}$	$\frac{3}{4}$	$1\frac{1}{2}$	$\frac{7}{8}$	$\frac{3}{8}$
5	3	$\frac{7}{8}$	$2\frac{1}{2}$	$1\frac{3}{8}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{4}$	$\frac{3}{8}$
4	$2\frac{1}{4}$	$\frac{7}{8}$	$2\frac{1}{4}$	$1\frac{1}{4}$	$\frac{5}{8}$	1	$\frac{9}{16}$	$\frac{1}{4}$

L	g_1	g_2	L	g_1	g_2
8	3	3	6*	$2\frac{1}{2}$	$2\frac{1}{4}$
7	$2\frac{1}{2}$	3	5	2	$1\frac{3}{4}$
6	$2\frac{1}{4}$	$2\frac{1}{2}$			

* When thickness is $\frac{1}{2}$ inch or over.

For convenience in designing, the values of $L \div r$ should be plotted as ordinates, and the resulting values of P as abscissæ, on cross-section paper, and the curve drawn in. Then the value of P for any given value of $L \div r$ may be taken at once from the diagram without the labor of substituting in the above formula.

The bearing value of a rivet in a plate of given thickness is equal to the thickness of the plate, *times* the diameter of the rivet, *times* the allowable unit bearing stress. The value of a rivet in single shear is equal to the area of the cross-section of the rivet, *times* the allowable unit shearing stress. The bearing values of rivets of different diameter in plates of different thickness, and the shearing values of rivets of different diameter, are given in Table X, the unit-stresses being as given above.

TABLE X

Bearing and Shearing Values of Rivets

Diameter of Rivet (Inches)	Single Shear (at 10 000 lbs. per sq. in.)	Bearing in Different Thicknesses of Plates (at 20 000 lbs. per sq. in.)							
		$\frac{1}{4}$ in.	$\frac{5}{16}$ in.	$\frac{3}{8}$ in.	$\frac{7}{16}$ in.	$\frac{1}{2}$ in.	$\frac{9}{16}$ in.	$\frac{5}{8}$ in.	$\frac{11}{16}$ in.
$\frac{1}{2}$	1 960	2 500	3 130	3 750					
$\frac{9}{16}$	2 480	2 810	3 520	4 210	4 920				
$\frac{5}{8}$	3 070	3 130	3 910	4 690	5 470				
$\frac{11}{16}$	3 710	3 440	4 290	5 160	6 010	6 880			
$\frac{3}{4}$	4 420	3 750	4 690	5 630	6 560	7 500	8 440		
$\frac{13}{16}$	5 180	4 070	5 080	6 090	7 110	8 120	9 150	10 160	
$\frac{7}{8}$	6 010	4 380	5 470	6 570	7 660	8 750	9 840	10 940	12 040

DESIGN OF A RIVETED ROOF TRUSS

14. Let it be required to design a Fink roof truss of 64 feet span and $\frac{1}{4}$ pitch, the distance between trusses being 16 feet. The roof covering is taken as 12 pounds per square foot of roof surface, and the total snow and wind load will be taken as 30 pounds per square foot of horizontal projection. The weight of the steel in the roof truss will be computed from Merriman's formula (see Art. 4, p. 6). The total weight is now found to be:

$$\text{Weight of truss, } \frac{3}{4} \times 16 \times 64 \left(1 + \frac{l}{10}\right) = 5\,580 \text{ pounds.}$$

$$\text{Weight of roof cover, } 35.6 \times 2 \times 16 \times 12 = 13\,650 \text{ pounds.}$$

$$\text{Weight of wind and snow } 64 \times 16 \times 30 = 30\,700 \text{ pounds.}$$

$$\text{Total} \quad 49\,930 \text{ pounds.}$$

Each apex load is therefore 49 930 ÷ 8 = 6 240 pounds. By multiplying this value by each of the stresses as given in Fig. 25, the stress in each member is computed as follows:

$L_0\ U_1 = 7.83 \times 6\,240 = 48\,800$ pounds
$L_0\ L_1 = 7.00 \times 6\,240 = 43\,700$ "
$U_1\ L_1 = 0.89 \times 6\,240 = 5\,580$ "
$U_1\ U_2 = 7.38 \times 6\,240 = 46\,000$ "
$L_1\ U_2$ and $U_2\ L_3 = 1.00 \times 6\,240 = 6\,240$ "
$L_1\ L_2 = 6.00 \times 6\,240 = 37\,450$ "
$U_2\ L_2 = 1.79 \times 6\,240 = 11\,150$ "
$U_2\ U_3 = 6.93 \times 6\,240 = 43\,200$ "
$L_2\ L_5 = 4.00 \times 6\,240 = 24\,950$ "
$L_2\ L_3 = 2.00 \times 6\,240 = 12\,475$ "
$U_3\ L_3 = 0.89 \times 6\,240 = 5\,580$ "
$L_3\ U_4 = 3.00 \times 6\,240 = 18\,725$ "
$U_3\ U_4 = 6.48 \times 6\,240 = 40\,500$ "

In the design of this truss, no material thinner than ¼-inch, and no angles smaller than 2½ by 2-inch, will be allowed.

Fig. 76 shows an outline diagram of the truss, with the stresses placed upon it. A positive sign signifies a tensile stress, and a negative sign signifies a compressive stress. The length of the top chords is $\sqrt{32^2 + 16^2} = 35.6$ feet; and the length of each panel is ¼ of this, or 8.9 feet. The horizontal projection of one panel is ¼ of half the span, or $32 \div 4 = 8$ feet.

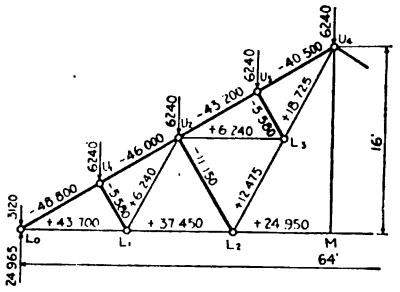


Fig. 76. Stresses in a Fink Truss.

Design of the Purlins. The distance between the trusses is 16 feet, and the distance between the purlins is 8.9 feet; therefore the load coming on one purlin is:

Roof covering, $8.9 \times 16 \times 12$	=	1 710 pounds
Snow and wind, $8 \times 16 \times 30$	=	3 840 "
Total	=	5 550 pounds

This should be resolved in two components, V and H, perpendicular and parallel to the truss chord. These are determined by the proportions of similar triangles, as follows:

$$V : 5\,550 = 32 : 35.6$$
$$V = 5\,080 \text{ pounds.}$$
$$H : 5\,550 = 16 : 35.6$$
$$H = 2\,490 \text{ pounds.}$$

The bending moment caused by V is $M_v = (5\,080 \times 16) \div 8 = 10\,160$ pound-feet. The bending moment caused by H is $M_H = (2\,490 \times 16) \div 8 = 4\,980$ pound-feet. The stress caused by V is $= \frac{M_v c}{I}$; and the stress caused by H is $\frac{M_H c^1}{I'}$; and there is also the condition that the sum of these two stresses shall not be greater than 15 000 pounds. Since the above formula involves the moment of inertia and half the depth of the beam, a beam must be chosen, and its moment of inertia and half-depth substituted in the above equation, and the equation solved. In case the sum of the stresses is in excess of 15 000 pounds, or very much smaller, a re-computation must be made, using a larger or a smaller beam.

A 15-inch 42-pound I-beam will be assumed, and will be examined to see if it fulfils the necessary conditions. The value of I and I' are taken from the Carnegie Handbook, p. 97. The value of c is $\frac{15}{2} = 7\frac{1}{2}$ in the first case, and $\frac{5.50}{2} = 2.75$ in the second case. The quantity 5.50 is the width of the flange of the I-beam. Substituting in the above formula, there results:

$$\frac{10\ 160 \times 12 \times 7\frac{1}{2}}{441.8} + \frac{4\ 980 \times 12 \times 2.75}{14.62} = 13\ 320 \text{ pounds.}$$

The above I-beam could be used; but in case the sheathing is laid closely and nailed tightly, we may consider it acting as a beam of a span of 16 feet, 8.9 feet deep, and of a thickness equal to that of the sheathing, which in this case will be assumed as $1\frac{1}{2}$ inches. The sheathing will then take up the moment caused by the force H; and the purlin will take up the vertical bending moment alone. The stress in the sheathing due to the force H is $\frac{M_H c}{I}$. Here $M = 4\ 980 \times 12$; $c = 8.9 \times 12 \div 2$; and $I = \frac{1.5\,(8.9 \times 12)^3}{12}$. Therefore,

$$S = \frac{4\ 980 \times 12 \times 8.9 \times 12 \times 12}{2 \times 1.5\,(8.9 \times 12)^3}$$

$= 20.95$ pounds per square inch, which is insignificant.

The vertical bending moment taken up by the purlin is $10\ 160 \times 12 = 121\ 920$ pound-inches, and this requires a section modulus of $121\ 920 \div 15\ 000 = 8.14$. By consulting pages 101 and 102 of the Carnegie Handbook, the following is found to be true:

An 8-inch 11.25-pound channel is just too small.

A 7-inch 17.25-pound channel gives the nearest section modulus.

An 8-inch 13.75-pound channel would be lighter and stiffer.

A 9-inch 13.25-pound channel would be still lighter and stiffer; and since it weighs less than any of the others, it will be more economical.

A 9-inch 13.25-pound channel will accordingly be used for the purlins.

On account of one half-panel load coming on the purlin at the ends and ridge of the truss, these purlins must theoretically be only one-half as strong as the other; but, on account of the fact that all purlins must be of the same height, these purlins are made of the lightest weight channel of the same height as the others. In this

case it happens that the lightest weight 9-inch channel is required for the intermediate purlins as well as for the end ones. To illustrate the above, suppose that the purlins were required to be 10-inch 25-pound channels, then the end purlins would be made of 10-inch 15-pound channels.

In case sheathing is not used, then some other method must be employed to take up the bending moment due to the force H. The usual method of doing this is to bore holes in the center of the purlins at the middle point of their span, and to connect them with rods which run from one eave up over the ridge and down to the other eave (see Fig. 22).

Design of Tension Members. *For Member* $L_0\ L_1$: The required net area is $43\,700 \div 15\,000 = 2.92$ square inches. By consulting the Carnegie Handbook, p. 118, it is seen that two 3 by 3 by $\frac{5}{16}$-inch angles give a gross area of $1.78 \times 2 = 3.56$ square inches. From this must be subtracted the rivet-hole made by a $\frac{3}{4}$-inch rivet. Since all rivet-holes are punched $\frac{1}{8}$ inch larger in diameter than the rivet, the amount to be substracted from the above gross area is $\frac{5}{16} \times (\frac{3}{4} + \frac{1}{8}) \times 2 = 0.54$, there being two rivet-holes taken out of the section. This gives a total net area of $3.56 - 0.54 = 3.02$ square inches. As this is but slightly larger than the required net area, these angles will be used for this member. Since the stress in this member is the greatest stress in the bottom chord, and since the bottom chord is made of the same section up to the splice at L_2, on account of economical construction, it being cheaper to run the same sized angle throughout than it would be to change the size of each panel and make a splice at each panel point, the size of angle as determined above will be used for the first two panels of the bottom chord at each end.

For Member $L_2\ L_5$: The required net area is $24\,950 \div 15\,000 = 1.67$ square inches. From Carnegie Handbook, p. 115, two angles $2\frac{1}{2}$ by 2 by $\frac{1}{4}$-inch give a gross area of $2 \times 1.06 = 2.12$ square inches; and taking out two $\frac{5}{8}$-inch rivets, the net area is $2.12 - \frac{1}{4}(\frac{5}{8} + \frac{1}{8}) \times 2 = 1.74$ square inches. This coincides very closely with the required area, and this angle will be used. Even if this angle should have been in excess of the required area, it would still be necessary to use it, since it is the smallest angle and of the least thickness allowed.

For Member $L_3 U_4$: The required net area is 18 725 ÷ 15 000 = 1.25 square inches. Two angles $2\frac{1}{2}$ by 2 by $\frac{1}{4}$-inch give a gross area of 2.12 square inches, and a net area of 1.74 square inches, as above computed. Although they give an area considerably larger than that required, nevertheless they must be used, since they are the smallest allowed.

For Members $L_1 U_2$ and *$U_2 L_3$:* The required net area is 6 240 ÷ 15 000 = 0.42 square inch. One angle $2\frac{1}{2}$ by 2 by $\frac{1}{4}$-inch gives a gross area of 1.06 square inches. The amount to deduct from this is $\frac{1}{4} \times (\frac{5}{8} + \frac{1}{8}) = 0.19$ square inch, one $\frac{5}{8}$-inch rivet-hole being taken from the section. This gives a net area of $1.06 - 0.19 = 0.87$ square inch, which shows this angle to be sufficient.

Since the member $U_4 M$ has no other use than to prevent the bottom chord from sagging, it will be made of the lightest angle allowed. It will therefore be made of one angle $2\frac{1}{2}$ by 2 by $\frac{1}{4}$-inch.

The member $L_2 L_3$ is made of the same section as the member $L_3 U_4$, since this is more economical than to change the section and to make a splice at L_3. It will be made of two angles $2\frac{1}{2}$ by 2 by $\frac{1}{4}$-inch.

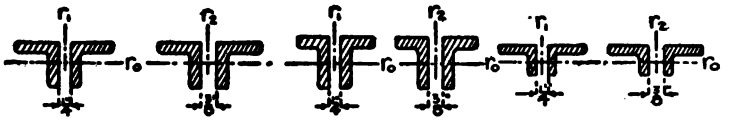
Design of the Compression Members. The general method of procedure in the design of compression members is, first, to assume a cross-section, and then to determine the unit compressive stress allowable by inserting the length of the member and the radius of gyration of the assumed section in the formula given for the unit allowable compressive stress; then divide the stress in the member by the unit allowable compressive stress determined as above. This will give the required area. If this required area is equal to, or slightly less than, the area of the cross-section assumed, the section assumed will be the correct one. If the required area as computed above is greater than the area of the section, then a larger section must be assumed and the operation repeated. Usually only two operations are required in order to obtain a section whose area is correct. It should be noted that the area of the rivet-holes is not deducted from the section in compression members, since the rivet fills up the rivet-hole and makes a section as strong in compression as it was in the first place. Care should be taken to assume a section whose radius of gyration is equal to or greater than the length of the member divided by 120. This is due to the fact that $l \div r$ should not be greater than 120. Compression members of roof trusses for the usual

spans are made of two angles placed back to back. The radius of gyration of such a section is equal to the radius of gyration of one angle, if it is referred to an axis perpendicular to the legs which are placed together. If it is referred to an axis through the center of the section and parallel to the legs which are placed together, it is equal to some value other than the radius of gyration of one angle. The radii of gyration for pairs of angles placed either directly back to back or a small distance apart, are given on pages 144 to 146 of the Carnegie Handbook and in Table XI, and should be used in the design. The value of the radius of gyration for sizes of angles other than those given, may be obtained by interpolation.

For example, let it be required to determine the radius of gyration of two 5 by $3\frac{1}{2}$ by $\frac{1}{2}$-inch angles placed $\frac{1}{2}$ inch apart and back to back, the 5-inch legs being horizontal (see p. 146, Carnegie Handbook). Since this value is not given in the tables, it must be interpolated from the values given for r_2 for the above sized angle, which are $\frac{5}{16}$ inch and $\frac{7}{8}$ inch thick. The difference between the two thicknesses given is $\frac{7}{8} - \frac{5}{16} = \frac{9}{16}$ inch. The difference between the two values given for the radius of gyration is $2.55 - 2.44 = 0.11$. This gives a difference of $.11 \div 9 = 0.0122$ for each $\frac{1}{16}$ inch difference in thickness in the angle. The difference between the thickest angle and the angle under consideration is $\frac{7}{8} - \frac{1}{2} = \frac{3}{8}$, or $\frac{6}{16}$. Therefore the amount to be subtracted from the radius of gyration of the thickest angle is $6 \times 0.0122 = 0.0732$; and the radius of gyration for two angles placed back to back as above stated is $2.55 - 0.07 = 2.48$. In case one angle is used for a member in compression, the least rectangular radius of gyration must be used; and if two angles are employed, placed back to back, care should be exercised to use the least radius of gyration; and if the angles have unequal legs, those legs should be placed back to back, which will make the rectangular radii of gyration as nearly equal as possible. The values of the radii of gyration will indicate whether the short legs or the long legs should be placed together. The tables given in the Carnegie Handbook give the radii of gyration for angles spaced at distances $\frac{1}{2}$ inch and $\frac{3}{4}$ inch apart; but since the connection plates of roof trusses are usually $\frac{1}{4}$ inch or $\frac{3}{8}$ inch thick, the values of the radii of gyration should be given for angles spaced $\frac{1}{4}$ inch and $\frac{3}{8}$ inch apart. Such values are given in Table XI.

TABLE XI

Radii of Gyration of Angles Placed Back to Back



Equal Legs			Unequal Legs				
Size (Inches)	r_1	r_0	Size (Inches)	r_1	r_2	r_1	r_0
2 × 2 × $\frac{3}{16}$	0.93	0.98	2½ × 2 × $\frac{3}{16}$	0.88	0.92	1.19	1.24
2 × 2 × $\frac{7}{16}$	0.98	1.03	2½ × 2 × ½	0.94	0.99	1.25	1.30
2½ × 2½ × ¼	1.14	1.19	3 × 2½ × ¼	1.09	1.13	1.40	1.45
2½ × 2½ × ½	1.19	1.24	3 × 2½ × $\frac{3}{16}$	1.15	1.20	1.46	1.51
3 × 3 × ¼	1.34	1.39	3½ × 2½ × ¼	1.04	1.09	1.67	1.72
3 × 3 × ⅜	1.41	1.46	3½ × 2½ × $\frac{11}{16}$	1.13	1.18	1.75	1.80
3½ × 3½ × ⅜	1.56	1.61	3½ × 3 × $\frac{5}{16}$	1.30	1.35	1.61	1.66
3½ × 3½ × $\frac{13}{16}$	1.65	1.70	3½ × 3 × $\frac{13}{16}$	1.40	1.45	1.71	1.76
4 × 4 × $\frac{5}{16}$	1.76	1.80	4 × 3 × $\frac{5}{16}$	1.25	1.30	1.88	1.93
4 × 4 × $\frac{13}{16}$	1.85	1.89	4 × 3 × $\frac{13}{16}$	1.35	1.40	1.97	2.02
6 × 6 × $\frac{7}{16}$	2.58	2.63	5 × 3 × $\frac{5}{16}$	1.17	1.22	2.42	2.47
6 × 6 × ⅞	2.66	2.70	5 × 3 × $\frac{13}{16}$	1.27	1.32	2.52	2.57
			5 × 3½ × ⅜	1.42	1.46	2.36	2.41
			5 × 3½ × ⅞	1.51	1.56	2.45	2.50
			6 × 3½ × ⅜	1.34	1.39	2.90	2.95
			6 × 3½ × ⅞	1.44	1.49	3.00	3.05
			6 × 4 × ⅜	1.58	1.62	2.83	2.87
			6 × 4 × ⅞	1.67	1.71	2.92	2.97

r_0 = in all cases, the radius of gyration of one angle referred to neutral axis parallel to the horizontal leg as shown above.

For Member $L_0 U_1$: Two angles 3½ by 3 by $\frac{5}{16}$-inch, long legs spaced back to back, and ¼ inch apart, will be assumed. The least radius of gyration is 1.10, and the length is 8.9 feet. The area of this section is 2 × 1.93 = 3.86 square inches. The unit allowable compressive stress is:

$$P = 24\,000 - \frac{110 \times 12 \times 8.9}{1.10} = 13\,400 \text{ pounds.}$$

The required area is 48 800 ÷ 13 400 = 3.65 square inches. Since the angles given are of somewhat larger area than that required, it might be well to examine the next smallest angle.

Two angles 3½ by 2½ by $\frac{5}{16}$-inch, with a radius of gyration

1.11 and a total area of 3.56 square inches, will be assumed. The unit allowable compressive stress is:

$$P = 24\,000 - \frac{110 \times 12 \times 8.9}{1.11} = 13\,510 \text{ pounds.}$$

The required area is 48 800 ÷ 13 510 = 3.61 square inches. Since the required area is greater than the given area, it shows that these angles are too small. Two angles $3\frac{1}{2}$ by 3 by $\frac{5}{16}$-inch will therefore be used for this member, and also for all the members of the top chord, since it is more economical to run the same size throughout than to change the section and make splices at all the upper chord panel points.

For Member $U_2\ L_2$: The length of this member is easily computed from similar triangles, and is found to be 8.9 feet. Two angles $2\frac{1}{2}$ by 2 by $\frac{5}{16}$-inch, with the long legs back to back, give a total area of 1.62 square inches and a radius of gyration of 0.78. The unit-stress is computed and found to be 8 950 pounds. The required area is 11 150 ÷ 8 950 = 1.25 square inches. These two angles would be used, but the least allowable radius of gyration is 8.9 × 12 ÷ 120 = 0.89. This is seen to be considerably greater than the radius of gyration given above, and therefore these angles cannot be used, according to Specifications. By consulting the tables, it is seen that two angles 3 by $2\frac{1}{2}$ by $\frac{1}{4}$-inch are the smallest angles that will give a radius of gyration nearest to the required amount (0.89) and still be standard size angles. Angles marked with a star in the tables are special angles, and can be procured only at a cost greatly in excess of the others, and then only with great delay in delivering except when large quantities are ordered. It may be said that standard angles should never be used.

For Members $U_1\ L_1$ and $U_3\ L_3$: The length of these members is 4.45 feet. The radius of gyration must therefore not be less than 4.45 × 12 ÷ 120 = 0.45. One angle $2\frac{1}{2}$ by 2 by $\frac{1}{4}$-inch, with an area of 1.06 square inches and a least rectangular radius of gyration of 0.59, will be assumed. The allowable unit compressive stress is:

$$P = 24\,000 - \frac{110 \times 12 \times \frac{8.9}{2}}{0.59} = 14\,050 \text{ pounds.}$$

The required area is 5 580 ÷ 14 050 = 0.40 square inch. The angle chosen gives a much larger area than that required; but since it is the smallest one allowed by the Specifications, it must be used.

Many designers do not place a limit on the value of the radius of gyration, but simply use the compressive formula, and any section whose radius of gyration will bring the required area near to its own area. This should not be the case, since the formula here given is not applicable when the value of $l \div r$ is greater than 120.

Top and Bottom Lateral Bracing. Since the stresses in the lateral bracing are not susceptible of a well-defined mathematical analysis, it cannot be rationally designed. Experience indicates that it should be as in Article 7. The lower chord bracing will therefore consist of single angles 3 by $2\frac{1}{2}$ by $\frac{5}{16}$-inch; and the upper chord bracing, of 3 by 3 by $\frac{5}{16}$-inch angles. This bracing should not be placed between every truss, but should be placed as indicated on the stress sheet, Plate I. If one $\frac{3}{4}$-inch rivet is taken out of the section of the bottom lateral bracing, it will give a net area of $1.62 - 0.27 = 1.35$ square inches; this could withstand a stress of $1.35 \times 15\,000 \times 1.25 = 27\,000$ pounds, which is the stress the bracing is assumed to carry, and which is to be used in determining the number of rivets for the connection. The stress in the top lateral bracing may be assumed to be the same.

Determination of Number of Rivets Required. It is to be remembered that $\frac{5}{8}$-inch rivets are to be used in the $2\frac{1}{2}$ and 2-inch legs of the angles, and $\frac{3}{8}$-inch rivets in all larger legs. Field rivets are to have a value equal to $\frac{3}{4}$ of a shop rivet. Connection plates $\frac{1}{4}$ inch thick are to be used in all cases, except where the number of rivets required will be greater than 10. In such cases, use a $\frac{3}{8}$-inch connection plate. The correct number of field rivets may be determined by multiplying the required number of shop rivets by $\frac{4}{3}$.

Whenever two angles back to back join on a plate, the number of rivets is governed by the bearing on the connection plate; and when one angle is joined to a plate, the number of rivets is governed by single shear if the rivet is $\frac{5}{8}$ inch in diameter, and by single shear if the rivet is $\frac{3}{4}$ inch in diameter and the plate is over $\frac{1}{4}$ inch thick. The bearing and shearing value of the rivets are taken from Table X, p.47.

Lower End of $L_0\ U_1$: Rivets $\frac{3}{4}$-inch. Plate $\frac{3}{8}$-inch.
48 800 ÷ 5 630 = 9 shop rivets required.

Upper End of $U_3\ U_4$: Rivets $\frac{3}{4}$-inch. Plate $\frac{3}{8}$-inch.
40 500 ÷ 5 630 = 8 shop or 10 field rivets

Upper End of $U_4\ L_3$: Rivets $\frac{5}{8}$-inch. Plate $\frac{3}{8}$-inch.
18 725 ÷ 4 690 = 4 shop or 6 field rivets.

Lower End of $L_2\ L_3$: Rivets $\frac{5}{8}$-inch. Plate $\frac{1}{4}$-inch.
12 474 ÷ 3 130 = 4 shop rivets.
Each End of $U_2\ L_2$: Rivets $\frac{5}{8}$-inch. Plate $\frac{1}{4}$-inch.
11 150 ÷ 3 130 = 4 shop rivets.
Each End of $L_1\ U_2$ *and* $U_2\ L_3$: Rivets $\frac{5}{8}$-inch. Plate $\frac{1}{4}$-inch.
6 240 ÷ 3 070 = 2 shop rivets.
Each End of $U_1\ L_1$ *and* $U_3\ L_3$: Rivets $\frac{5}{8}$-inch. Plate $\frac{1}{4}$-inch.
5 580 ÷ 3 070 = 2 shop rivets.

Where U_1L_1 and U_3L_3 join the top chord, two rivets will be required in the top chord.

Since the components of the two diagonals meeting at U_2 are parallel and equal, and opposite to the stress in $U_2\ L_2$, no rivets will be required, theoretically, to hold the plate to the top chord. A sufficient number, however, must be put in to take up the vertical reaction of the purlin. This number is 5 550 ÷ 3 130 = 2 shop rivets. In practice a greater number are usually put in to prevent vibration and to fill out the plate.

At L_3 a sufficient number of rivets must be placed in $L_2\ U_4$ to take up the difference in stress between $L_3\ U_4$ and $L_2\ L_3$. The number required is (18 725 − 12 475) ÷ 3 130 = 3 shop rivets.

At the end L_0 of the member $L_0\ L_1$, there is a horizontal stress of 43 700 pounds, and a vertical force equal to the reaction, which is 49 930 ÷ 2 = 24 965 pounds (see Fig. 76). The force acting on the rivets in this member is the resultant of these two forces, and is:

$$\sqrt{43\,700^2 + 24\,965^2} = 50\,300 \text{ pounds.}$$

Since the rivets are $\frac{3}{4}$-inch and the plates $\frac{3}{8}$-inch, the number of rivets required is 50 300 ÷ 5 630 = 9 shop rivets. This number should be placed symmetrically with respect to the intersection of the two chords. In case the point of application of the reaction had not coincided with the intersection of the chords, the number of rivets must be computed according to the formula on page 36.

For the joint at L_1, a sufficient number of rivets must be put in, in order to take up the difference in stress between the members $L_0\ L_1$ and $L_1\ L_2$. The number required is (43 700 − 37 450) ÷ 3 750 = 2 shop rivets.

The purlins have a horizontal shear at each end, of H ÷ 2 = 2 490 ÷ 2 = 1 245 pounds. This requires 1 245 ÷ 4 420 = 1 shop rivet or 1 field rivet, to keep them from sliding down on the top chord. Clip angles 5 by $3\frac{1}{2}$ by $\frac{3}{8}$-inch will be used as shown in Plate I.

These help in the erecting of the purlins, since they are shop-riveted to the truss and therefore hold the purlins in place while they are being field-riveted to the truss and to the clip angles (see Fig. 77).

Rivets in Lateral Bracing. The plates of the lateral bracing should be made $\frac{1}{4}$ inch thick. The 3-inch leg of the angle will be placed against the plate. Rivets $\frac{3}{4}$ inch in diameter can then be used, and the strength of the joint will be governed by bearing in the $\frac{1}{4}$-inch plate. The stress for which the rivets are to be determined is given on p. 55. It is 27 000 pounds. The number of field rivets in bearing in $\frac{1}{4}$-inch plate, required to withstand the stress, is $(27\,000 \div 4\,420) \times \frac{4}{3} = 9$. The size and shape of the plate can be determined only while making the detailed drawing (see Plate III, p. 61).

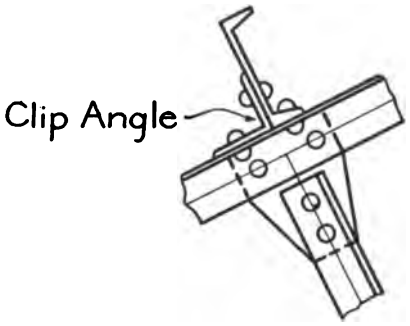


Fig. 77. Detail Showing Clip Angle Connection.

Design of the Splice. The general details of the splice will be as shown in Fig. 68. The plate underneath will be made $\frac{1}{4}$ inch thick, the same thickness as the vertical connection plate at this point. Note that the member on the left-hand side of the splice must have $\frac{3}{4}$-inch shop rivets, and the member on the right-hand side must have $\frac{5}{8}$-inch field rivets. The total number of rivets on either side of the splice must be sufficient to take up the entire stress of the member through which they are driven. If eight $\frac{5}{8}$-inch field rivets are driven through the horizontal legs and the bottom splice plate, and five $\frac{5}{8}$-inch field rivets are driven through the vertical plate and legs of the angles (see Fig. 78), the total strength of the joint, remembering that the rivets are $\frac{5}{8}$-inch, will be:

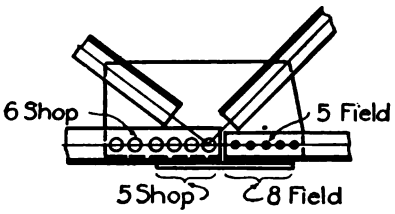


Fig. 78. Detail of Lower Chord Splice.

$$8 \times \tfrac{3}{4} \times 3\,070 = 16\,370 \text{ pounds.}$$
$$5 \times \tfrac{3}{4} \times 3\,130 = 10\,430 \text{ pounds.}$$
$$\text{Total} \quad 26\,800 \text{ pounds.}$$

Note that the rivets through the bottom splice plates are governed by single shear; and those through the vertical plate, by bearing in the plate. Since 16 370 pounds is the value of the rivets through the bottom splice plate, this amount will be transmitted to the other side, where it must be taken up by shop rivets. Bearing in the plate governs the number of $\frac{3}{4}$-inch shop rivets required. This number is $16\,370 \div 3\,750 = 5$. Since 16 370 pounds of the stress in the member $L_1 L_2$ is taken up by these 5 shop rivets, the remainder, $37\,450 - 16\,370 = 21\,080$ pounds, must be taken up by the rivets through the vertical connection plate. This requires $21\,080 \div 3\,750 = 6$ shop rivets.

Since 16 370 pounds is transmitted from one side of the splice to the other by means of the bottom splice plate, this plate should be $16\,370 \div 15\,000 = 1.09$ square inches in net section. The net width, the plate being $\frac{1}{4}$ inch thick, is $1.09 \div 0.25 = 4.36$ inches. If two $\frac{3}{4}$-inch rivet-holes are taken out of the section, the entire width of the plate must be $4.36 + 2\,(\frac{3}{4} + \frac{1}{8}) = 6.11$, say 7 inches wide. The length of the plate must be sufficient to get in the number of rivets, and this length is determined in detailing.

Design of the Masonry Plate. If this truss rested upon a masonry wall, it would require a bearing of $(49\,930 \div 2) \div 250 = 100$ square inches. The width of the plate cannot be less than twice the width of the legs of the bottom chord angle, nor should it extend outside the legs of the chord angle more than 3 inches on each side. The masonry plate will be assumed as 12 inches wide, in which case it must be $100 \div 12 = 8.34$, say $8\frac{1}{2}$ inches long. The thickness should be $\frac{1}{2}$ inch.

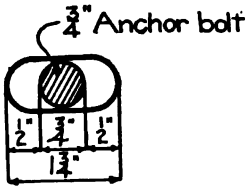


Fig. 79. Slotted Hole for Truss of Fig. 76.

Temperature Allowance. Slotted holes must be put in one end of the truss, to allow for a variation of 150 degrees in temperature. A common rule is to allow $\frac{1}{8}$-inch expansion for every ten feet of span. The total allowance for expansion is $64 \times \frac{1}{8}$ = say, 1 inch. Since the bolts which go through this hole are $\frac{3}{4}$ inch in diameter, the hole must be long enough to allow for $\frac{1}{2}$ the expansion on each side. The width of the hole should be $\frac{1}{4}$ inch greater than the diameter of the bolt (see Fig. 79).

Connections to the Posts. If the truss rests upon posts at the end,

sufficient rivets must be driven through the posts and the end connection plates to take up the end reaction, which (see page 38) is 24 970 pounds. Since the rivets are field rivets, this will require 24 970 ÷ 3 750 = 7. This number is to be used in case the posts are built in masonry walls. In case the truss has knee-braces and the walls of the building consist of steel framework, the reaction due to the wind must be added to the above.

15. **The Stress Sheet.** This should also be somewhat of a general drawing, showing the details. It should give an outline sketch of the building, showing bays, the distances between trusses, and the bracing in the plane of the top and bottom chords. See Plate I, p. 43, which is a stress sheet of the truss designed in Article 14. While not necessary, it is very convenient to have the required number of rivets noted upon the stress sheet.

16. **The Detail Drawing.** The stress sheet, in the matter of sizes, gives general dimensions only. It would be impossible for the shop men to make a truss from the stress sheet.

The shop or detailed drawings must be prepared by the draftsman. These drawings must show the exact number of rivets, and their positions, the dimensions of every plate, member, and purlin. The placing of the dimensions so that it will be unnecessary to add or subtract in order to get another desired dimension, is quite an art, and can be attained only through experience or from the study of correctly detailed work. Plates II and III give the shop drawings for a roof truss and the bracing. These are made according to the latest and best practice, and a thorough study of them will be a help to an intelligent design of the trusses.

All members and plates which are to be riveted together in the field should be given a mark. This mark should be painted on the member or plate, and also marked on the *Marking* or *Erection Diagram* (see Plate IV). This diagram is a sketch, with the pieces in their proper position and the correct mark placed upon them. For example, if it is desired to rivet into place the first panel of the lower lateral system, the men look on the marking diagram and see that plates Pl_7, Pl_8, and Pl_9, and the laterals $BL1$, $BL2$, and $BL3$ are required. They would then go to the place where all the trusses are piled up, and pick out the plates and members with these marks upon them. They would then rivet Pl_7 at L_0, Pl_8 or Pl_9 at L_2, then $BL1$,

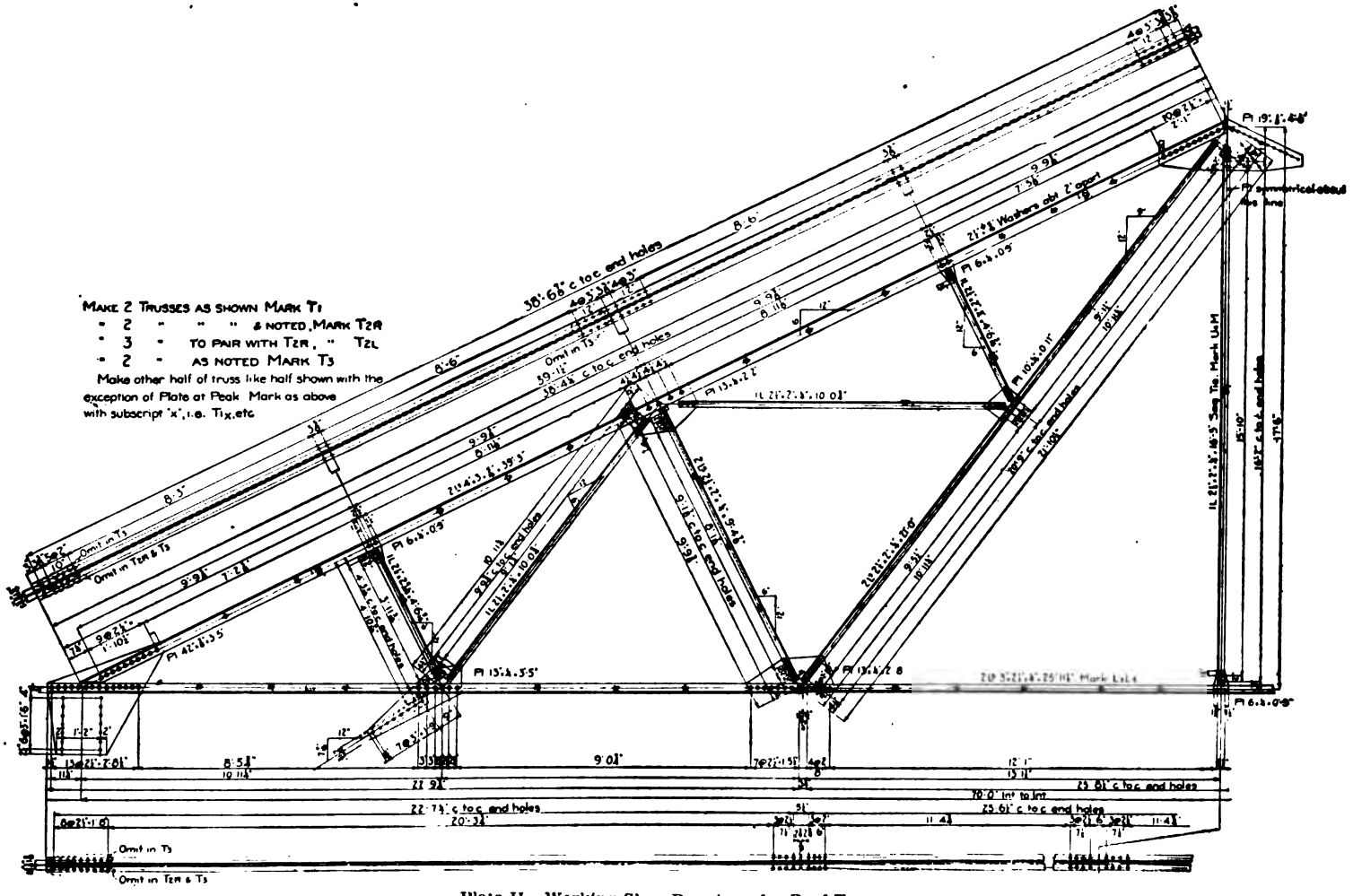


Plate II. Working Shop Drawing of a Roof Truss.

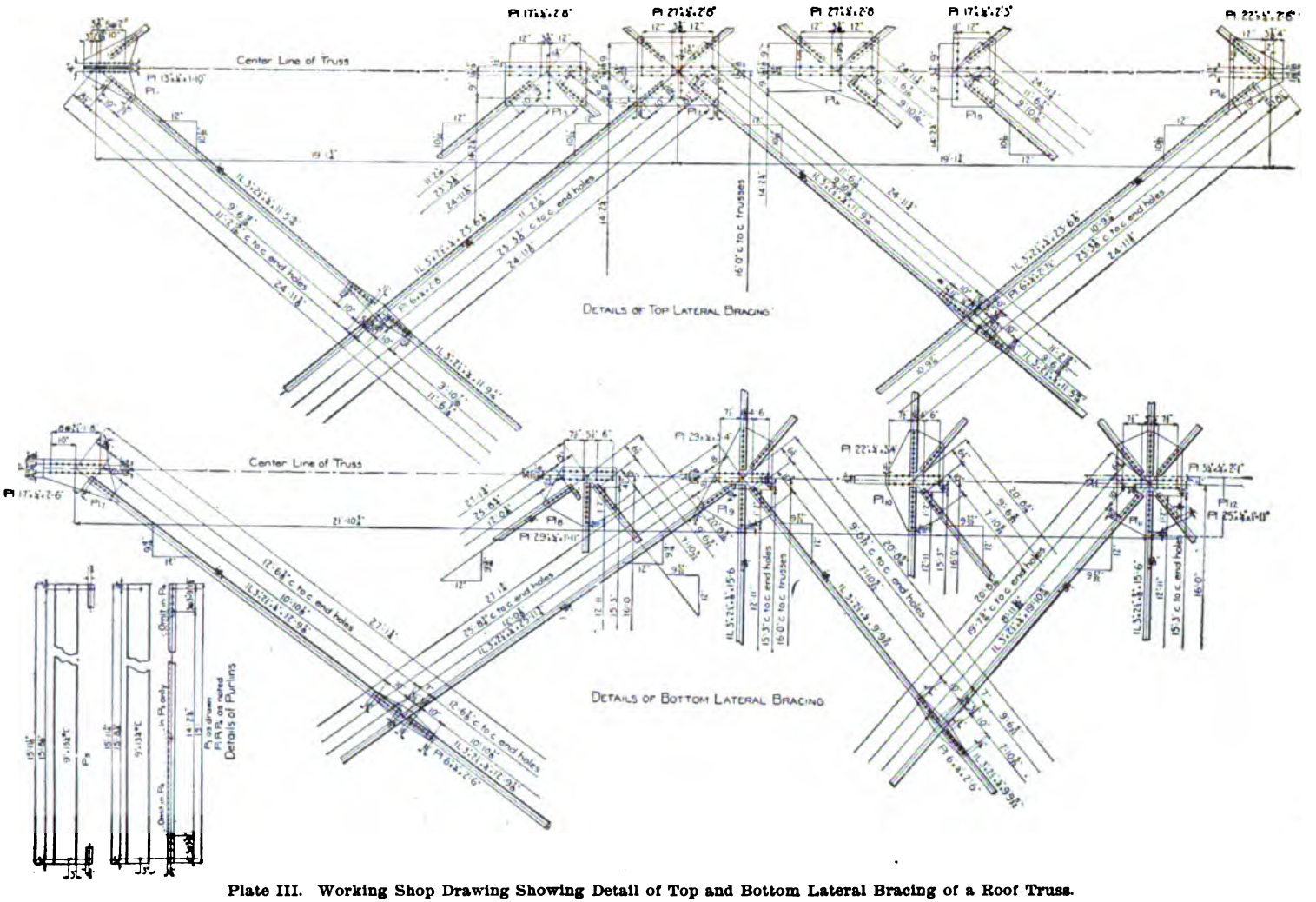


Plate III. Working Shop Drawing Showing Detail of Top and Bottom Lateral Bracing of a Roof Truss.

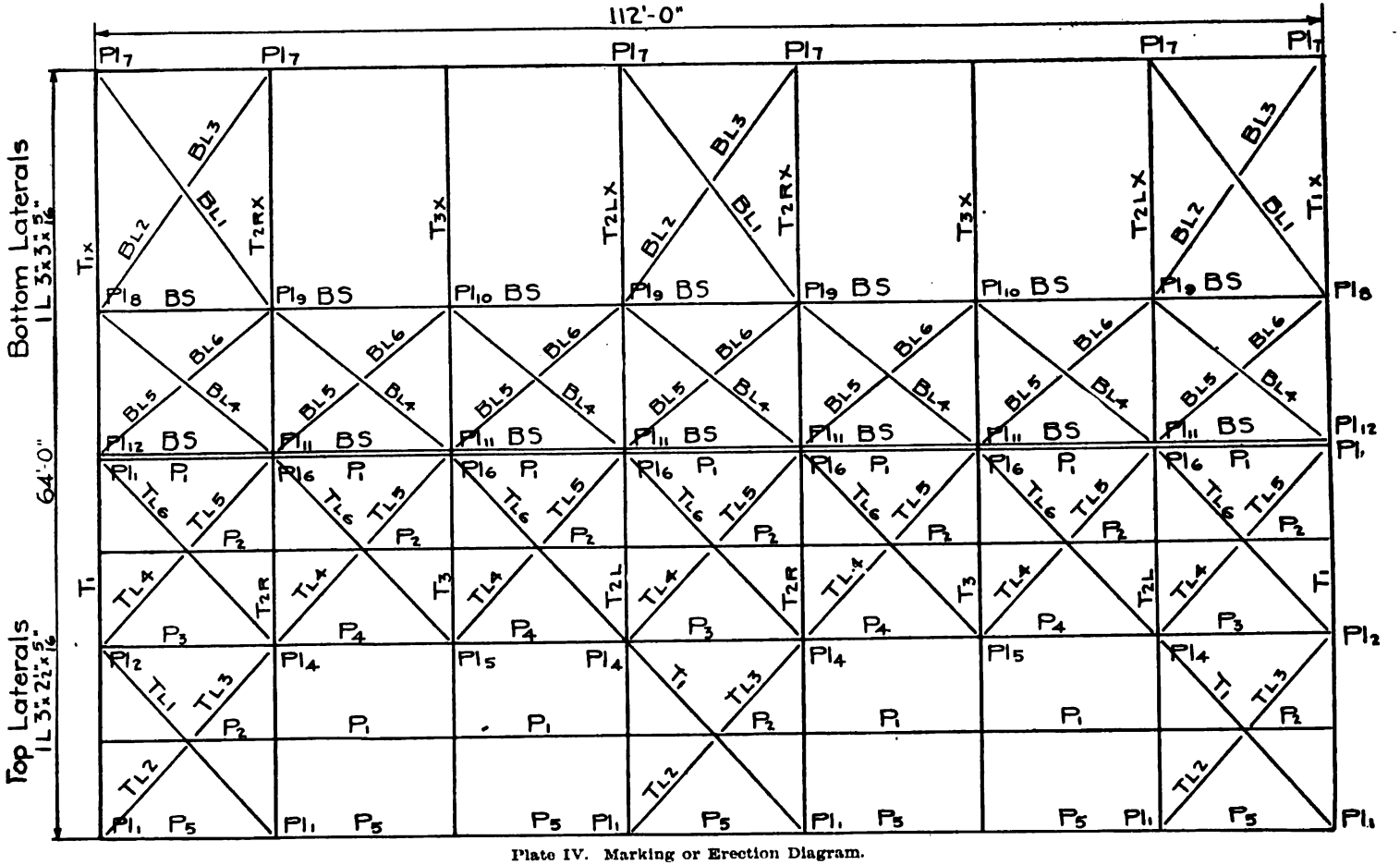


Plate IV. Marking or Erection Diagram.

then $BL3$, and finally $BL2$, all of which are shown on the Marking or Erection Diagram.

Care should be taken to give each piece that is different from others in any way whatsoever, a different mark. For instance, the purlins are the same size, and differ only in length and on account of the fact that one has holes in the bottom flange (see Plate III).

Plate IV gives the roof marking and erection diagram for the roof trusses of Plates II and III. Note that the roof truss on Plates II and III is not the same as that of which Plate I is a stress sheet.

17. **Estimate of Cost.** A rough estimate of the cost of steel in the roof may be obtained by multiplying the weight of the purlins, in pounds, by 2½ cents; then adding to this the result obtained by multiplying the weight of all the steel in the trusses and the bracing by 3½ cents. This will give the cost of the steel work in place with two coats of paint. This will give the cost closely enough for an Engineer's estimate; but should a Contractor desire to bid, a detailed estimate should be made as indicated in the remainder of this article.

The cost of the roof covering may be approximately determined according to the prices given in Article 5, but may be more accurately obtained by asking a Contractor for a figure which his experience will indicate as correct.

Paint of various kinds may be bought in open market. Table XII gives some of the kinds used in painting structural steel, together with the amount of surface one gallon will cover.

TABLE XII

Surface Covered per Gallon of Paint*

Paint	Square Feet	
	1 coat	2 coats
Iron Oxide (powdered)	600	350
" " (ground in oil)	630	375
Red Lead (powdered)	630	375
White Lead (ground in oil)	500	300
Graphite " " "	360	215
Black Asphalt	515	310
Linseed Oil	875	...

*Pencoyd Handbook, 1898, p. 293.

One gallon of paint will give two tons of structural steel the first coat, or 2½ tons the second coat. The cost of one coat of paint in the shop is 45 cents, and two coats after erection $1.80 per ton of structural steel.*

The detailed estimate of the cost of steel includes several items which are given in Table XIII. In each case the weight of the steel on which the work is done must be multiplied by the unit-cost, and the sum total of all the costs will be the total cost of the entire roof or building. Table XIII gives the various operations which go to make up the cost, and also the unit-costs. Note that the costs vary considerably. This table is given as a rough guide. In order to analyze intelligently the cost in this manner, great experience or access to the cost records of some structural steel company is necessary.

TABLE XIII

Analysis of Cost of Roof Trusses and Mill Buildings†

Operation		Cost per Ton
Raw Material		$37.00 to $40.00
Work done at Rolling Mills (mill work)		3.00 " 7.00
Work done in Bridge Shops	Columns	14.00 " 20.00
	Trusses	12.00 " 25.00
	Girders	12.00 " 25.00
Work done in Drafting Room	Purlins	.30 " 1.00
	Trusses and Buildings	2.00 " 8.00
Painting		1.50 " 3.00
Shipping (depends upon freight rates)		.....
Erection		5.00 " 15.00

It is not to be supposed that all of the operations indicated in Table XIII are made on one piece. Usually pieces which have mill work done on them require no shop work. In such cases a saving of freight is effected, since the material may be shipped directly from the mill to the place of erection.

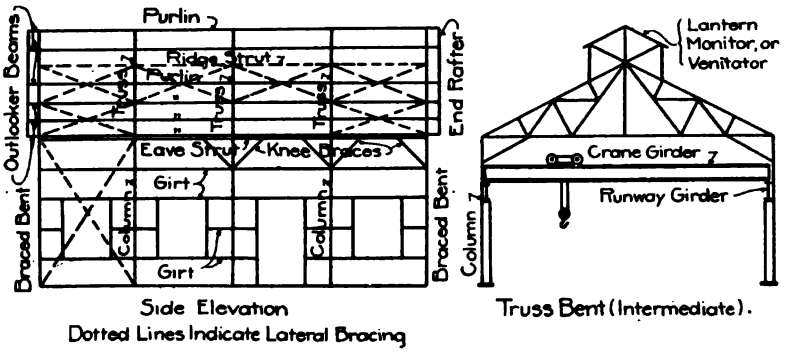
MILL BUILDING

18. **Definitions and Description.** A *mill building* consists of a roof supported either on steel columns, on steel columns built in

*Pencoyd Handbook, 1898, p. 293.

†Compiled from Ketchum's "Steel Mill Buildings."

masonry walls, or on masonry walls alone. The roof may consist of any of the forms of roof trusses that have previously been mentioned; and the roof covering, which may rest on purlins, or on rafters and purlins, may consist of any of the roof coverings mentioned in Article 5. In case the roof is supported on steel columns, the columns are connected at their tops by a strut called the *eave-strut*; and they are



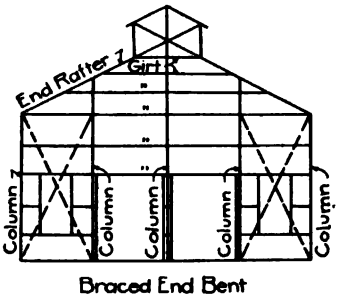


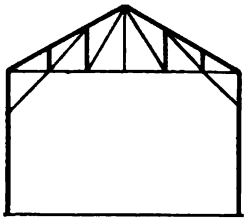
Fig. 80. Physical Analysis of a Mill Building.

also connected at certain distances throughout their height by horizontal members called *girts*. The building may or may not have a monitor ventilator on top. See Fig. 80 for general form of mill buildings, together with the names of the various parts.

The eave-struts and the girts are used as a framework on which to place the covering for the walls of the building. This covering may be of wood, of wire lath and plaster, or of corrugated steel. The eave-strut may also act as the end purlin.

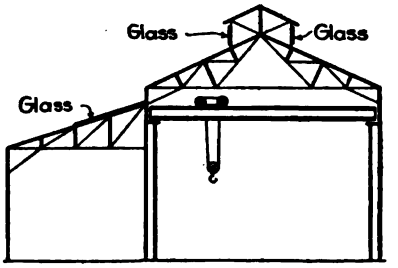
Since the majority of mill buildings have their roofs and sides covered with corrugated steel, the remainder of this text will be devoted to mill buildings with this kind of covering.

19. **Types of Buildings.** Mill buildings may be classified according to their width and the number of bays which they have. A building may consist of one center bay (see Fig. 81). In this case



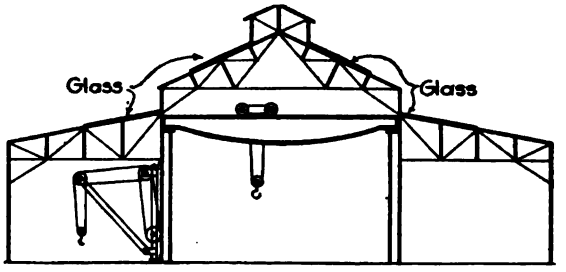
30 to 100 feet.

Fig. 81.



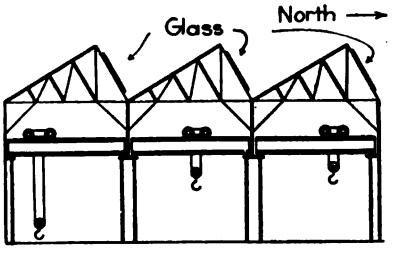
30 to 40 feet 30 to 60 feet

Fig. 82.



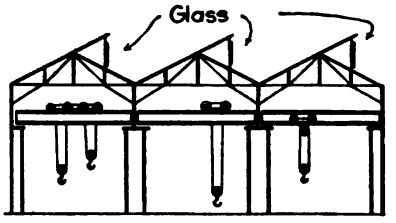
30 to 40 feet 30 to 60 feet 30 to 40 feet

Fig. 83.



Saw Tooth

Fig. 84.



20 to 40 foot spans.
Ketchum's Modified Saw Tooth

Fig. 85.

Cross-Sections of Mill Buildings.

the span may vary from 30 to 100 feet. Usually side windows give sufficient light, no skylights being required in the roofs or monitor if the span is less than 80 feet.

The building may consist of one center bay and one or two side

bays, as shown in Figs. 82 and 83. The truss of the center bay is usually of the Fink type, and in most cases is supplied with skylights and lights in the monitor. The side trusses for the most part consist of that type in which the chords are nearly parallel. The center bay is generally not more than 60 feet in span. This is due to the fact that the crane girder would be unnecessarily heavy if a longer span were used. The side spans are usually from 30 to 40 feet.

In case it is desirable to have the building wider than 150 feet, and still have it lighted by natural light, the common saw-tooth roof (Fig. 84) or Ketchum's modified saw-tooth (Fig. 85) is used. In such cases the bays are seldom greater than 40 feet. Cranes may be placed in one or all of the bays. One great advantage of this type of roof is that it gives a good light uniformly throughout the entire shop; and at any time it is desired to widen the shop, additional bays may be added. The shop may also be lengthened by adding additional trusses at the end. Of course, shops of the first two types mentioned may be widened by addition of extra bays; but the connection to the old work will be unsatisfactory, and skylights will have to be placed in the roofs both of the old bays and of the new ones. For views showing the interior of shops, see pages 77 to 84.

20. **General Requirements.** The general requirements of a mill building depend in detail on the purpose for which it is intended. The requirements which are common to all classes of buildings are ventilation, good light, and transportation facilities both inside and outside the shop. The question of light and ventilation is discussed on pp. 42 and 66. In regard to transportation facilities, it may be said that either the building should be placed so close to a railway track that the material may be unloaded by means of a crane and hauled into the shop, or the track must run into the shop so that the material may be unloaded and placed on the stock floor by means of a crane in the center bay and wall jib-cranes (see Fig. 109, p. 83) or by means of hand trucks.

21. **Layout.** The purpose for which the building is intended, and its relative location in regard to transportation facilities, will determine its layout. For manufacturing purposes, it should be so laid out that the materials will always pass *forward* in going from the raw material to the finished product. In general it may be said that the engines, machines (lathes, milling machines, drill presses, shears,

punches, etc.), and stock room, should be in the side bays; and the laying out, erection, and shipping floors should be placed in the center bay in the order mentioned. Fig. 86 gives a layout of a concern manufacturing frogs and switches.

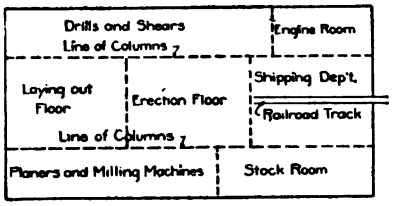


Fig. 86. Layout of a Frog and Switch Company's Building.

22. **Framing.** The framing of a mill building consists of the *roof framing*, which has been discussed in the preceding articles; the *columns*, which will be discussed in the next article; and the *girts* and *eave-struts*.

Eave-struts are a detail of cornice design. Various forms and methods of connections are shown in Article 29, p. 95, and the student is referred to this article.

Girts may be made of wood, angles, or channels. They should be designed for a pressure of from 20 to 30 pounds to the square foot on the side of the building. The spacing of the purlins depends upon the thickness of the corrugated steel used. On account of the fact that corrugated steel can be procured in lengths up to 10 feet and for spans of 5 feet, the stress per square inch due to 30 pounds per square foot is about 25 000 pounds. In No. 24 gauge corrugated steel, the spacing of the girts is limited to 5 feet or less.

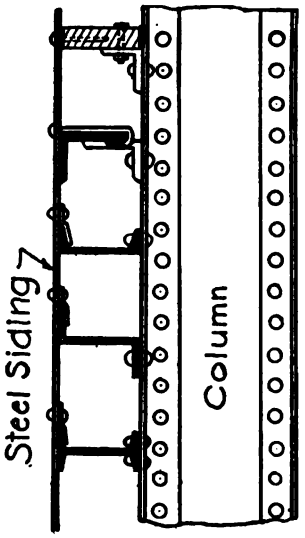


Fig. 87. Methods of Connecting Corrugated Steel to Girts.

Corrugated steel may be fastened to the girts by barbed roofing nails in case the girts are wood, or by clinch nails in case the girts are angles, or by clips fastened with rivets or $\frac{3}{16}$-inch stove bolts $\frac{3}{8}$ inch long. Nails and clinch nails should be spaced about 8 to 12 inches apart. Clips are made of No. 16 gauge steel from $\frac{5}{8}$ inch to $2\frac{1}{2}$ inches wide, and are spaced 8 to 12 inches apart. Fig. 87 shows girts, together with the method of attaching the corrugated steel. The number of nails, rivets, and

stove bolts in a pound is to be found in the handbooks of various manufacturers.

Window-frames in mill buildings are, in general, similar to those placed in frame or brick buildings. These frames are fastened either directly to an iron framing or to wood nailing-pieces placed on the iron framing. The windows may be glazed in the usual fashion by means of putty, or may have the glass held in place by some of the methods shown in Fig. 73, p. 44. Windows in the side of the shop may be so fixed that they may be raised and lowered as the ordinary dwelling window; or they may slide horizontally; or, again, they may be fixed so that they cannot be moved. The windows in the monitors are usually fixed with a swinging sash which can be operated from the floor of the shop (see Fig. 89).

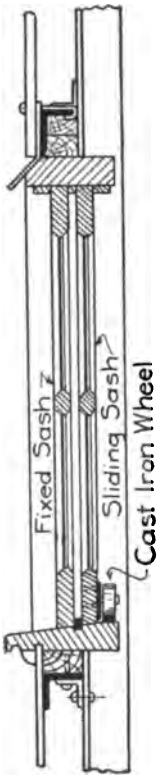


Fig. 88. Section of a Sliding-Sash Window.

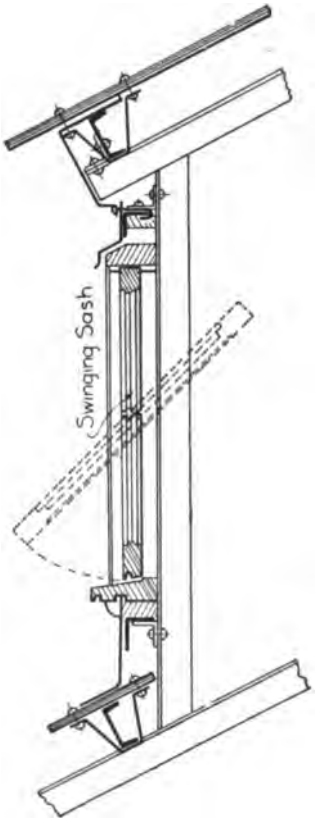


Fig. 89. Section of a Monitor Swing Window.

The glass in the windows may be the common window glass, common plate glass, ribbed or corrugated glass, wire glass, or prisms. Of these varieties, the prisms and the ribbed or corrugated are the best, since they give a more uniform light and are not easily broken. Wire glass, which is made of wire netting moulded in the middle of the sheet of glass, gives a very good light, and has the additional advantage that it does not crack and fall out under the action of fire and water. It is considered fireproof. Common window glass does not diffuse the light so well as most of the other glasses. It is

very liable to fracture, and for this reason the inside of the window should be covered with wire netting. Prisms are made by the American Luxfer Prism Company, of Chicago. They may be obtained up to 84 inches in width and 36 inches in height. The width is parallel to the saw teeth. Figs. 88 and 89 give sections of windows, showing the framing. Attention is called to the fact that the roof on the monitor should overhang sufficiently to prevent the water from dropping upon the swinging window when it is fully opened.

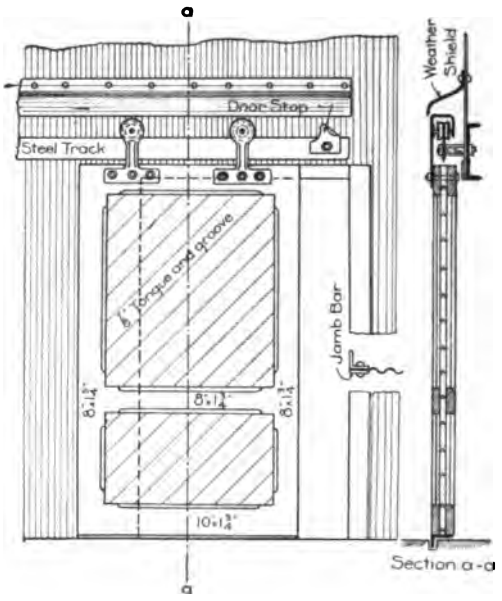


Fig. 90. Detail of a Wooden Door.

Doors may consist entirely of wood, of a frame of angles covered with corrugated steel, or of corrugated steel alone. The first two classes may be so fixed that they will slide, as in the folding doors of residences; open outward like a common door; lift vertically; or, in case they are made entirely of corrugated steel, roll up like a window-shade. This latter door is a patented one. Shop doors are seldom made to open outward or inward, on account of the space required—a space which can be devoted to better purposes. Figs. 90, 91, and 92 show details of the above doors.

Let it be required to design the girt when the trusses are 16 feet apart and the girts are 5 feet center to center. The moment is $\frac{5 \times 16 \times 30 \times 16 \times 12}{8} = 57\,600$ pound-inches; and the required section modulus is $\frac{57\,600}{15\,000} = 3.84$. By inspecting the tables in the Carnegie Handbook, pp. 97 to 119, it is found that the following shapes will be sufficient:

Shape	Section Modulus
One 5-inch 9.75-pound I-beam	4.80
One 6-inch 8.00-pound channel	4.30
One $4\frac{1}{16}$ by $3\frac{1}{8}$-inch 10.3-pound zee-bar	3.91
One 6 by 4 by $\frac{7}{16}$-inch 14.3-pound angle	3.83

From this it is evident that the channel is the most efficient and economical.

23. **Columns.** Columns may consist of almost any combination of shapes, either latticed or connected by plates. Some of the most common cross-sections are shown in Fig. 93, those illustrated in *b* and *c* being used to a great extent. The advantage of these forms is that they give a small radius of gyration about the axis *b–b*, and a larger one about the axis *a–a* (see Fig. 94). This is especially desirable, since, in addition to the direct stress due to the weight of the cranes, roof truss, and covering, the column must withstand the moment due to the wind and to the eccentricity of the runway girder. Both of these moments tend to bend the column around the axis *a–a*. The bending moment due to the eccentricity of the runway girder is equal to the reaction of the girder, *times* the distance from the center of the column (see Figs. 95 and 96). In case the details of the column are as given in Fig. 96, the direct load due to the reaction of the truss and its covering produces a moment due to its eccentricity. This moment is $R_1 \times e_1$. Since R_1 acts on the opposite side of the center of the column from the point of action of R_2, it tends to counteract the effect of the moment due to the eccentricity of the runway girder. The total moment due to eccentricity is $M_e = R_1 \times e_1 - R_2 \times e$. If

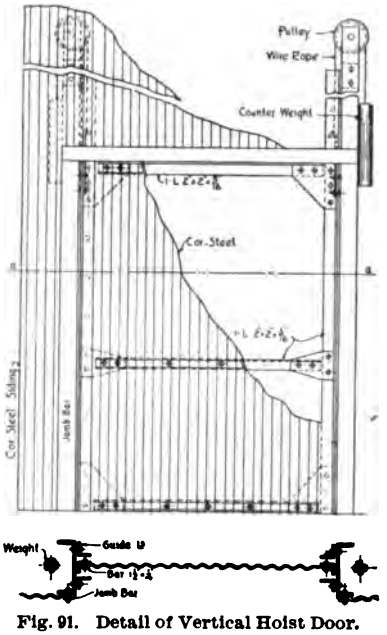


Fig. 91. Detail of Vertical Hoist Door.

the first term of this equation is less than the last term, the compressive stress on the side of the column with the runway girder is increased, and *vice versâ*. The stress in the column from the runway girder

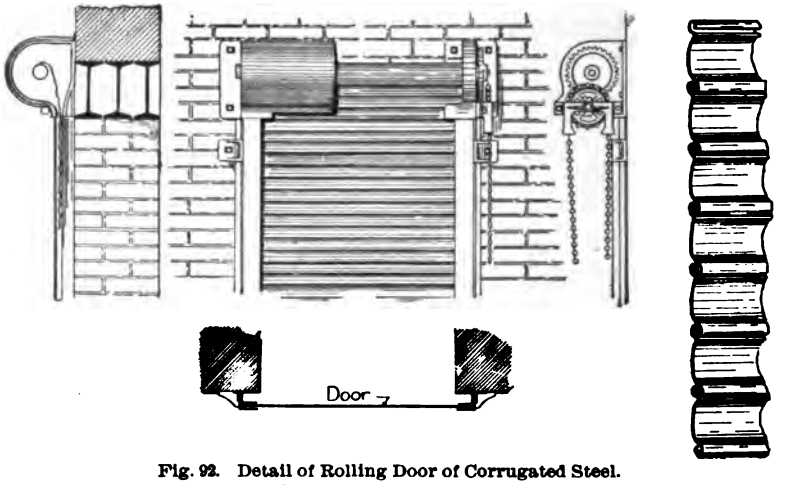


Fig. 92. Detail of Rolling Door of Corrugated Steel.
Courtesy of Kinnear Mfg. Co., Columbus, Ohio.

to the roof is that due only to the vertical reaction of the roof and the bending due to the wind. In that part of the column below the crane girder, the stress is that due to the direct action of the weight of the roof; its eccentricity, if there be any; the direct action and eccentricity of the runway girder; and the bending moment due to the wind. The bending moment due to the wind is less in this part of the

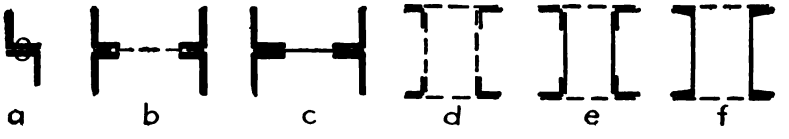


Fig. 93. Cross-Sections of Columns.

column than it is at the foot of the knee-bracing, but it is customary to consider it the same.

In order to prevent eccentric stresses due to the reaction of the runway girder, an extra column to carry the crane girder is placed alongside the roof column (see Figs. 100 and 117, pp. 75 and 88). This is much used by A. F. Robinson, Bridge Engineer of the Atchison, Topeka & Santa Fé Railroad System, who claims it to be a very

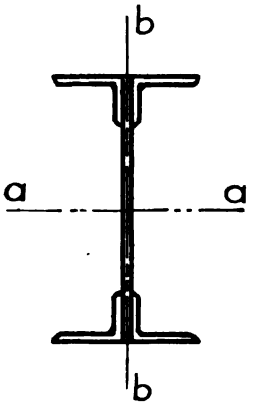


Fig. 94. Illustrating the Two Radii of Gyration.

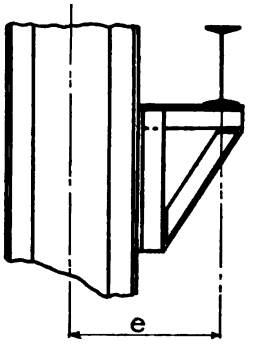


Fig. 95. Runway-Girder Eccentricity.

economical and efficient detail. One advantage of this is, that, if at any time it is desirable to use a heavier crane, this column can be removed and a stronger one put in its place, without in any way affecting the remainder of the building.

In order to illustrate the design of a column, let it be required to design a column with detail as shown in Fig. 95, the height being 20 feet, the distance of the runway girder from the face of the column 8 inches, the direct stress 15 600 pounds, and the bending moment due to the wind 924 000 pound-inches. The reaction of the runway girder is 20 000 pounds. The stress due to the bending moment caused by the wind and the eccentricity of the runway girder must be worked out by formula 8, "Strength of Materials," p. 86; and to this must be added the direct stress caused by the weight of the roof and the crane-girder reaction.

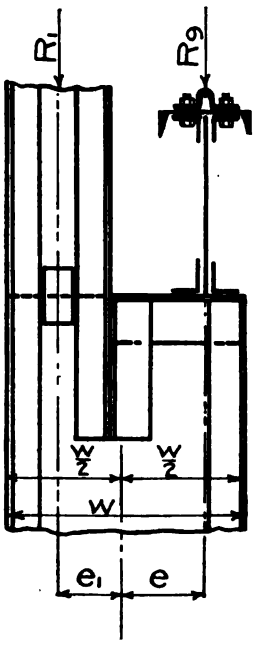


Fig. 96. Runway-Girder and Roof-Truss Eccentricity.

Since, according to Article 13, the unit-stress must be reduced one-half in determining the section to withstand stresses due to crane loads, the moment due to the crane loads and also its direct action must be

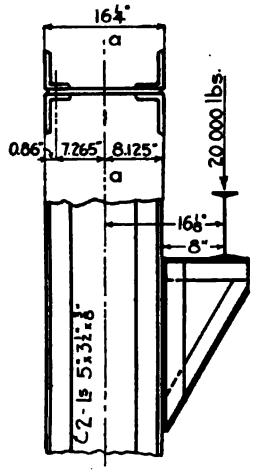


Fig. 97. Illustrating Problem on Page 78.

multiplied by 2 in order that the same formula for the unit-stress may be used throughout in the design of the column.

Let four 5 by 3½ by ⅜-inch angles with a 16 by ⅜-inch web plate be assumed, and placed as shown in Fig. 97. These angles have an area of 3.05 square inches each; and a moment of inertia parallel to the long leg, of 3 18. Then (see "Strength of Materials," pp. 48–53), the moment due to crane reaction is 20 000 × (8.125 + 8) = 322 500 pound-inches. Accordingly, in using this in the formula, it will be 2 × 322 500 = 645 000 pound-inches; and this, added to the 924 000 pound-inches due to the wind, will make a total bending moment of 1,569,000 pound-inches.

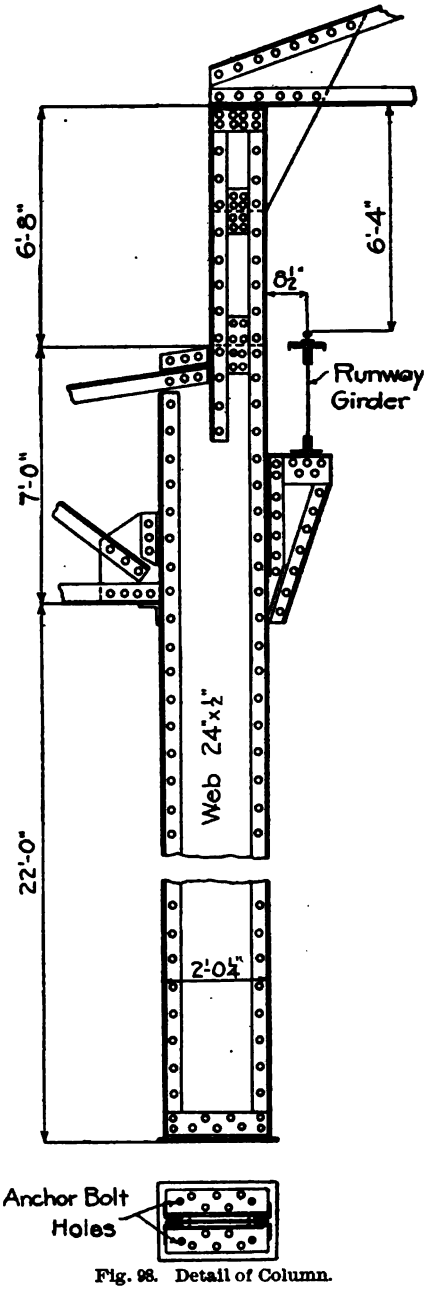


Fig. 98. Detail of Column.

$$I_{a-a} = 4 \times 3.18 + 4 \times 3.05 \times 7.265^2 + \frac{\frac{3}{8} \times 16^3}{12} = 784.72,$$

$$r_{a-a} = \sqrt{\frac{784.72}{4 \times 3.05 + \frac{3}{8} \times 16}} = 6.56$$

The allowable unit-stress is:

$$P = 24\,000 - 110 \times \frac{20 \times 12}{6.56} = 19\,975 \text{ pounds per square inch.}$$

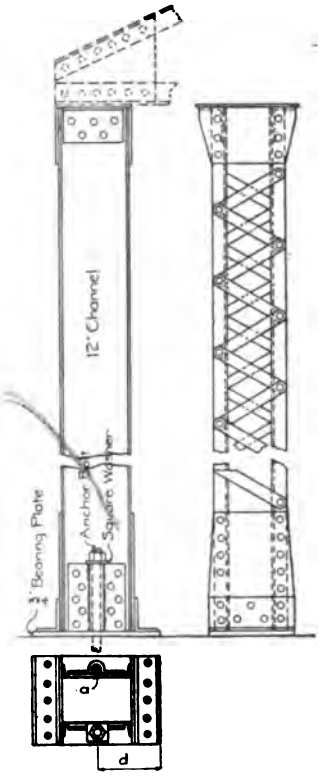


Fig. 99. Detail of Column.

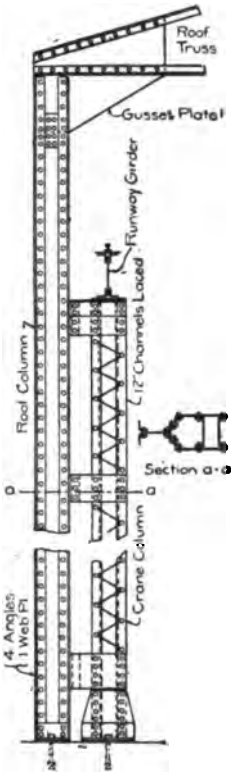


Fig. 100. Detail of Column.

The actual unit-stress (see "Strength of Materials," p. 86), is:

$$S = \frac{15\,600 + 2 \times 20\,000}{4 \times 3.05 + \frac{3}{8} \times 16} + \frac{1\,569\,000 \times \frac{16.25}{2}}{784.72 - \frac{(15\,600 + 2 \times 20\,000)(20 \times 12)^2}{10 \times 28\,000\,000}}$$

$$= 3\,024 + 16\,420$$

$$= 19\,444 \text{ pounds per square inch.}$$

Since this is slightly less than the allowable stress, 19 975 pounds per square inch, this section is the correct one.

Details of columns are shown in Figs. 98, 99, and 100. In case the column is considered fixed at its base, the base of the column is usually made as shown in Fig. 99. Long bolts deeply imbedded in the masonry are run up through the holes a; a heavy washer is placed over the bolt, and the nut screwed down tightly. Each bolt must be designed to withstand a stress of $H_2 \times n \div 2d$ (see Figs. 43 and 99).

24. **Knee-Braces.** The determination of the stresses in knee-bracing has been made in Article 9. Knee-braces consist of two angles placed back to back, and are joined to the column and roof truss as shown in Fig. 101 and in the figures showing cranes. They must be designed to withstand the greatest compressive stress; and must also be examined to see if they are safe in tension, since they are under either tensile or compressive stresses according to the direction in which the wind blows.

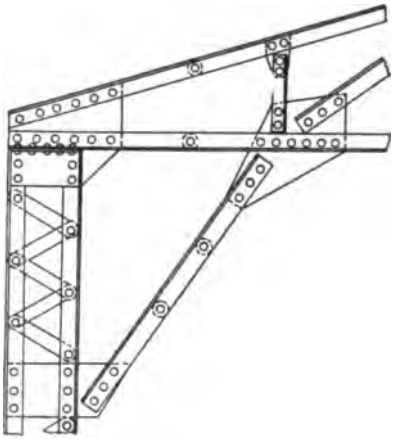


Fig. 101. Detail of Knee-Brace and Connections.

The knee-brace for the truss-bent of Article 9 will now be designed. The maximum compressive stress is 21 440 pounds. The radius of gyration must be at least $131 \div 120 = 1.09$. Two angles $3\frac{1}{2}$ by 3 by $\frac{5}{16}$-inch, placed back to back with their longer legs $\frac{1}{4}$ inch apart, will be assumed, since they are the smallest size to be used with $r = 1.09$ or greater. The radius of gyration about an axis perpendicular to the longer legs is 1.10; and the allowable unit-stress is $P = 24\,000 - \frac{110 \times 131}{110} = 10\,900$ pounds per square inch, the length being 131 inches. The required area is $\frac{21\,440}{10\,900} = 1.97$ square inches. Since this is less than the given area, and since the angles are the smallest allowed, these angles are sufficient. The maximum tensile stress is 8 640 pounds, and the required net area is $8\,640 \div 15\,000 = 0.58$ square inch. The net area given

by the angles is 3.86 − 0.55 = 3.31 square inches, two $\frac{3}{4}$-inch rivet-holes being taken out. This shows the angles to be amply safe in tension, and they will therefore be used for the section of the knee-braces.

25. **Runway Girders.** The runway girders extend from column to column on each side of the bay in which the girder runs. An

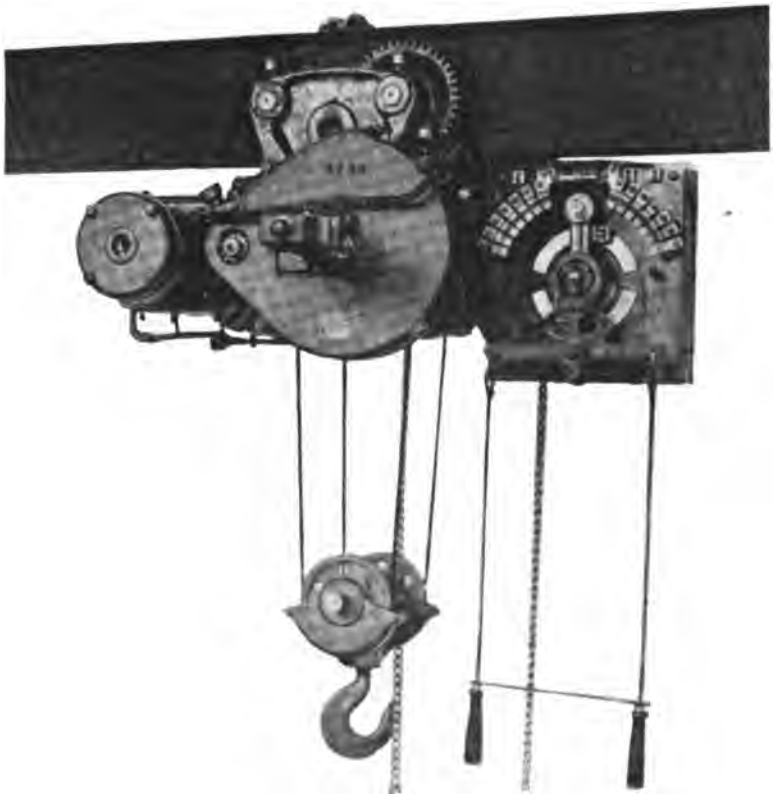


Fig. 102. Electric I-Beam Hoist, One-Motor.
Courtesy of Case Manufacturing Company, Columbus, Ohio.

inspection of the figures of this article will give a clear idea of their position and their details. Along these girders run the wheels which support the end of the crane. The crane may be a small hoist, as indicated in Figs. 102, 103, and 104, in which case the crane girder consists of a simple I-beam supported by two wheels at the ends, and these are placed close together. In other cases the crane consists of

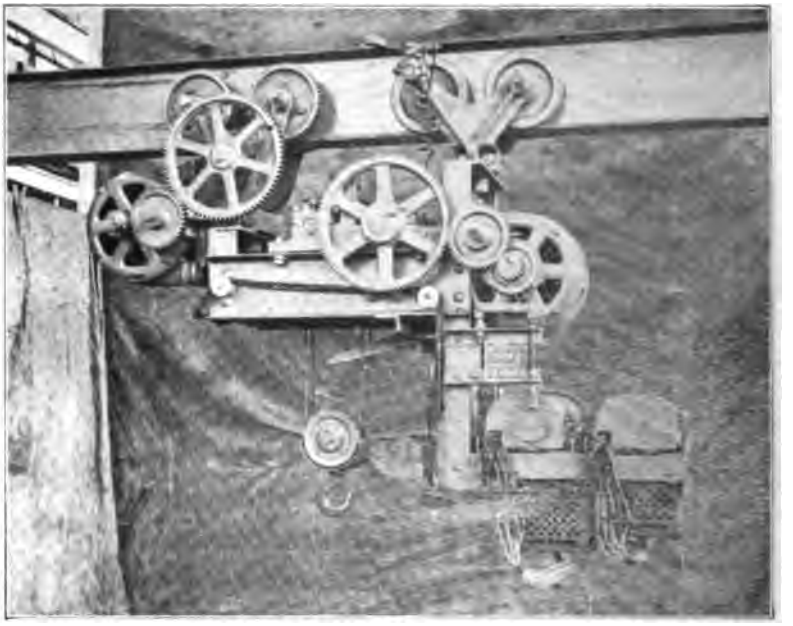


Fig. 103. Electric I-Beam Hoist.
Courtesy of Maris Brothers, Philadelphia, Pennsylvania.

two girders placed side by side, upon which runs the carriage carrying the hoist. This type of crane is supported upon four to eight wheels (see Figs. 105, 106, 107, 108, 109, and 110).

The maximum bending moment and shear in a runway girder

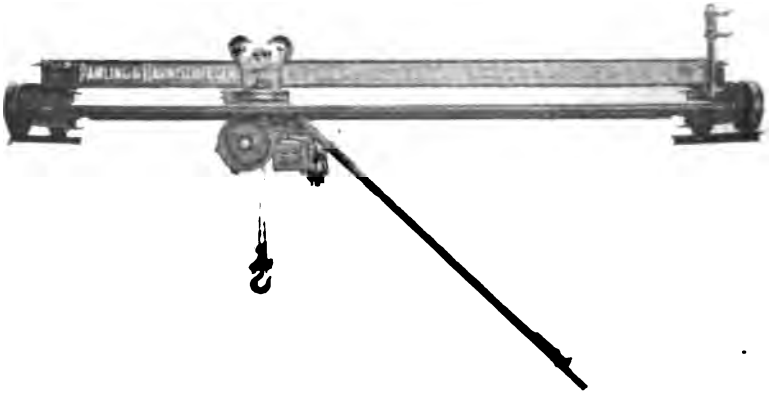


Fig. 104. Small Electric I-Beam Hoist; Capacity 500 Lbs.
Courtesy of Pawling & Harnischfeger, Milwaukee, Wis.

Fig 105. Ten-Ton Three-Motor Electric Traveling Crane.
Courtesy of Case Manufacturing Company, Columbus, Ohio.

Fig. 106. Five-Ton Three-Motor Electric Traveling Crane, Operated from Floor.
Courtesy of Case Manufacturing Company, Columbus, Ohio.

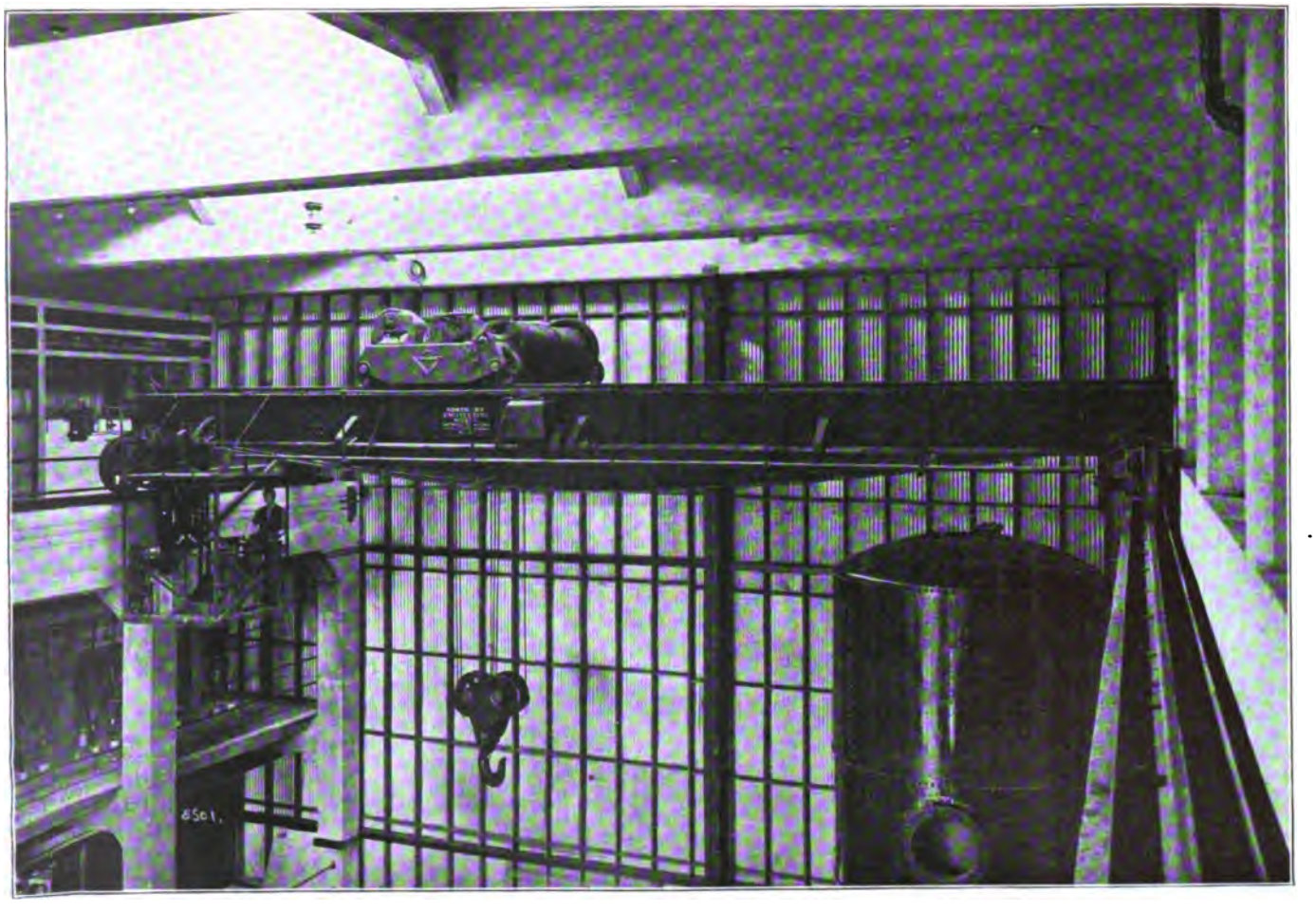


Fig. 107. Fifty-Ton Three-Motor Electric Traveling Crane.
Courtesy of Northern Engineering Works, Detroit, Mich.

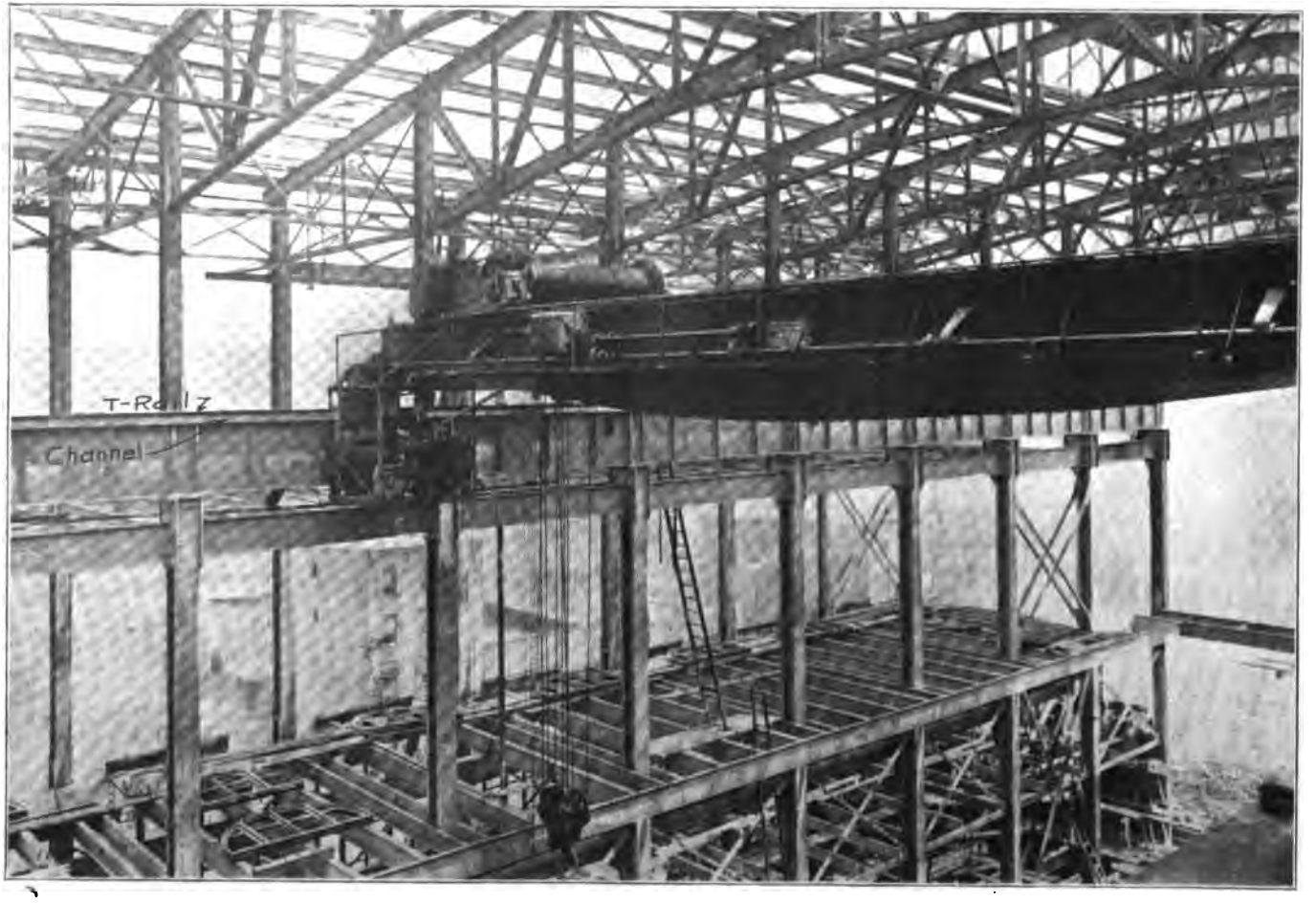


Fig. 108. Fifty-Ton Four-Motor Electric Traveling Crane.
Courtesy of Northern Engineering Works, Detroit, Mich.

will depend upon the capacity and span of the crane, and also upon the distance apart of the wheels at its ends. Where the bending moment is not too great, the runway girders may be composed of channels or I-beams (see Figs. 103, 104, and 106). In case the moment is too great to make the use of these possible, the runway girders are composed of plate-girders (see Figs. 105, 107 to 110).

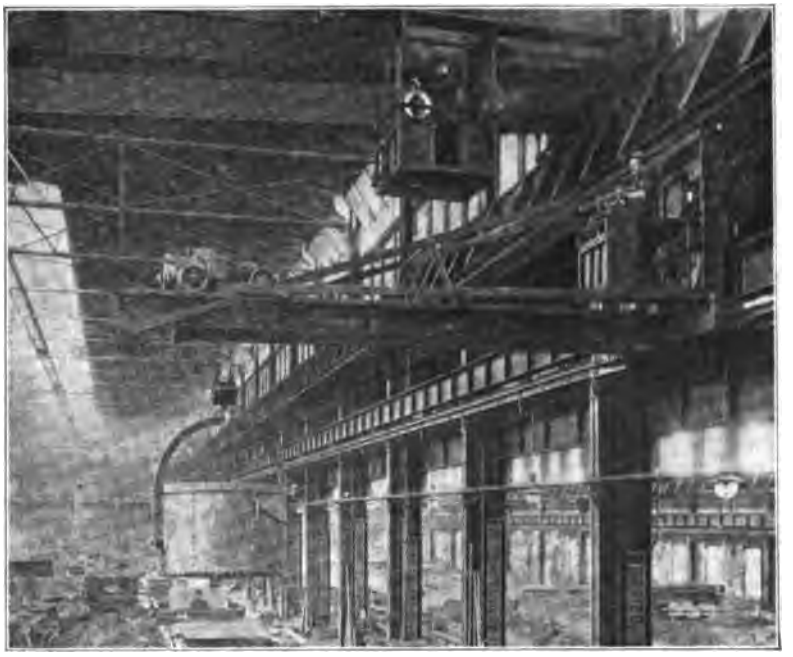


Fig. 109. Wall Jib-Crane, Electrically Operated; Capacity, 10,000 Lbs.
Courtesy of Pawling & Harnischfeger, Milwaukee, Wisconsin.

Plate-girders consist of a flat plate called a *web plate,* which has riveted to it at its upper and lower edges two angles, or two angles and one plate, called the *cover-plate.* The angles are called *flange angles*; and the two angles together, and the cover-plate when used, are called the *flanges.* At certain distances along its length, equal to or less than its depth, vertical angles are riveted on opposite sides of the web plate. These are called *stiffeners,* their function being to stiffen the web under the action of the shear. See Fig. 111 for a general view of a plate-girder, together with the names of the various parts.

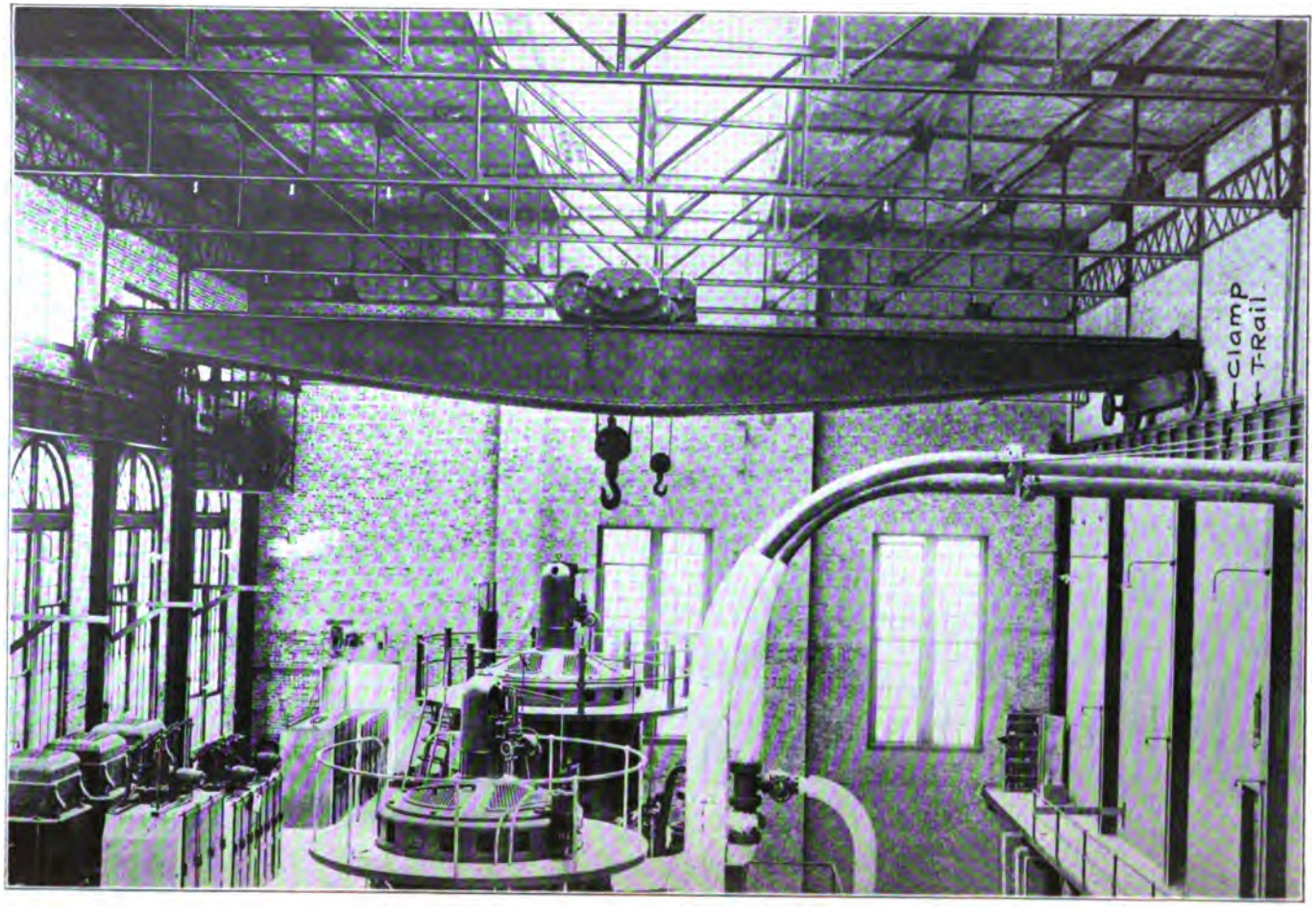


Fig. 110. Thirty-Ton Four-Motor Electric Traveling Crane in Power Station.
Courtesy of Northern Engineering Works, Detroit, Mich.

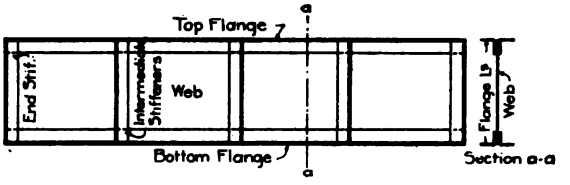


Fig. 111. Plate-Girder Notation.

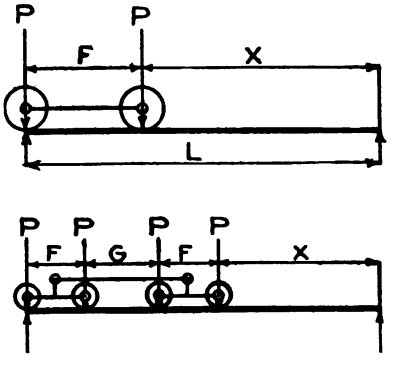


Fig. 112. Position of Crane Truck for Maximum End Shear of Runway Girder.

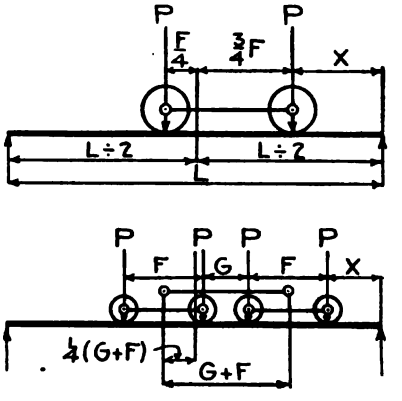


Fig. 113. Position of Crane Truck for Maximum Moment in Runway Girder.

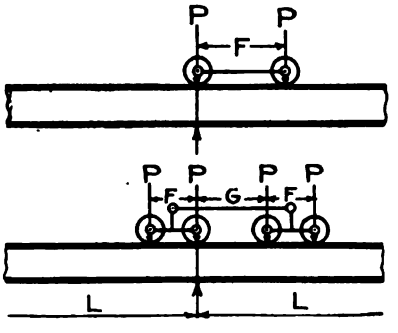


Fig. 114. Position of Crane Truck for Maximum Reaction on Column.

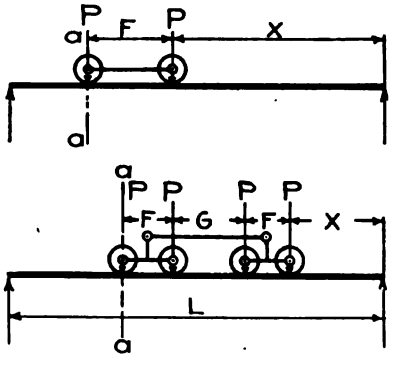
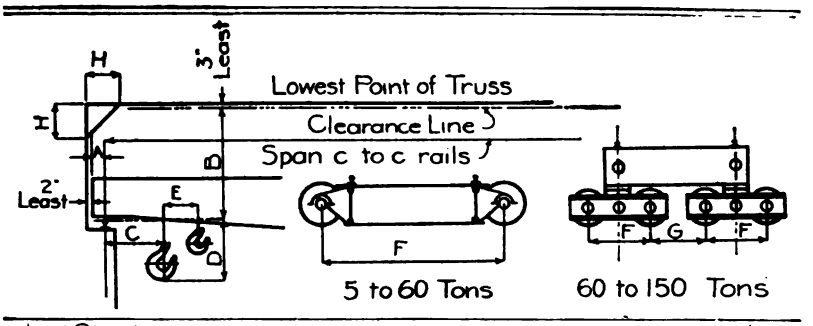


Fig. 115. Position of Crane Truck for Maximum Shear at any Section.

The maximum shear in the runway girder will occur when the crane wheels are in the position shown in Fig. 112; and the maximum moment will occur under the wheel nearest the middle of the span, when the wheels are in the position shown in Fig. 113. The maximum reaction of the runway girders on the column will occur when the wheels are in the position shown in Fig. 114. In order to

TABLE XIV
Maximum Crane Reaction



	Capacity (tons)	Span (feet)	A in.	B ft. in.	C ft. in.	D ft. in.	E ft. in.	F ft. in.	G ft. in	Wheel Load (in lbs.)	Tram Rail Am. Std.	H ft. in.
SINGLE DRUM. TYPE C	5	40	7	4- 5½	1- 8 2- 0	7		7- 6½		10 000	3½-1⅞- 40	1- 5
		60	8	4- 7½	1-11 2- 1	5		9- 0		12 500		
		80	8	4- 8½	1-11 2- 2	4		9- 3½		15 500		
	10	40	8	5- 0½	1- 0 2- 4	9		8- 5½		16 500	3½-1⅞- 40	1- 0
		60	8	4-11¼	1-11½ 2- 4	10		9- 7¼		19 500		
		80	8	5- 0½	1-11½ 2- 4½	9		9- 9½		23 000		
	15	40	8	5- 4¼	2- 0 2- 9	1-3½	2- 0	9- 6		24 500	3⅞-2⅛- 50	1- 3
		60	8	5- 4¼	2- 0 2- 9	1-3½	2- 0	10- 7½		28 000		
		80	8	5- 5½	2- 0 2- 9	1-2¼	2- 0	10- 9½		31 500		
	20	40	8	5- 5½	2- 0 2- 9	1-6¼	2- 0	9- 6		30 000	3⅞-2⅛- 50	1- 3
		60	8	5- 5½	2- 0 2- 9	1-6¼	2- 0	10- 7½		34 000		
		80	9	5- 6½	2- 0½ 2- 9	1-5¼	2- 0	11- 1½		38 000		
DOUBLE DRUM. TYPE A	25	40	9	6- 0¼	2- 6	3-9¼	2- 7½	11- 0½		38 600	4¼-2⅜- 60	1- 3
		60	9	6- 0¼	2- 6	3-9¼	2- 7½	11- 3½		42 400		
		80	10	6- 2¼	2- 6	3-7¼	2- 7½	11- 8½		47 000		
	30	40	10	6- 3¾	2- 7	3-8½	2-10	11- 4½		45 600	4¼-2⅜- 60	1- 3
		60	10	6- 3¾	2- 7	3-8½	2-10	11- 6½		49 600		
		80	10	6- 3¾	2- 7	3-8½	2-10	12- 0		54 600		
	40	40	11	7- 4½	3- 1½	4-2½	3- 8½	12- 8		59 600	5 -2½- 80	1- 6
		60	11	7- 4½	3- 1½	4-2½	3- 8½	13- 0		64 000		
		80	11	7- 4½	3- 1½	4-2½	3- 8½	13- 5		70 000		
	50	40	13	7-11½	3- 6	4-0½	3- 9	13-10½		77 000	5¾-2¾-100	1- 6
		60	13	7-11½	3- 6	4-0½	3- 9	13-10½		84 600		
		80	13	7-11½	3- 6	4-0½	3- 9	14- 0		92 000		
	60	40	14	8- 7¼	4- 1	4-3¾	4- 4½	15- 2		88 000	5¾-2¾-100	1-11
		60	14	8- 7¼	4- 1	4-3¾	4- 4½	15- 2		94 000		
		80	14	8- 7¼	4- 1	4-3¾	4- 4½	15- 4		103 000		
	60	40	12½	10- 3¼	4- 0	2-7¾	4- 4½	3- 6	5-4	44 000	5¾-2¾-100	
		60	12½	10- 3¼	4- 0	2-7¾	4- 4½	3- 6	5-4	47 000	6 -4 -150	
		80	12½	10- 3¼	4- 0	2-7¾	4- 4½	3- 6	5-4	51 500		
	75	40	12½	11- 3	4- 6	3-8½	4- 0½	5- 0	6-0	55 000	5¾-2¾-100	2- 2
		60	12½	11- 3	4- 6	3-8½	4- 0½	5- 0	6-0	60 000	6 -4 -150	
		80	12½	11- 3	4- 6	3-8½	4- 0½	5- 0	6-0	64 000		
	100	40	16½	13- 2¼	4- 1	2-0	SPECIAL	5- 0	6-0	83 000	6 -4 -150	4- 7
		60	16½	13- 2¼	4- 1	2-0	"	5- 0	6-0	86 000		
		80	16½	13- 2¼	4- 1	2-0	"	5- 0	6-0	89 000		
	150	40	17	15- 6⅛	6- 0	3-2	"	6- 0	6-0	130 000	6 -4 -150	4- 7
		60	17	15- 6⅛	6- 0	3-2	"	6- 0	6-0	134 000		
		80	17	15- 6⅛	6- 0	3-2	"	6- 0	6-0	139 000		

TABLE XV
Typical Electric Cranes

Capacity (Tons)	Span (Ft.)	Wheel Base F	Wheel Load P	$A+2$ in.	B	Weight of Runway Rail	
						For Plate-Girders	I-Beams
5	40	8 ft. 6 in.	12 000	10 in.	7 ft.	40 lbs. per yd.	40 lbs.
	60	9 " 0 "	13 000	10 "	7 "	40 "	40 "
10	40	9 " 0 "	19 000	10 "	7 "	45 "	40 "
	60	9 " 6 "	21 000	10 "	7 "	45 "	40 "
15	40	9 " 6 "	26 000	10 "	7 "	50 "	50 "
	60	10 " 0 "	29 000	10 "	7 "	50 "	50 "
20	40	10 " 0 "	33 000	12 "	8 "	55 "	50 "
	60	10 " 6 "	36 000	12 "	8 "	55 "	50 "
25	40	10 " 0 "	40 000	12 "	8 "	60 "	50 "
	60	10 " 6 "	44 000	12 "	8 "	60 "	50 "
30	40	10 " 6 "	48 000	12 "	8 "	70 "	60 "
	60	11 " 0 "	52 000	12 "	8 "	70 "	60 "
40	40	11 " 0 "	64 000	14 "	9 "	80 "	60 "
	60	12 " 0 "	70 000	14 "	9 "	80 "	60 "
50	40	11 " 0 "	72 000	14 "	9 "	100 "	60 "
	60	12 " 0 "	80 000	14 "	9 "	100 "	60 "

obtain the maximum shear at any section, as a–a, the load should be placed as shown in Fig. 115; and the maximum shear will then be the left reaction, which is $R = 2P\ (x + \frac{F}{2}) \div l$, for two wheels; and $R = 4\,P\ (x + F + \frac{G}{2}) \div l$, for four wheels.

The values of P for traveling cranes of various capacities and spans may be obtained upon writing to the various crane manufacturing companies, whose addresses will be found in the advertising sections of the engineering periodicals. The distances between wheels may be obtained from their catalogues, which may be had upon application. The values of P, and the distances between wheels for cranes of various spans and capacities, are given in Table XIV, which is made from information furnished through the courtesy of Pawling & Harnischfeger, Milwaukee, Wisconsin.

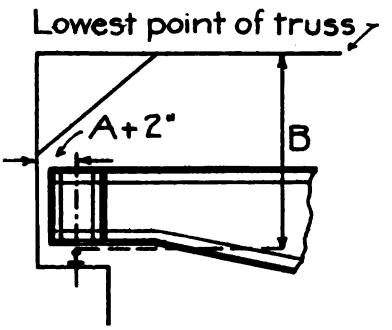


Fig. 116. Showing Notation used in Table XV.

The values in Table XV are taken from the "Transactions" of

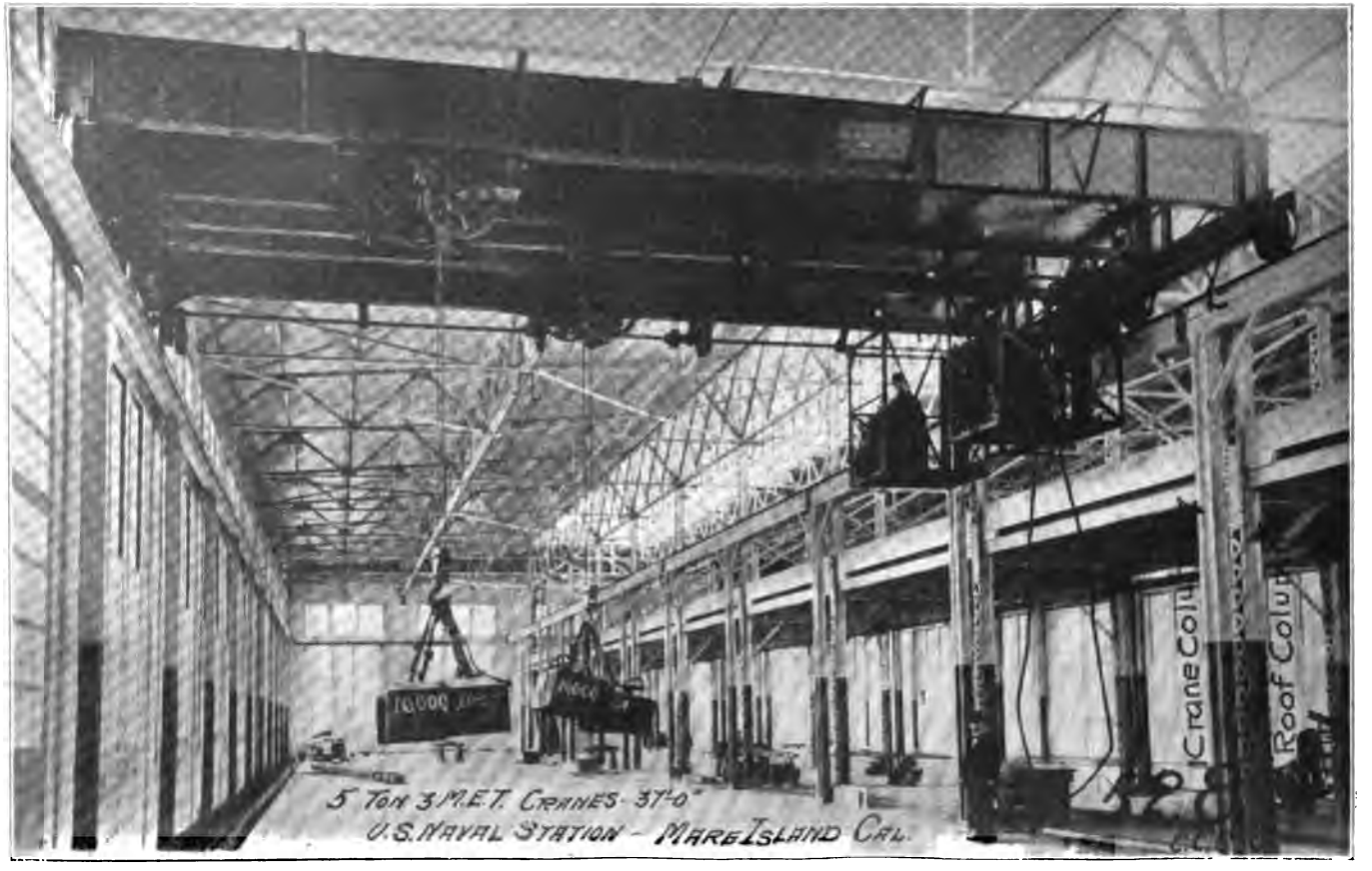


Fig. 117. Five-Ton Electric Traveling Cranes Installed at U. S. Naval Station, Mare Island, California.
Courtesy of Cleveland Crane & Car Company, Cleveland, Ohio.

the American Society of Civil Engineers, Vol. 54, p. 400, 1905. They are for typical traveling electric cranes, and are proposed by Mr. C. C. Schneider, one of the most distinguished of structural engineers.

The side clearance B from the center of the rail, and the vertical clearance of the beam from the top of the rail, are given in this table (see Fig. 116). These values for the cranes of different manufacturers may be obtained from their catalogues; and they must be known, in order that the crane shall not interfere with the columns or the roof trusses.

If the runway girder is composed of an I-beam, a channel is usually riveted to its top; and on this the rail on which the crane wheels move is fastened down at intervals (see Fig. 107) of about $2\frac{1}{2}$ or 3 feet. Figs. 106 and 117 show details of this kind of girder. Note that the rails are U-shaped (see Fig. 105). This rail is used extensively, although in many cases the common T-rail is used and is fastened down by means of clamps around the edge of the flange of the girder (see Fig. 110).

In case plate-girders are necessary for runway girders, they must be designed. The depth of these girders should be $\frac{1}{10}$ to $\frac{1}{6}$ of the distance between trusses or columns—that is, $\frac{1}{10}$ of their span; The depths must be in the even inch. For example, if the trusses were 16 feet apart, the depth of the girder would be $16 \div 10 = 1.6$ feet, which is equal to 19.2 inches. The depth of the girder must then be made 20 inches, since, if it were made 19 inches, it would be difficult to obtain a web plate 19 inches wide, for the mills do not as a rule have plates of odd-inch widths in stock.

The thickness of the web plate is given by the formula:

$$t = \frac{V_0}{S_s\, d};$$

but in no case shall it be less than $\frac{5}{16}$ inch. In this formula, V_0 is the maximum end reaction of the runway girder. It is equal to R as given by the formula on p. 87, when x is equal to $l - F$, and d is the depth of the girder, which is equal to the depth of the web plate, and S_s is the unit allowable shearing stress.

The flanges are composed of two angles, placed with the long legs horizontal in case unequal-legged angles are used. The required net area of *one flange* is given by the formula:

$$A = \frac{M_m}{S_t\,(d-2)},$$

in which M_m is the moment obtained when the wheels are in the position shown in Fig. 113, S_t is the unit allowable tensile stress, and d is the width of the web plate. If the area A has been computed, two angles must be found from the tables in the Carnegie Handbook, such that when one $\frac{7}{8}$-inch or $\frac{3}{4}$-inch rivet-hole, as the case may be, is taken out, each angle will give a net area equal to or slightly in excess of the area A.

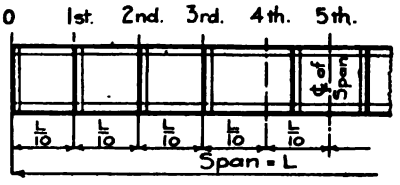


Fig. 118. Position of Tenth-Points.

These flange angles must be riveted to the web by rivets placed a certain distance apart. For convenience of manufacture, the girder is divided into ten equal parts, and the rivet spacing between any two of these divisions—or *tenth-points*, as they are called—is kept the same. These tenth-points are numbered (see Fig. 118). The rivet spacing in the first division is the same as that computed for the end of the girder, which is the zero tenth-point; the rivet spacing in the second division is the same as that computed for the 1st tenth-point; and so on. The rivet spacing at any point is given by the formula:

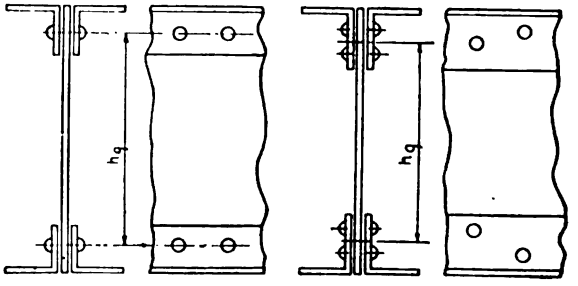


Fig. 119. Determination of Distance between Gauge Lines.

$$S = \frac{v}{\sqrt{\left(\frac{V_x}{h_g}\right)^2 + \left(\frac{P}{30}\right)^2}},$$

in which,

V_x = Maximum shear at the point;
v = Maximum allowable stress on one rivet; this will be the bearing value of the rivet in the web plate (see Table X, p. 47);
P = Maximum reaction of one crane wheel (see Table XIV or XV);
h_g = Distance between gauge lines of the angles.

In case there are two gauge lines on the angle, then the distance h_g is the distance between centers of these gauge lines (see Fig. 119).

Table IX, p. 46, gives the gauge lines for different lengths of angle legs. If S gives a value less than $2\frac{5}{8}$ inches, the leg of the angle against the web must be 5 inches or more, on account of practical limitations of manufacture.

26. *Examples.* In order to illustrate the preceding methods, two problems will be worked out.

1. Design a runway girder for a 5-ton crane of 40-foot span, the wheel loads and wheel base being as given in Table XV, p. 87, and the distance between trusses 20 feet.

In order to produce the maximum moment, the wheel must be placed as shown in Fig. 120. The left reaction is 12 000 (2.125 + 10.00 + 3.625) ÷ 20 = 9 450. The moment under wheel 1 is 9 450 × 7.875 × 12 = 894 000 pound-inches, which requires a section modulus of 894 000 ÷ 15 000 = 59.60. Looking in the Car-

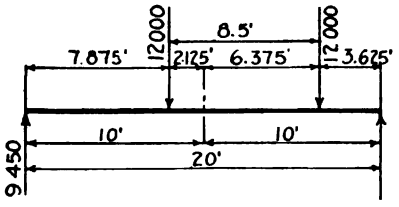


Fig. 120. Position for Maximum Moment for Problem 1 on Page 91.

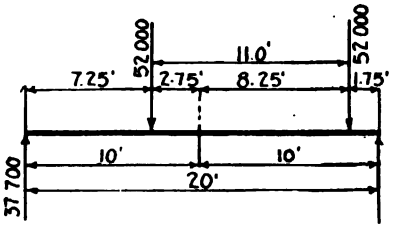


Fig. 121. Position for Maximum Moment for Problem 2 on Page 91.

negie Handbook, pp. 97 and 98, it is seen that a 15-inch 42-pound I-beam with a section modulus of 58.9 will be sufficient, since the section modulus is less than $2\frac{1}{2}$ per cent under that required.

2. Design a runway girder for a 30-ton crane of 60-foot span, the wheel loads and wheel base being as given in Table XV, and the distance between trusses 20 feet.

The wheels are placed in position as shown in Fig. 121. The left reaction is $\frac{52\,000\,(12.75 + 1.75)}{20} = 37\,700$ pounds; and the maximum moment, which occurs under wheel 1, is 37 700 × 7.25 × 12 = 3 285 000 pound-inches. The maximum shear occurs when the wheels are in position as shown in Fig. 112, p. 85, and is 75 400 pounds. The required thickness of the web is $\frac{75\,400}{10\,000 \times 24} =$ 0.314 inch, the depth being 20 ÷ 10 = 2 feet = 24 inches. The web will be made 24 inches wide and $\frac{3}{8}$ inch thick.

The required net flange area is $\frac{3\,285\,000}{15\,000 \times (24 - 2)} = 9.97$ square inches for two angles, or 4.99 square inches for one angle. An angle 6 by 6 by $\frac{1}{2}$-inch gives a gross area of 5.75 square inches and a net area of $5.75 - 0.50 = 5.25$ square inches, one $\frac{7}{8}$-inch rivet-hole being taken out of the section. Since this area coincides quite closely with the required area and is larger, it will be used. A 6 by $3\frac{1}{2}$ by $\frac{3}{8}$-inch angle would have been better in regard to area, but the rivet spacing is less than $2\frac{5}{8}$ inches at the end, and this required a double gauge line and therefore a leg 5 inches or over.

The maximum shears at the tenth-points are now computed, and are tabulated as follows:

$V_0 = 75\,400$ pounds.
$V_1 = 65\,000$ "
$V_2 = 54\,600$ "
$V_3 = 44\,200$ "
$V_4 = 33\,800$ "
$V_5 = 26\,000$ "

The value of the shear to be used in any particular case is given

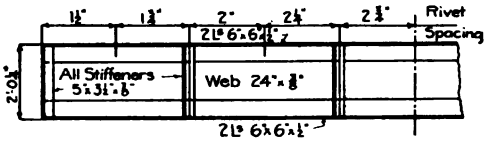


Fig. 122. Stress Sheet of Runway Girder of Problem 2 on Page 91.

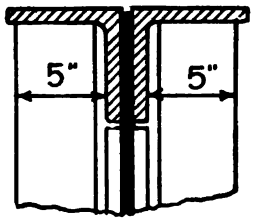


Fig. 123. Determination of Size of Stiffener.

above. In this case, $P = 52\,000$ pounds; $v = 6\,570$ pounds, $\frac{7}{8}$-inch rivets being used; and h_s is $24\frac{1}{4} - (2 \times 2\frac{1}{4} + \frac{2\frac{1}{2}}{2}) = 18\frac{1}{2}$ inches. The rivet spacing for the first division or first two feet of the span is:

$$S = \frac{6\,570}{\sqrt{\left(\frac{75\,400}{18\frac{1}{2}}\right)^2 + \left(\frac{52\,000}{30}\right)^2}} = 1.495, \text{ say } 1\tfrac{1}{2} \text{ inches.}$$

The rivet spacing for the other divisions may be computed by the student. It is given in Fig. 122. The web of the girder should be stiffened as shown in the figure, by angles placed as there indicated. The thickness of the angles should not be less than $\frac{5}{16}$ inch, nor greater than $\frac{1}{2}$ inch. The size of the angles should be such that the

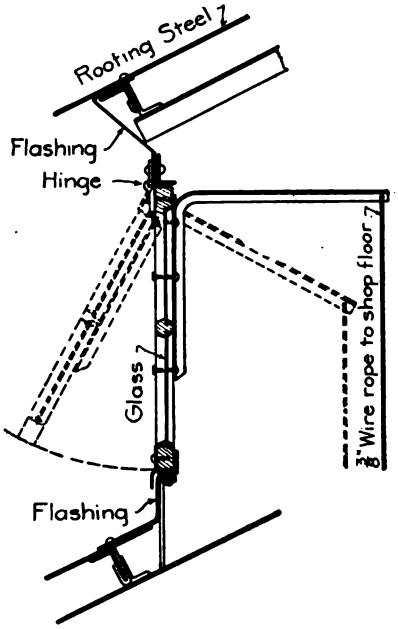


Fig. 124. Section of Glass Louvres in Monitor.

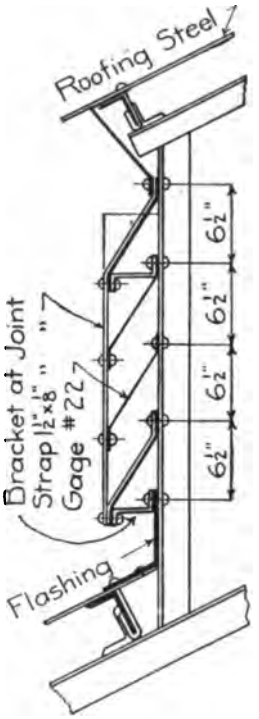


Fig. 125. Section of Metal Louvres in Monitor.

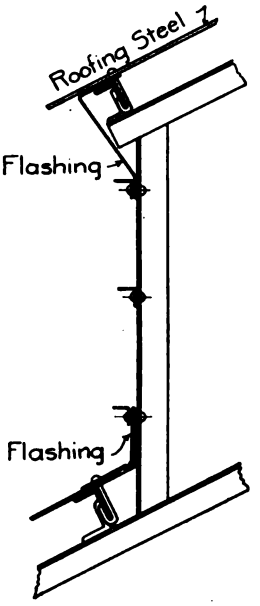


Fig. 126. Section of Open Monitor.

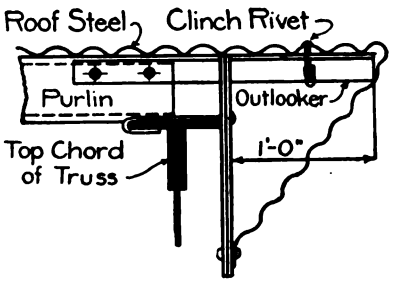


Fig. 127. Gable Details for Corrugated Steel.

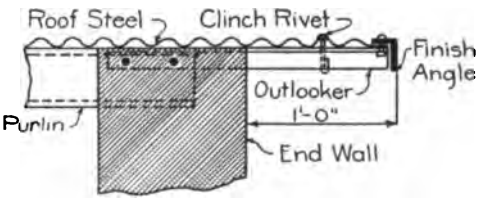


Fig. 128. Gable Details for Corrugated Steel.

outstanding leg does not reach beyond the leg of the flange angle (see Fig. 123). This makes their size as shown in Fig. 122. The crane rail may be connected directly to these and the flange angles; or a channel may be placed over the flange angles and riveted to them in a manner similar to that employed in the case of I-beams, the crane rail being fastened to that. If this latter detail is employed, the area of the channel is reckoned as forming part of the upper flange; and the net area of the angle must then be equal to the required net area, less the net area of the channel.

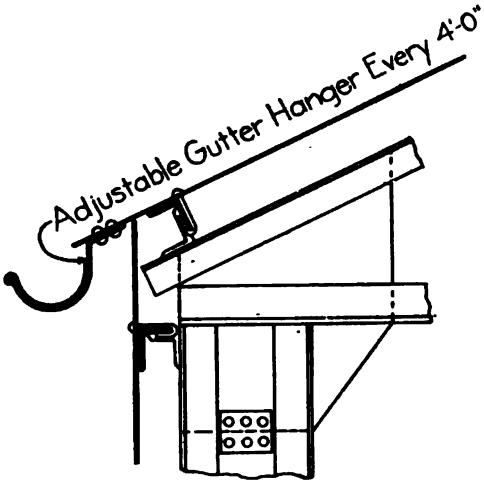


Fig. 129. Cornice Details for Steel Roof. See also Figs. 130 and 131.

27. **Ventilators.** Mill buildings may be ventilated by means of small circular ventilators such as shown in Fig. 69, p. 41, placed at certain intervals along the ridge or peak of the roof, or by means of monitors as shown in Fig. 71. The sides of these monitors may be fitted with swinging glass windows, with wooden or metal louvres, or, in case a large amount of ventilation is required, may be simply left open. Figs. 124, 125, and 126 give details of monitors, and show how they are connected to the trusses.

28. **Gable Details.** The gable is the end of the roof at that end of the building which is parallel to the roof trusses. Since this extends beyond the plane of the side of the building, some method must be employed in connecting the outer edge with the wall of the building, in order to keep out the rain and wind. Figs. 127 and 128 give several details which are efficient and at the same time economical.

29. **Cornice Details.** The cornice is that edge of the roof which is perpendicular to the planes of the roof trusses. In addition to being necessarily so constructed as to keep out the wind and the elements, it must have in many cases some form of gutter connected to it, which takes the water off the roof. This gutter should be con-

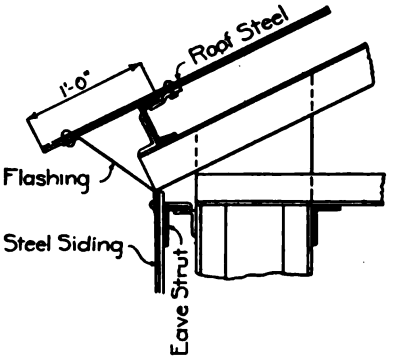


Fig. 130. Cornice Details for Steel Roof. See also Figs. 129 and 131.

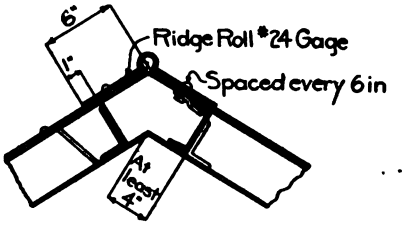


Fig. 133. Ridge Roll.

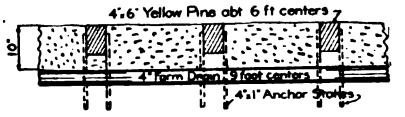


Fig. 134. Detail of Cinder Floor.

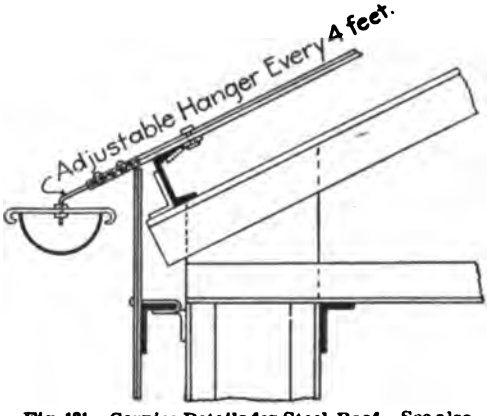


Fig. 131. Cornice Details for Steel Roof. See also Figs. 129 and 130.

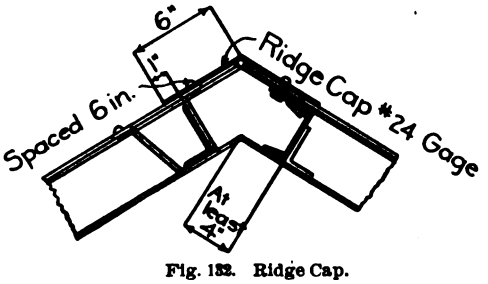


Fig. 132. Ridge Cap.

nected at intervals of every three bays—or, in case this exceeds 50 feet, every two bays—with a pipe or conductor to carry the water to the ground. Gutters, as a general thing, are semicircular or nearly so; and for ordinary spans they should not be less than 6 inches wide. Conductors should not be less than 5 inches in diameter. It is not to be supposed that the water entirely fills either the conductors or the gutters. The sizes are made so as to allow for any obstruction such as dirt or ice. Gutters should preferably have a pitch of one inch in every 10 feet. Figs. 129 to 131 give details of cornices with various forms of gutters attached.

The ridge, or peak of the roof, is usually covered with a plain sheet of metal, in which case it is called the *ridge cap*; or with a

metallic roll with flared sides, in which case it is called a *ridge roll.* Figs. 132 and 133 show cross-sections of a ridge cap and a ridge roll.

30. **Floors.** The floor of the shop depends very largely upon the purpose for which the building is intended. It may consist of earth,

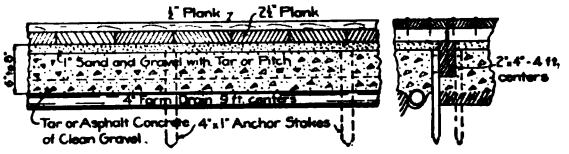


Fig. 135. Detail of Asphalt or Coal-Tar Concrete Floor.

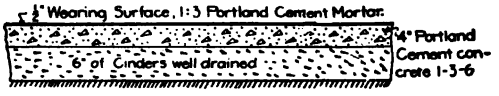


Fig. 136. Detail of Concrete Floor.

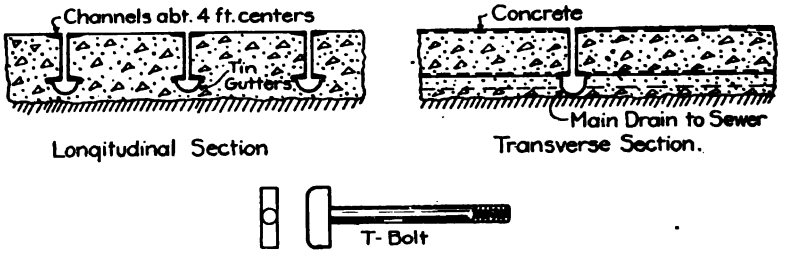


Fig. 137. Detail of Floor of Steam Laboratory of the University of Illinois.

cinders, boards, concrete, or sheet steel. In cases where men are required to work standing, cinders or boards give the best results. Earth floors will wear into holes in places where the men stand, and concrete or steel makes them foot-weary on account of its inelasticity. Where heavy machinery is installed, and men are seldom present except for a short time at certain periods, concrete makes an ideal floor. Figs. 134, 135, and 136 show details of various kinds of floors. Fig. 137 gives a detail of the floor in the Steam Engineering Laboratory of the University of Illinois. This consists of channels imbedded in concrete. These channels, which are placed in pairs a small distance apart, run both lengthwise and crosswise of the shop. The advantage of this form of construction is that machinery can be placed anywhere on the shop floor and quickly bolted into place by means of T-bolts, a detail of which is shown in the figure.

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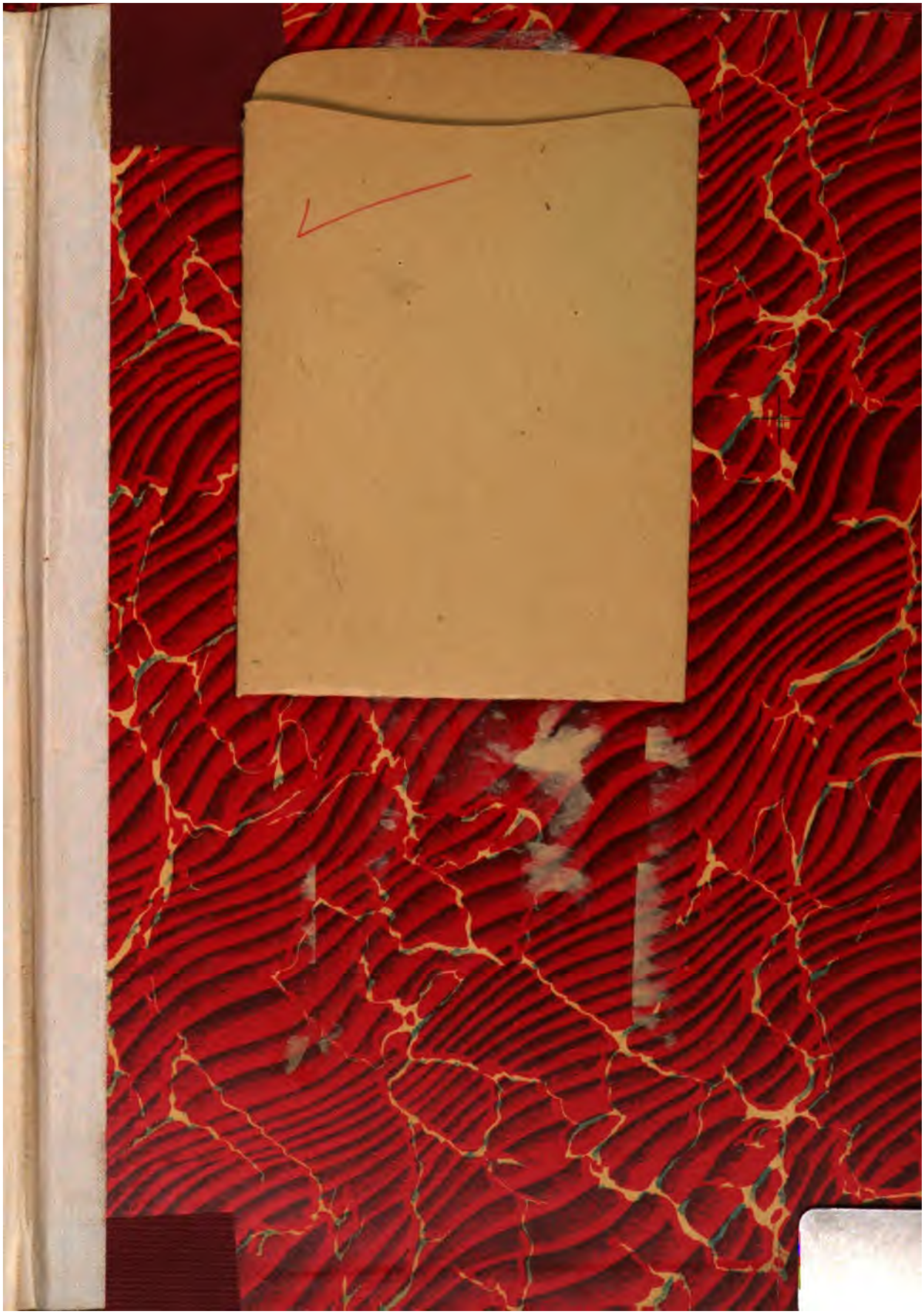
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